MATH 1 HOLIDAY REVISION SET 2

1. Solve the simultaneous equations using row reduction to echelon form.

   $x+3y-3z=-4, 3x-y+2z=1, -2x+y+z=7$

2. If α and β are roots of the equation $x^2 + px + 45 = 0$ and $(\alpha - \beta)^2 = 144$, find the value of p. Hence solve the equation.
3. Solve for θ: $4sin2x - 3cos2x = 3$ for $0^0 \leq X \leq 360^0$
4. Find the equation of the circle which passes through A (5,0), B (5,2) and C (-3,-4).
5. If $y = mx - 5$ is a tangent to the circle $x^2 + y^2 = 9$, find the possible values of m, hence find the equations of the tangents.
6. Find the equation of a normal at (2, 1) to the curve $y^2 + 3xy = 2x^2 - 1$.
7. If $x^2 + 2xy + 3y^2 = 1$ prove that $(x + 3y)^3 \frac{d^2y}{dx^2} + 2 = 0$.
8. $If\ (x+1)^2\ is\ factor\ of\ 2x^4 + 7x^3 + 6x^2 + Ax + b$, find the value of A and b.
9. Find the values of λ for which $10x^2 + 4x + 1 = 2\lambda x(2 - x)$ has equal roots.
10. If $y = a^{log_a x}$ prove that $y = x$. Hence without using tables or calculators, evaluate $(5^{log_{10} 4})(50^{log_{10} 25})$.
11. Given that $cos\ 2A - cos\ 2B = -p$ and $sin\ 2A - sin\ 2B = q$, prove that $sec\ (A + B) = \frac{1}{q}\sqrt{p^2 + q^2}$
12. A right pyramid having a square base is inscribed in a sphere of radius R, all five vertices of the pyramid lying on the sphere. The height of the pyramid is x; show that the four vertices forming the base of the pyramid lie on a circle of radius r, where $r^2 = 2Rx - x^2$. Hence or otherwise, show that the volume V of the pyramid is given by the formula $V = \frac{2}{3}x^2(2R - x)$. If R is fixed but x may vary. Find the greatest possible value of V.

"Mathematics, in its way, reveals patterns and truths unseen by ordinary thinking — keep learning, keep exploring."