UGANDA ADVANCED CERTIFICATE OF EDUCATION
PRINCIPAL MATHEMATICS
Paper 1
Time: 3 Hours

INSTRUCTIONS TO CANDIDATES
1. Answer ANY FIVE questions.
2. All questions carry equal marks.
3. Show all necessary working clearly.
4. Non-programmable calculators may be used.
5. Credit will be given for clear reasoning and correct mathematical presentation.

SECTION A

Item 1
(Algebra: Equations and Inequalities)
A cooperative society plans to distribute relief food to families affected by floods. Each family receives a combination of maize flour and beans. The total mass of food given to a family must not exceed 25 kg due to transport limitations.

The mass of maize flour per family is x kg, while the mass of beans is $(2x - 3)$ kg.

Task
(a) Form an inequality representing the condition that the total mass of food does not exceed the allowed limit.
(b) Solve the inequality and interpret your solution.

Due to shortages, the cost per kilogram of maize flour is 2000 shillings, while beans cost 3000 shillings per kilogram.

(c) Write an expression for the total cost of food per family in terms of x.
(d) Find the value of x that minimizes the total cost, given the constraint in part (b).

Item 2
(Trigonometry: Identities, Equations and Proofs)
A surveyor is mapping a triangular piece of land. Two sides of the triangle meet at a point where an observation tower is constructed. The angles between the sides vary depending on wind direction and observation time.

The angles A and B of the triangle satisfy the condition that sin A = 2 sin B, where A + B + C = 180 degrees and angle C is fixed.

Task
(a) Show that tan A = 2 tan B, stating clearly any identities used.
(b) Given that angle B is acute, find the possible values of angle B.

The surveyor later measures the height of the tower using angles of elevation from two points on level ground.

(c) Derive a formula for the height of the tower in terms of the measured angles and distance between the observation points.
(d) State one assumption made in your calculations.

Item 3
(Coordinate Geometry I)
A town council plans to construct a straight road connecting two trading centres represented on a coordinate grid. The first centre is at point A (2, −1) and the second at point B (8, 5).

Task
(a) Find the equation of the straight line joining A and B.
(b) Determine the midpoint of the road segment AB.
A hospital is to be built at a point P on the line AB such that AP : PB = 1 : 2.
(c) Find the coordinates of point P.
A second road passes through point A and is perpendicular to AB.
(d) Find the equation of this second road.

Item 4
(Differentiation II: Applications and Curve Sketching)
The profit P (in thousands of shillings) made by a small manufacturing company depends on the number of units x produced per day and is given by

$P = 2x^3 - 15x^2 + 36x - 10.$

Task
(a) Find the first derivative of P with respect to x.
(b) Determine the stationary points of the curve.

The company manager wants to know whether these points represent maximum or minimum profit.

(c) Use the second derivative test to classify each stationary point.
(d) Hence state the number of units that should be produced per day to maximize profit.

Item 5
(Integration I and II)
A vehicle moves along a straight road and its velocity v (in metres per second) at time t seconds is given by

$v = 6t^2 - 4t$.

Task
(a) Find an expression for the displacement of the vehicle at time t.

Given that the vehicle passes through the origin at t = 0,

(b) Find the displacement of the vehicle after 3 seconds.

Later, the engine performance changes and the velocity varies irregularly. The following table shows velocity readings at equal time intervals.

Time t (s):
0, 2, 4, 6, 8

Velocity v (m s-1):
0, 10, 18, 20, 16

(c) Use the trapezium rule to estimate the distance travelled in the first 8 seconds.
(d) State one reason why this method gives only an approximate value.

Item 6
(Series)
A student saves money weekly to buy a laptop. In the first week, she saves 5000 shillings. Each week thereafter, the amount saved increases by 1500 shillings.

Task
(a) Write down the first four terms of the sequence.
(b) Find an expression for the nth term of the sequence.

After several weeks, the student decides to calculate the total amount saved.

(c) Derive a formula for the sum of the first n terms of the sequence.
(d) Find the minimum number of weeks required for the student to save at least 500000 shillings.

Item 7
(Probability II)
A box contains 6 red balls, 4 blue balls, and 5 green balls. Two balls are selected at random, one after the other, without replacement.

Task
(a) Find the probability that both balls selected are red.
(b) Find the probability that the two balls are of different colours.

The experiment is repeated many times.

(c) Explain what is meant by the term expected value in this context.
(d) State one practical application of probability in decision-making.

Item 8
(Random Variables and Distributions)
The random variable X represents the number of defective items found in a batch of four items selected at random from a large production process. The probability distribution of X is given as follows:

X: 0, 1, 2, 3, 4
P(X): k, 4k, 6k, 4k, k

(a) Find the value of k.
(b) Calculate the mean value of X.

The manufacturer uses the mean value to assess quality control.

(c) Explain why the variance is also important in quality assessment.
(d) State one limitation of using probability distributions to model real-life situations.

For inquiries, contact me on: joelamanyire3@gmail.com or 0788477520
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UGANDA ADVANCED CERTIFICATE OF EDUCATION
PRINCIPAL MATHEMATICS
MARKING GUIDE / SOLUTIONS

QUESTION 1: ALGEBRA (EQUATIONS AND INEQUALITIES)
(a)
Mass of maize flour = x kg
Mass of beans = (2x - 3) kg

Total mass = x + (2x - 3)
Total mass = 3x - 3

Condition:
$3x - 3 \leq 25$

(b) Solving the inequality

$3x - 3 \leq 25$
$3x \leq 28$
$x \leq 28 / 3$

$x \leq 9\ 1/3$

Interpretation:
The mass of maize flour per family must not exceed 9 1/3 kg.

(c) Cost expression

Cost of maize = 2000 sh per kg
Cost of beans = 3000 sh per kg

Total cost
$= 2000x + 3000(2x - 3)$
$= 2000x + 6000x - 9000$
$= 8000x - 9000$

(d) Minimizing the cost

The cost function increases as x increases.
Therefore, the minimum cost occurs at the smallest allowable value of x that satisfies the inequality.

QUESTION 2: TRIGONOMETRY

(a)
Given: sin A = 2 sin B

Divide both sides by cos A cos B

sin A / cos A = 2 sin B / cos B

tan A = 2 tan B

Hence proved.

(b)
Since B is acute and sin A = 2 sin B,
a possible value is B = 30 degrees.

(c)
Let distance between observation points = d
Let angles of elevation = alpha and beta

Height h = (d tan alpha tan beta) / (tan alpha - tan beta)

(d)
Assumption:
The ground is level and the angles are measured accurately.

QUESTION 3: COORDINATE GEOMETRY
Given points:
A (2, -1)
B (8, 5)

(a) Equation of line AB

Gradient m

= (5 - (-1)) / (8 - 2)
= 6 / 6
= 1

Equation:
y - (-1) = 1(x - 2)
y + 1 = x - 2
y = x - 3

(b) Midpoint of AB

x-coordinate = (2 + 8) / 2 = 5
y-coordinate = (-1 + 5) / 2 = 2

Midpoint = (5, 2)

(c) Point P dividing AB in ratio 1 : 2

x-coordinate
= (1 × 8 + 2 × 2) / 3
= 12 / 3
= 4

y-coordinate
= (1 × 5 + 2 × (-1)) / 3
= 3 / 3
= 1

P = (4, 1)

(d) Equation of perpendicular line through A

Gradient of AB = 1
Gradient of perpendicular line = -1

Equation:
y - (-1) = -1(x - 2)
y + 1 = -x + 2
y = -x + 1

QUESTION 4: DIFFERENTIATION II
Given:
P = 2x^3 - 15x^2 + 36x - 10

(a) First derivative

dP/dx = 6x^2 - 30x + 36

(b) Stationary points

6x^2 - 30x + 36 = 0

Divide by 6

x^2 - 5x + 6 = 0

(x - 2)(x - 3) = 0

x = 2 or x = 3

(c) Second derivative

d2P/dx2 = 12x - 30

At x = 2
12(2) - 30 = -6
Maximum point

At x = 3
12(3) - 30 = 6
Minimum point

(d)
Maximum profit occurs when x = 2 units.

QUESTION 5: INTEGRATION AND TRAPEZIUM RULE
(a)
v = 6t^2 - 4t

Displacement s

= integral of v dt
= 2t^3 - 2t^2 + C

(b)
Given s = 0 when t = 0
C = 0

At t = 3

s = 2(3^3) - 2(3^2)
s = 54 - 18
s = 36 m

(c) Trapezium rule

Step size h = 2

Distance
= h/2 [v0 + v4 + 2(v1 + v2 + v3)]

= 2/2 [0 + 16 + 2(10 + 18 + 20)]
= 1 [16 + 96]
= 112 m

(d)
Velocity does not change linearly between intervals.

QUESTION 6: SERIES
(a) First four terms

5000, 6500, 8000, 9500

(b) nth term

a = 5000
d = 1500

Tn = 5000 + (n - 1)1500

(c) Sum of first n terms

$$Sn = n/2 [2a + (n - 1)d]$$

$$Sn = n/2 [10000 + (n - 1)1500]$$

(d)
Minimum n such that Sn ≥ 500000

n = 24 weeks

QUESTION 7: PROBABILITY II
Total balls = 15

(a)
P(both red)
= (6/15)(5/14)
= 30/210
= 1/7

(b)
P(different colours)
= 1 - P(same colour)

(c)
Expected value is the long-run average outcome of an experiment.

(d)
Used in decision making and risk analysis.

QUESTION 8: RANDOM VARIABLES
Given:
P(X) = k, 4k, 6k, 4k, k

(a)
k + 4k + 6k + 4k + k = 16k = 1

k = 1/16

(b) Mean

Mean
= 0(k) + 1(4k) + 2(6k) + 3(4k) + 4(k)

= 32k
= 2
(c)
Variance measures spread around the mean.
(d)
Real-life data may not follow exact probability models.

Designed and authored by Teacher JoelPCM.
Physics, Mathematics and Chemistry – UACE New Curriculum Item Banks.
For inquiries, contact me on: joelamanyire3@gmail.com or 0788477510

1. "Stay calm, trust your preparation, and give every question your best."
2. "Every step you have written shows effort — be proud of your work."
3. "Great minds are built through challenge. You have done well."
4. "Exams test knowledge, but perseverance builds success."
5. "Believe in yourself — the future belongs to those who try."