

COMPETENCE BASED ASSESSMENT



UGANDA ADVANCED CERTIFICATE OF EDUCATION

ITEM BANK FOR SENIOR FIVE

COMPOSED BY: **KIN-MATICS**

SURDS

ITEM 1. Imagine being a worker in a company determining workers' payments by multiplying \$2 on the squares of the total number of hours worked per worker. One day, each of the workers Alex, Beckie and Charles were paid \$54, \$96 and \$150 respectively. As you passed by, the finance manager called you to simply make him know the exact number of hours corresponding to the total payments which was given to these workers that day.

Task. With evidence, respond to the finance manager's questions by determining the exact number of hours worked by each employee.

ITEM 2. Alex noted that a wire bent to form a right-angled triangular design of base 4cm and height 5cm cost \$4. He is planning to use a longer wire to create a different design and wishes to determine the cost implications before proceeding. In particular, he wishes to know the exact price of a 10cm wire, assuming cost is directly proportional to the length of the wire used.

Task. Determine the exact price of a 10cm wire, clearly justifying each step in your reasoning.

ITEM 3. Mike is a carpenter planning to inscribe an equilateral triangular design into a circular wooden door. In this design, he wishes to ensure that the diameter of the door is exactly two meters longer than the base of the triangular design. In order to avoid costly construction errors, Mike wants to determine the exact area occupied by the triangular design. However, he is unable to carry out the necessary computations.

Task. Help Mike determine the exact area of the triangular design, expressing your answer in its simplest form and clearly showing all working.

ITEM 4. Paradox: "My father's age is the square of mine and my mother's is the square of my brother's. My father is fifteen years older than my mother. Our sister's age is the sum of both my brother and I." The speaker challenges you to express the sister's age in terms of the mother's age and hence determine the actual age of the father, given that the elder sister is exactly fifteen years old.

Task. Take on the challenge by forming appropriate equations, expressing relationships clearly, and determining the father's age.

ITEM 5. A civil engineer is designing a square platform to be constructed from two different prefabricated square slabs whose areas are 18m^2 and 32m^2 respectively. Due to transportation and structural limitations, the slabs must be merged without cutting, forming a single square platform. For precision in construction, the engineer insists that the side length of the resulting square must be expressed exactly in surd form to eliminate cumulative rounding errors.

Task. Determine the exact side length of the resulting square platform in simplest surd form, and explain why using approximations would lead to structural inconsistencies.

ITEM 6. A precision engineer is calibrating a laser-cutting machine using a rectangular metallic plate whose sides measure $\sqrt{50}$ cm and $\sqrt{72}$ cm. The calibration requires determining the diagonal length exactly, since even minor approximations could distort future automated cuts. However, the engineer is uncertain how to simplify the radical expression obtained.

Task.

Determine the exact length of the diagonal of the plate and express your answer in simplest radical form, clearly showing all steps.

ITEM 7. A farmer is designing a triangular irrigation layout where two sides of the field measure $(3 + \sqrt{5})$ m and $(3 - \sqrt{5})$ m respectively. Due to financial constraints and the need for precise budgeting, the farmer must determine the exact perimeter of the field without using decimal approximations.

Task.

Determine the exact perimeter of the triangular field and simplify your answer fully.

ITEM 8. A metal fabricator produces square frames where each side is given by $\sqrt{11 + 6\sqrt{2}}$ cm. Before proceeding with large-scale production, the fabricator needs to determine the total length of material required per frame in exact form in order to minimize wastage and ensure accurate costing.

Task. Express the perimeter of the square frame in simplest surd form, clearly showing the simplification process.

ITEM 9. During a mathematics seminar, a student claims that the expression $\sqrt{7 + 4\sqrt{3}}$ can be rewritten as the sum of two surds of the form $\sqrt{a} + \sqrt{b}$. The instructor challenges the class to verify this claim algebraically rather than numerically.

Task. Verify whether the expression can be written in the stated form and determine the values of a and b if possible.

◆ INDICES AND LOGARITHMS

ITEM 10. In a certain experiment, it was observed that the mass of a particle increases at the rate of 30% per week. Since the initial mass of the particle is 40 grams, the researchers are puzzled about how to determine the exact time period it will take the particle to weigh a kilogram.

Task. Help the researchers determine the required duration, clearly showing all working.

ITEM 11. A deadly virus has attacked a certain country, and initially ten males and six females were infected. The number of male patients increases at a rate of 20% per week, while the number of female patients increases at a rate of 44% per week. Analysts wish to determine how long it will take for the total number of infected individuals to reach one thousand and also determine the percentage of males at that point.

Task.

- Determine the time required for the total number of infected individuals to reach 1000.
- Determine the percentage of males among the infected population at that time.

ITEM 12. In two different years, Mugagga started a shoe-making business and a wholesale business with initial capital of UGX 10 million and UGX 20 million respectively. Both businesses grow at an average rate of 20% per annum. Currently, the shoe business has UGX 14 million more capital than the wholesale business, and together they have been operating for 35 years.

Task.

- Determine the year Mugagga started the shoe-making business.
- Determine the expected capital of the wholesale business in 2027.

ITEM 13. A microbiologist observes that a culture of bacteria doubles every 3 hours. Initially, there are 500 bacteria. Due to safety concerns, the experiment must be stopped once the population exceeds one million.

Task. Determine the exact time required for the population to exceed one million using logarithmic methods.

ITEM 14. A machine purchased at UGX 8,000,000 depreciates at a rate of 15% per year. The company wishes to determine how long it will take for the value of the machine to reduce to UGX 2,000,000.

Task. Determine the required time, showing all steps clearly.

ITEM 15. An investment grows continuously and triples in value after 5 years. The investor wishes to determine the constant rate of growth.

Task. Determine the growth rate of the investment.

ITEM 16. An audio system produces sound intensity given by $L = 10 \log_{10}(I/I_0)$. During testing, the intensity increases by a factor of 100.

Task. Determine the increase in decibel level.

ITEM 17. A radioactive substance decays such that only 25% of its original mass remains after 8 years.

Task. Determine the half-life of the substance.

QUADRATICS

ITEM 18. Lyto enterprises sell its commodities in large containers. In every container sold, the company earns \$40 million and also receives donations worth \$5 million per year. Since the business is entitled to a total expense which is \$10 million times the square of the number of containers sold throughout the year, the business manager seeks knowledge about the total number of containers to sell so as to gain the maximum profit each year.

Task. Guide the manager on how to make maximum profit every year.

ITEM 19. A company sells quantity (x) of a commodity at $\$(3x - 18)$ million while spending \$21 million on production through the year. The company wishes to know how much must be sold to avoid losses and also determine the biggest loss that can ever occur.

Task.

- Determine the minimum production level to avoid losses
- Determine the maximum possible loss and how it occurs

ITEM 20. A farmer has 80 meters of fencing material to enclose a rectangular garden along a river where one side requires no fencing.

Task. Determine the dimensions that maximize the enclosed area.

ITEM 21. A ball is projected vertically upward from ground level such that its height after t -seconds is given by

$$h(t) = 20t - 5t^2$$

A coach analyzing the motion wants to determine not only the maximum height reached but also the exact time at which this occurs, since this information is critical for training optimization and timing of performance drills. However, the coach insists that the result must be obtained analytically rather than graphically.

Task. Determine the maximum height reached by the ball and the exact time at which this occurs, clearly justifying each step.

ITEM 22. A manufacturing company models its annual profit using the function

$$p(x) = -2x^2 + 40x - 150$$

where (x) represents the number of units produced (in thousands). The management seeks to determine the level of production that maximizes profit and to understand the implications of operating above or below this level.

Task.

Determine the production level that yields maximum profit and compute the corresponding maximum profit, explaining the significance of your result.

ITEM 23.

The product of two consecutive integers is known to be 1320. A learner attempts to determine these integers but is required to avoid guesswork and instead construct a mathematical model that reflects the relationship accurately.

Task.

Form an appropriate quadratic equation representing the situation and determine the two integers, showing all working.

ITEM 24.

A rectangular plot of land has an area of 96 m^2 , and its length exceeds its width by 4 meters. A surveyor must determine the exact dimensions of the plot before it is subdivided for development.

Task.

Form a quadratic equation representing the situation and determine the exact dimensions of the plot.

◆ POLYNOMIALS AND EQUATIONS

ITEM 25. Company A pays workers based on a combined system: during the day, workers earn an amount equal to \$3 times the square of the number of hours worked, while during the evening session they earn \$2 times the number of hours worked. Alex works the same number of hours in both sessions.

Company B uses a different system. Peter earns \$8 plus the square of Alex's total daily earnings, while Tom earns nine times Alex's total daily earnings. It is observed that Peter and Tom earn exactly the same amount per day.

Task. Form suitable equations and determine:

- The number of hours Alex works per day
- How much more money Tom earns compared to Alex

ITEM 26. On a back-to-school shopping day, a shop sells combinations of books, pens, and pencils at different prices. Specifically:

- 5 books, 7 pens, and 3 pencils cost UGX 37,000
- 3 books, 12 pens, and 4 pencils cost UGX 45,000
- 7 books, 2 pens, and 12 pencils cost UGX 44,000

Alice purchases 6 books, 7 pens, and 9 pencils and pays UGX 50,000.

Task. Form a system of equations and determine the exact change Alice should receive.

ITEM 27. For the past three times, your father managed to purchase the following items with exactly Ugx.50,000/=

- 5kg of sugar, 3kg of posho and 8kg of rice
- 4kg of sugar, 8kg of posho and 6kg of rice
- 7kg of sugar, 5kg of posho and 6kg of rice

Your father wanted to buy 9kg of sugar, 4kg of rice and 3kg of rice but he is not sure whether he will be granted all these items at the same amount of money.

Task. Help your father to know whether he can buy the required items at Ugx.50,000/=

ITEM 28. A production system is modeled by a cubic polynomial whose output becomes zero at three critical operating levels due to mechanical shutdown conditions. Engineers identify these shutdown points as occurring at $x = 1, 2, 3$

Task. Construct a cubic polynomial representing this system and explain the significance of each root in the context of production.

ITEM 29. A student is required to construct a polynomial equation whose roots are 2, 3, and 5. Beyond forming the equation, the student must demonstrate how the structure of the polynomial arises from its roots.

Task. Construct the polynomial and clearly explain the relationship between its roots and coefficients.

ITEM 30. The perimeter of two square shapes is 40m, and the difference between their total areas is $52m^2$. A student is required to translate this relationship into algebraic form and determine the numbers.

Task. Form appropriate equations and determine the two numbers, clearly explaining your reasoning.

ITEM 31. A factory's production levels satisfy the equation

$$x^3 - 17x^2 + 80x - 100 = 0$$

Management wishes to determine all feasible production levels that satisfy this condition and interpret their practical meaning.

Task. Solve the equation completely and interpret each solution in context.

◆ PARTIAL FRACTIONS

ITEM 32. A box contains 4 mangoes and n -apples. Two fruits are selected at random without replacement. A student claims that the probability of selecting two apples can be expressed as

$$1 - \frac{20}{n+4} + \frac{12}{n+3}$$

Task. Verify this claim by deriving the probability expression and simplifying it using appropriate algebraic techniques.

ITEM33. A probability analyst encounters the following rational expressions

$$f(x) = \frac{2x^2 - 4}{x^2(x+2)}, \quad g(x) = \frac{x^5 - x^3}{x^3 + 1} \text{ and } h(x) = \frac{x^2 + 14}{(x+2)(x^2 + 2)}$$

arising from a sequential selection process. The expressions were all complex and obscures the interpretation of individual probability components. The analyst suspects that decomposing it into partial fractions will reveal meaningful structure.

Task.

Decompose the given rational expressions into partial fractions to ease interpretation

INEQUALITIES

ITEM 34. A student is enrolled in a course where performance is evaluated based on five equally weighted tests. The institution requires that a student must maintain an overall average score of at least 70% to qualify for certification. After completing four tests, the student has obtained scores that yield an average of 65%.

Task.

Formulate an inequality representing the situation and determine the minimum score required in the final test

ITEM35. Alata and Banabba are investment groups. The total membership of Alata is six people more than Banabba. One day, the government gave UGX.1million to each member of Alata. Banabba received a total of UGX.24 million which was equally distributed among all group members. However, it was observed that the total amount of money received by Alata plus that of one individual from Banabba was less than UGX.20million, hence the auditors wish to know the minimum and maximum number of people from both groups who benefited in this arrangement.

Task.

Help the auditors in determining the range of people who benefited in this arrangement.

ITEM 36. In certain community, the number children, men and women are in millions. Usually, the total number of children is the square of women and men are fifteen million less than the women. Since the total number men and women is usually one-seventeenth of the population, the inspectors wish to determine the possible number of men, women and children in the community.

Task.

Help the inspectors to determine the possible number of men, women and children in the community.

ITEM 37.

The Red cross organization is planning to donate some funds to country A, B and C to help those facing political instabilities. The targeted people in each country are in millions with country A having the largest population, country B has a population four million less than A while country C has 10million people. In the recent post forwarded online via social media, it was clear that country A, B and C was to receive 60%, 30% and 10% of the donations respectively, while ensuring that funds received by country A do not exceed those received by both country B and C. One man asked a question through the comment section regarding the range of people in each country who may benefit in this arrangement. However, no one has responded to him since then.

Task. Give a detailed response to the man's question on social media.

ITEM 38.

A manufacturing firm produces goods on a daily basis, where production must lie between 100 and 300 units due to contractual obligations. Each unit requires 2 hours of labor, but the total labor available per day is constrained between 300 and 500 hours due to workforce scheduling and shift limitations.

Furthermore, operational efficiency policies require that production must not fall below half of the maximum capacity on any given day, while safety regulations prevent labor utilization from exceeding 90% of the available maximum labor hours. The management seeks to determine feasible production levels that satisfy all operational and labor-related constraints.

Task.

Formulate the system of inequalities representing the situation and determine the feasible range of daily production levels.

ITEM 39. Two quantities, x and y , represent the number of units produced by two different machines in a factory. The total output must not exceed 50 units due to storage limitations, while machine A must produce at least twice as many units as machine B to maintain efficiency.

Additionally, machine B must produce no fewer than 5 units to justify its operational cost, and machine A must not exceed 40 units due to maintenance constraints. The production manager seeks to determine all feasible combinations of outputs that satisfy these conditions.

Task.

Formulate and solve the system of inequalities representing this situation and describe the feasible region of solutions.

ITEM 40.

A cylindrical storage container initially holds a volume of liquid between 10L and 25L. Water is added at a constant rate of 2L per minute. Due to safety regulations, the container must never exceed its maximum capacity nor fall below its minimum operational level.

In addition, a monitoring system triggers an alert if the volume remains above 20L for more than 3 consecutive minutes. The operator seeks to determine the time intervals during which the system operates safely without triggering any alerts.

Task.

Formulate inequalities representing the volume over time and determine the intervals during which all safety and monitoring conditions are satisfied.

TRIGONOMETRY

ITEM 41. A vertical communication mast is erected on level ground and casts a shadow whose length varies throughout the day depending on the position of the sun. At a particular moment, the angle of elevation of the sun is measured to be 35° , and the length of the shadow is found to be 18 meters.

Due to signal calibration requirements, engineers must determine the exact height of the mast, ensuring that all calculations are based on trigonometric principles without approximation errors that could affect signal alignment.

Task.

Determine the exact height of the mast, clearly showing all trigonometric reasoning and justifying each step.

ITEM 42. A surveyor is tasked with determining the height of an inaccessible tower located across a river. From one point on the riverbank, the angle of elevation to the top of the tower is measured as 40° . After moving 10 meters closer to the tower along a straight line perpendicular to the river, the angle of elevation increases to 55° .

The surveyor must determine the exact height of the tower and the distance from the initial observation point to the base of the tower.

Task. Formulate appropriate trigonometric equations and determine the required measurements.

ITEM 43. A ladder of fixed length leans against a vertical wall, forming an angle of 60° with the ground. The base of the ladder rests on a horizontal surface that is subject to slight displacement due to uneven terrain, and engineers must determine both the vertical height reached by the ladder and the horizontal distance from the wall.

Additionally, safety guidelines require that the angle of inclination must not fall below 55° to prevent slipping.

Task. Determine the height reached and the base distance from the wall, and discuss whether the setup satisfies the safety condition.

ITEM 44. Pauline is a professional interior designer. She is to fix a triangular design of base 40cm on a flat wall while ensuring one side of this triangle is 17cm and makes an angle of 68° with the base. To increase beauty, the artist is to surround the triangular design with a circular ring touching its three corners. She is interested in knowing radius of the circular ring and the vertical height of the proposed triangular design.

Task. Help Pauline to determine radius of the circular ring and the vertical height of the proposed triangular design.

ITEM45. Alex wishes to come up with a construction plan where two pillars A and B are to be raised from the same fixed point on the level ground towards the vertical wall. In this, A is to touch at the top of the wall while B touches a quarter way the height of the wall from the level ground. He was advised to incline A at an angle which is twice B hence wishes to come up with the alternative directions with in a full rotation pillar A and pillar B can face.

Task. Help Alex to come up the alternative directions with in a full rotation pillar A and pillar B can face.

ITEM46. John an architect wishes to develop two walls A and B bending towards each other from opposite sides. The two walls rise from the same level ground but the angle of inclination of wall B from the ground is twice wall A. The distance between walls A and B on the level ground is equivalent to the length of wall A from the ground the point they meet with wall B while the length of wall B from the ground to the point it meets wall A is ten meters. John plans to fix a vertical pillar to hold the two walls at their meeting point. However, he is failing to determine the minimum height of a pillar and the actual distance between the two walls on the level ground satisfying the above conditions.

Task. Use the above information and help John to determine

- a. The required height of the pillar
- b. The distance between the two walls on the level ground

ITEM47. Lule the builder wishes to come up with a construction plan where two pillars A and B are to be raised from the same fixed point on the leveled ground and touch the vertical wall as supports. In this, A is to touch at the top of the wall while B touches at any other point of the wall. Since the length of A is to be six times that of B, he was advised to incline A at an angle which is twice B hence seeks help on how to determine various possible angles at which A and B can be inclined.

Task. Find the angle of inclination for each of the two pillars.

ITEM48. Joseph the welder was given a project of making triangular designs with two sides A and B rising from the same level at the base. The distance between side A and B from the base is 30cm and the length of side B from the base to the point it meets side A is 36cm. Side A must be inclined at an angle twice of B. For better planning, Joseph wishes to determine to actual length of side A of the designs, and also knowing whether such a design can be cut from a 42cm by 22cm rectangular metal sheet. He is however still puzzled on how to derive solutions to these questions.

Task.

- a. Use the above information and help Joseph to determine the actual length of side A.
- b. Mathematically show whether such triangular designs can be cut from the given rectangular metal sheet.

ITEM49. Alex rotated a point A(9,5) through a center C(4,6) and its new location after rotation was $A'(5,11)$. However, he forgot the direction in which he rotated the point. The only thing he remembers is the angle of rotation was less than a thousand degrees.

Task. Help Alex to determine the possible direction through which he may have rotated point A

ITEM50. A square block is placed on a level ground while ensuring that strong pipes A and B rise from opposite sides on the same ground level of the block, to torch on the top ends of the block so as to keep at the same fixed position. The ground distance between the pipes must be thrice the ground distance between pipe A and the block. However, each side of the block is two meters less than the ground distance between pipe A and it. Since the sum of the angles of inclination for pipe A and B with the ground is 45° , the engineer wishes to determine the minimum length of pipe A and B, and how he should set the pipes towards the block.

Task.

- a. Determine the minimum length of pipe A and B.
- b. How should the engineer incline pipe A and B towards the square block?

ITEM51. Mukasa owns a farm having three corners A, B and C where A is western of B and C is northwards of B. Mukasa takes 16minutes to move around the entire farm at a uniform speed of 56m per minute. He however noted that he can walk the shortest route from A to C in 8minutes at a uniform speed of 50m per minute. He is now interested in knowing the area occupied by his farm.

Task. Compute the total area occupied by Mukasa's farm.

END

"Every battle is won or lost before it is fought"