

SCIENCE SCHOLARS ACADEMY | SSA

S.5 PRINCIPAL MATH EXAM [COMBINED]

END OF TERM 1

*2 hours, 45 minutes*

**Instructions:**

- *Attempt all problems!*

**ITEM 1:**

In the fast-growing metropolis of Emerald City, the Department of Urban Transport is launching an ambitious project: constructing **two major arterial roads** to reduce congestion and link newly developing neighborhoods.

**Road A: From Westhaven Point to Eastmoor Terminal**

According to the planning grid:

- **Westhaven Point** is at coordinate **(8, 40)**
- **Eastmoor Terminal** lies at **(22, 18)**

**Road B: From Northbridge Park to Southgate Circle**

- **Northbridge Park** is located at **(10, 6)**
- **Southgate Circle** sits at **(4, 28)**

Before breaking ground, engineers must analyze how these roads will align within the city's layout. To ensure safety and ease of navigation, they must determine whether the roads **intersect** and, if so, at **what angle**. A sharp angle could result in a risky junction – or a future hotspot for traffic jams and accidents.

Meanwhile, local delivery riders – always the first to test new shortcuts – are watching the surveyors with keen interest.

**Your Task:**

To support Emerald City's planners, do the following:

1. Find the **equation** of each road segment (*as a straight line*) and hence deduce the **y-intercept**.
2. Determine if the roads **intersect**, and if they do, compute the **point of intersection**
3. Determine the **midpoint** and **length** of each road segment.

**ITEM 2:**

In a high-security pharmaceutical laboratory, scientists are testing a sensitive compound used in the development of antiviral medication. This compound contains a radioactive isotope that gradually **loses its potency over time** when dissolved in solution.

At the **start** of the experiment, the **concentration** of the isotope is **4 moles per liter**. Researchers are tracking its decay to determine safe handling protocols and predict when reinforcement or neutralization might be necessary. The decay follows the mathematical model:

$$N = 4(0.76)^t$$

Where:

- **N** is the *concentration in moles per liter* and **t** is *time in seconds*

**Tasks for the Research Team:**

- (a) **Predict** the **concentration** of the isotope after **4 seconds**. Round your answer to **1 decimal place**.
- (b) Determine **how long** it will take for the **concentration to drop to half of its original level**.
- (c) In the rare event of a reaction causing the isotope concentration to **triple**, immediate isolation of the solution is required. Advise **how long** it would take for the **concentration to reach three times its starting amount** if such a reaction occurred.

*As a mathematics student, assist the research team to accomplish the tasks!*

**ITEM 3:**

In a high-tech drone flight research center, engineers are testing the altitude stability of an advanced surveillance drone during its early flight stage over simulated terrain.

The **altitude deviation**, denoted by  **$h$  (in meters)**, from the drone's ideal cruising level is modeled as a function of **time,  $r$  (in minutes)** after launch. The deviation follows this mathematical model:

$$h = 6\cos(r) - 3\sin^2(r) + 1$$

**Note to students:** *For this investigation, assume that  $180^\circ = 1$  radian ( $\pi$ ) to simplify calculations when interpreting trigonometric values.*

This model helps the flight team assess and correct any instability that could interfere with the drone's stealth or energy efficiency during missions.

**Tasks for the Engineering Team:**

(a) Determine the **time,  $r$**  when the drone is exactly at its programmed cruising altitude (i.e., when the **deviation  $h = 0$** ). Round your answer to **3 sfs**.

(b) Identify the **time(s),  $r$  (if any)** when the drone descends to **7 meters below** its intended altitude.

***Assume you are part of the engineering team and respond!***

**ITEM 4:**

To study the performance of a specialized solar panel, its **brightness  $Y$** , measured at **time,  $t$**  hours after sunrise, is modeled by the formula:

$$Y = 3 \left( \frac{\sqrt{32t}}{4^t} \right)$$

**Task:**

(a) **Simplify** the expression for the brightness. Your final answer should express the brightness using a **single power of 2 and a surd**.

(b) At a moment when the brightness is observed to be **3.0 arbitrary units**, **prove** that the equation:

$$\log_t 2 = \frac{1}{4t - 5}$$

*is satisfied.*

- (c) Hence, or otherwise, determine the **value** of  $\log_2 \sqrt{8t}$  at that same time.

**ITEM 5:**

As part of a training exercise for emergency response engineers, a test was conducted using a steel rescue stretcher suspended in mid-air by six tension cables.

Each cable pulled the stretcher with a different force and direction:

- **Two upward vertical** pulls of **8 N** and **16 N**
- **One downward** pull of **20 N**
- **Two horizontal** pulls to the **left** with magnitudes **9 N** and **7 N**
- **One horizontal** pull to the **right** with a magnitude of **18 N**

Your task is to determine the **resultant force** acting on the stretcher and the **net effect** this would have on its motion.

**END!**

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