

NAME:.....COMB:.....

4th, Monday May 2026  
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**Tr joelPCM ACADEMIC COUNCIL**  
**S.5 PRINCIPAL MATHEMATICS**  
2 Hours 15 Minutes

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**INSTRUCTIONS:**

*Attempt any **FOUR** items of your choice.*

**Item 1**

Uganda Wildlife Authority (UWA) has commissioned a scientific study tracking the migratory movement of a newly discovered insect species found along the shores of Lake Victoria. Field researchers have collected data on the displacement of individual insects and have modelled their movement patterns using two equations involving trigonometric and algebraic expressions. After careful analysis of flight trajectories and positional data over several observation sessions, the research team established that the displacement components of the insect follow the equations;

$$24\operatorname{cosec}\theta = 56/x^2 \quad \text{and} \quad 2/3 y + 42\sin\theta = 25$$

For the broader ecological simulation model to be validated and declared a success, the lead statistician requires verification that the following trigonometric relationship holds true for any value of  $x$  within the domain of definition;

$$(1 + \cos x) / \sin x + \sin x / (1 + \cos x) = 2\operatorname{cosec} x$$

**Task:**

- (a) Show that the Cartesian equation of motion of the insect is given by;

$$y = \frac{1}{2}(75 - 54x^2)$$

- (b) **Verify that the study program is a success. (15 scores)**

**Item 2**

During the inter-school mathematics competition organised by the Uganda Mathematics Society (UMS), a highly enthusiastic S.4 student approached an S.5 Principal Mathematics student for assistance with two challenging problems that had appeared in the qualifying round of the African Olympiad Mathematicians' programme. The S.4 student explained that he had attempted both questions multiple times but had been unable to arrive at the correct answers. He requested that the S.5 student guide him through the full working, showing all relevant steps and clearly explaining the reasoning at each stage.

**Question 1:** Solve  $8^{x-y} = 4^{x+y}$  and  $5^{x^2-y^2} = 15625$

**Question 2:** Prove that  $(n + m)/(n - m) = \log_3 75$ , if  $\log_5 x = n$  and  $\log_{15} x = m$

**Task:**

Help him show how this can be solved.

**(15 scores)**

**Item 3**

During a geography and physics fieldwork exercise at Ssesse Islands on Lake Victoria, a group of S.5 students from PCM Academy observed a boat sailing in a straight line directly towards a steep cliff face along the shoreline. Using a clinometer and measuring tape, two students positioned at the front of the boat carefully measured the angle of elevation to a clearly visible point at the top of the cliff directly ahead. They recorded that as the boat travelled a distance of exactly 50 metres towards the cliff, the angle of elevation of the top of the cliff increased from  $10^\circ$  to  $15^\circ$ . The students were tasked with estimating the height of the cliff using trigonometric principles.

Separately, during a sports science lesson, a student named Ahmed was investigating the physics of projectile motion. A ball was kicked from ground level and just landed on a flat rooftop 5 metres high. Using basic knowledge of projectile motion, Ahmed correctly derived that the launch angle  $\theta$  satisfies the equation:

$$2 = 4\tan\theta - \sec^2\theta$$

Ahmed is curious about all possible angles at which the ball could have been kicked, within the range  $0^\circ \leq x \leq 360^\circ$ .

**Task:**

- (a) Show that the height of the cliff is approximately  $h \approx 25.783$  metres.
- (b) Help determine all the possible angles at which the ball was kicked.

**(15 scores)**

**Item 4**

Kira Motors Corporation, Uganda's pioneering electric vehicle manufacturer, is in the advanced stages of designing and fabricating precision components for their latest electric bus model. The

engineering department has been tasked with computing exact material dimensions for a triangular mounting bracket that will be used in the vehicle's suspension system. Precision is critical because any deviation in measurement could affect the structural integrity and safety certification of the vehicle.

The chief mechanical engineer has provided the following specifications: the diagonal support brace of the triangular bracket must measure exactly  $\sqrt{50}$  cm. The two known sides of the triangular bracket are given in surd form as  $(5 + 3\sqrt{2})$  cm and  $(7 - 2\sqrt{2})$  cm. The full perimeter of the bracket must be exactly 25 cm to meet the design template requirements.

**Task:**

- (a) Express the length of the bracket diagonal support in the simplified form  $a\sqrt{b}$ , where  $a$  and  $b$  are integers and  $b$  is not divisible by a perfect square. Hence state the values of  $a$  and  $b$ .
  - (b) Establish the exact total length of material needed for both given sides combined.
  - (c) If the perimeter of the entire triangular bracket must be exactly 25 cm, calculate the length of the third side.
- (15 scores)**