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A-LEVEL MATHEMATICS

HOLIDAY WORK – S.5

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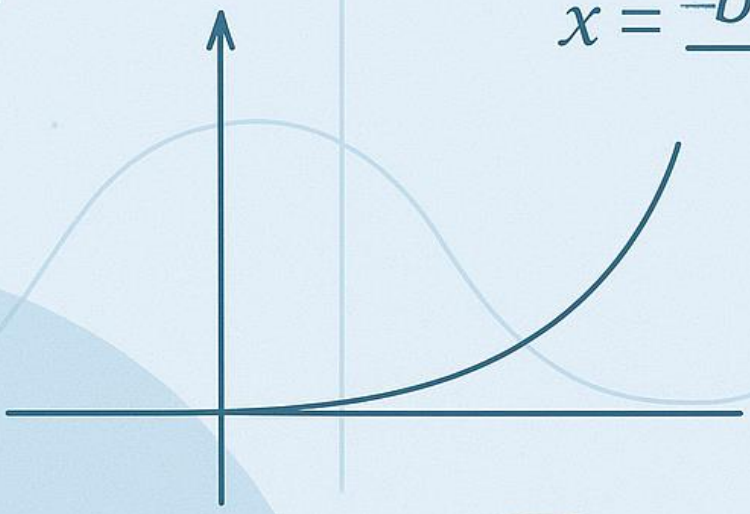
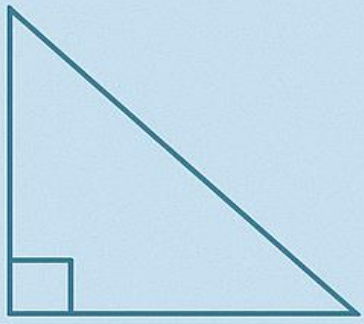
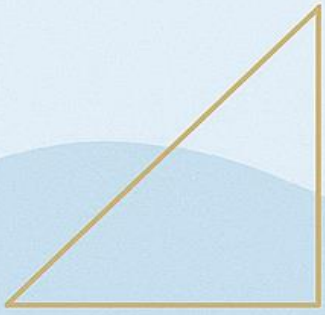
$\sqrt{a}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$





$\frac{a}{b}$



Contents

ALGEBRA, ANALYSIS, TRIGNOMETRY AND GEOMETRY

ITEM ONE

At Iceme Secondary School, the Math Club is working on a coordinate geometry challenge. The students are studying points, lines, and distances on a graph.

(a) Sarah is given a point P(2, 1) and told that there's a straight line L which is perpendicular to the line OP (where O is the origin) and which passes through point P.

(a) She is asked to find the equation of L.

(b) The teacher then says that line L meets the x-axis at point A and the y-axis at point B. Sarah must calculate:

(i) The area of triangle OAP.

(ii) The area of triangle OBP.

(iii) The ratio of the area OAP to that of OBP.

(b) In the next round, the teacher challenges the class to find the shortest distance between a given point and a given line:

(a) The point (2, 4) and the line $3x - 4y + 8 = 0$.

(b) The point (5, -1) and the line $12x + 5y - 3 = 0$.

(c) The point (9, -3) and the line $y = x$.

ITEM TWO

(a) A delivery company handles three types of packages: Small, Medium, and Large.

On a particular day, the company receives three separate orders. From Client A, the company makes £1 profit per Small package, £3 profit per Medium package, but incurs a £3 penalty for each Large package delivered. The total profit from this order was a loss of £4.

From Client B, the company makes £3 profit per Small package, loses £1 per Medium package, and makes £2 profit per Large package. The total profit from this order was £1.

From Client C, the company loses £2 per Small package (due to returns), makes £1 profit per Medium package, and makes £1 profit per Large package. The total profit from this order was £7.

Using x for the number of Small packages, y for the number of Medium packages, and z for the number of Large packages, form a system of simultaneous equations to represent the information above, and solve to determine how many packages of each type were involved that day.

(b) Later, the company reviews three further performance indicators for the same day:

The difference between the number of Small and Medium packages, when halved, is equal to three times the number of Medium packages minus the number of Large packages.

One third of the number of Large packages is equal to twice the number of Small packages minus the number of Large packages.

Three times the number of Small packages plus twice the number of Medium packages minus twice the number of Large packages equals 10.

TASK

Again using x, y, and z to represent the number of packages of each type, form a new system of simultaneous equations from the information above, and solve to find the values of x, y, and z under these conditions.

ITEM THREE

(a) A model rocket is launched from the ground. Its height, y metres, after t seconds is related to its motion by the expression:

$$y = x^2 - 2x$$

The path of the rocket also satisfies the quartic equation:

$$x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$$

Use the substitution to solve the quartic equation and determine all possible values of x.

(b) A roller coaster's speed, v metres per second, after x seconds from the start of a drop is modelled by:

$$v = 1 - 5x - 2x^2$$

Hence, determine the maximum possible speed of the roller coaster and the time x at which this occurs.

(C) A financial analyst models expected returns from two linked investment funds using the quadratic equation

½x² – 3x + 4 = 0

The roots of this equation are called α and β. The analyst wants to design a new combined product whose benchmark return rates are 5α + β and 5β + α.

Form an equation, with integer coefficients in standard form, whose roots are 5α + β and 5β + α.

(c) A teacher uses a polynomial function f(x) to model the overall performance score of a student, where x represents the number of study hours per week.

When the score model is checked for a student who studies 2 hours per week, the predicted result is 2 marks lower than the actual score.

When it is checked for a student who studies 3 hours per week, the predicted result is 3 marks higher than the actual score.

Find the expression for the difference between the model and the actual score for students who study x hours per week, when this difference is expressed as the remainder after dividing f(x) by (x – 2)(x – 3).

ITEM FOUR

(a) A chemist is preparing a special solution for an experiment. The concentration of the active ingredient, C, in moles per litre, is calculated using the formula:

$$C = \frac{5 + \sqrt{8}}{2 - \sqrt{3}}$$

The laboratory computer software requires all concentration formulas to be expressed without surds in the denominator before they can be used in automated calculations.

(i) Rationalise the denominator of C and simplify the expression fully.

(ii) Hence, determine the exact value of C if the result is required in the form $p + q\sqrt{r}$

(b) A company tracks the number of online visitors to its website, n, using a data model:

$$\log_3 n - 2\log_n 3 = 1$$

Solve this equation to determine the value of n.

ITEM FIVE

(a) A teacher is designing a set of questions for a maths competition. She writes down three expressions to represent the first, second, and third terms of a sequence:

$(2x + 1)$, $(x^2 + x + 1)$, and $(3x^2 – 3x + 3)$.

She wants these three terms to be consecutive terms of an arithmetic progression.

Determine the value of x that makes these expressions consecutive terms of an arithmetic progression.

(b) In another competition question, an arithmetic progression (A.P.) has a common difference of 3, and a geometric progression (G.P.) has a common ratio of 2.

A third sequence is formed by subtracting each term of the A.P. from the corresponding term of the G.P.

It is known that the third term of this new sequence is 4 and the sixth term is 79.

Find the original A.P. and G.P.

(c) A group of students is designing a large decorative arch for their school fair. The estimated cost (in pounds) for the materials is modelled by the expression:

$(2 – 3x + x^2)(1 + 2x)^4$

where x represents a scaled change in the size of the arch from its original design.

Find the coefficient of x^2 in the expansion of this expression.

(d) In a statistics project, the probability of a certain sequence of events can be modelled by the binomial expansion of $(1 + x)^n$.

The students notice that the coefficients of x, x^2, and x^3 are in arithmetic progression for a certain value of n.

Find the value of n for which the coefficients of x, x^2, and x^3 in the expansion of $(1 + x)^n$ form an arithmetic progression.

ITEM SIX

(a) In a physics experiment, a student is tracking the motion of a small metallic ball rolling on a tilted table. The relationship between its horizontal position x (in metres) and its vertical position y (in metres) is modelled by:

$y^2 + 3xy = 2x^2 - 1$

At one point, the ball passes through the position (2, 1). To analyse its motion at this moment, the student wants to determine how steeply the path is changing.

Find the equation of the tangent to the path at the point (2, 1).

(b) A team of engineers is designing a new roller coaster. The vertical height y (in metres) of the track above the ground is modelled by the equation:

$y = 2 - 4x^2 + x^3$

where x is the horizontal distance (in metres) from the starting point of this section of the track.

At the point (1, –1), the engineers want to determine the gradient of the track to ensure a smooth transition. They also need to know where the tangent at this point will meet the track again to plan structural supports.

Find the equation of the tangent to the track at the point (1, –1) and determine the coordinates of the other point where this tangent line meets the track.

(c) A biologist is monitoring the growth of a rare plant in a greenhouse. The height y of the plant (in centimetres) after x weeks is modelled by:

$y = \sqrt{(2 + x)}$

The biologist wants to calculate the instantaneous rate of growth at different stages to decide on optimal watering intervals.

Differentiate y with respect to x and give your answer in simplified form.

(d) A company produces and sells a certain product. The profit y (in pounds) from selling x hundred units is modelled by the equation:

$y = 600x + 15x^2 – x^3$

The marketing team needs to determine the production level that will yield the maximum possible profit, as well as the value of that maximum profit, to set sales targets for the upcoming quarter.

Find the value of x that maximises the profit and state the maximum profit.

(e) A small drinks company is designing a new closed cylindrical can (with both a top and a base). The design team must use as little metal as possible while keeping the can volume fixed at 54π cm^3, to reduce manufacturing costs and minimise waste.

Let r be the radius of the circular base (in cm) and h be the height of the can (in cm).

(a) Express the surface area S (in cm^2) of the closed cylinder in terms of r and h.

(b) Using the fixed volume condition $54\pi = \pi r^2 h$, eliminate h to write S as a function of r alone.

(c) Find the value of r that minimises the surface area. Hence find the corresponding value of h.

(d) State the minimum surface area and give units.

You should show all calculus steps clearly (including differentiation and justification that your critical point gives a minimum).

ITEM SEVEN

(a) An architect is designing a staircase in a modern house. One of the sloping handrail supports makes an angle of –15° with the horizontal when measured in a certain reference frame.

Without using tables or calculators, show that:

$\tan(-15°) = -2 + \sqrt{3}$

(b) A civil engineering team is calibrating a vibration sensor mounted on a bridge. The sensor records two independent signal components from a passing vehicle: a vertical oscillation modelled by sin x and a rotational component related to the tilt angle modelled by cot x. During calibration tests the engineers derive the following relation between the squared vertical signal, the squared rotational signal and the squared reciprocal of the vertical component:

$4 \sin^2 x + 8 \cot^2 x = 5 \operatorname{cosec}^2 x$

Here x denotes the phase angle (in degrees) associated with the oscillation at a particular sensor position.

(i) Solve the equation
$4 \sin^2 x + 8 \cot^2 x = 5 \operatorname{cosec}^2 x$
for x in the interval $0° \leq x \leq 360°$. State all solutions and give reasons for rejecting any values that are not physically meaningful (for example where functions are undefined).

(ii) For each valid solution, comment on what the angle implies about the sensor's state (e.g. near-vertical motion, near-horizontal tilt) and whether the calibration point should be accepted or re-tested.

(c) Two sound waves of the same frequency are combined in a physics lab. The measured instantaneous amplitudes at a certain reference time satisfy the trigonometric relation:

$\sin x + \sin 5x = \sin 2x + \sin 4x.$

The students are asked to find which phase values x (measured in degrees) on the interval $0° < x < 90°$ make the equality true.

Task: Solve sin x + sin 5x = sin 2x + sin 4x for x in the interval $0° < x < 90°$. (Give all solutions in that open interval.)

(d) A surveying team models the effect of a lateral load on a retaining wall by an expression involving cosines of the wall angle theta. The equation that must be solved is

$2\cos^2 x - 5\cos x = 4$,

for $0° \leq x \leq 360°$.

Task: Solve the equation and give the answer within the required interval.

ITEM EIGHT

A group of A-level physics students are analysing a data set from an experiment on fluid resistance.

In their lab, they find that the expression describing the rate of change of velocity vvv with respect to time t is given by:

$$\frac{dv}{dt} = \frac{4t^2 + 7t - 3}{(t-1)(t+2)^2}$$

To solve the differential equation and predict the velocity at various times, they first need to break this expression into simpler terms so that it can be integrated more easily.

Task:

Resolve

$$\frac{dv}{dt} = \frac{4t^2 + 7t - 3}{(t-1)(t+2)^2}$$ into partial fraction

ITEM NINE

(a) A student engineering team at Makerere High is designing a short access ramp for a school go-kart project. The safety rule requires the ramp height h (metres) to satisfy

$$\frac{2h^2 - 5h - 3}{h^2 - 4} \geq 0.$$

i. Find the critical values of h

ii. Using a sign–line or interval test, determine all intervals of h that satisfy the inequality.

(b) A Form 6 entrepreneurship group in Mbarara models daily profit P (in thousands of Uganda shillings) for the sale of x cups of a new drink by

$$\frac{x^2 - 8x + 12}{x^2 - x - 6} \geq 0.$$

i. Find the critical values of x.

ii. Using those critical values, determine the intervals of x for which the profit is non-negative.

(c) During a Physics practical at Jinja Senior School the students record the voltage V (volts) across a resistor as a function of time t (seconds). The model is

$$\frac{t^2 - 3t - 10}{t^2 + t - 6} < 0.$$

(I) Determine the critical values of t.

(I) Using the critical values, find the intervals of time for which the voltage is fully satisfied

STATISTICS

ITEM TEN

(a) A school statistics teacher collected the marks of 9 students in a short quiz to demonstrate how quartiles can be calculated. The results, in ascending order, were:

10, 12, 13, 15, 19, 19, 24, 26, 26

Using the given data, determine the lower quartile and upper quartile.

(b) Later, the same teacher decided to compare this with the students' heights. The heights (in cm) of all 400 students in the school were recorded and grouped as follows:

Height (cm)	Number of students
$100 \leq x \leq 110$	27
$110 \leq x \leq 120$	85
$120 \leq x \leq 130$	215
$130 \leq x \leq 140$	320
$140 \leq x \leq 150$	370
$150 \leq x \leq 160$	395
$160 \leq x \leq 170$	400

Tasks:

(i) Calculate the mean height of the students.

(ii) Draw a cumulative frequency curve for the data and from the graph estimate:

the median height,

the quartile deviation,

the 10th percentile and the 90th percentile.

ITEM ELEVEN

(a) On a local farm, an agricultural researcher wanted to compare two different methods of rearing calves to see if they affected meat palatability. Eight pairs of identical twin calves were reared, with one from each pair randomly assigned to Method A and the other to Method B. After slaughter, each sample was scored for palatability.

Twin Pair	1	2	3	4	5	6	7	8
Method A	27	37	31	38	29	35	41	37
Method B	23	28	30	32	27	29	36	31

Task: Test whether there is a significant difference in palatability scores between the two rearing methods at 5%.

(b) During the COVID-19 pandemic, two locally developed herbal antiviral drugs were tested on 12 symptomatic patients. The percentage effectiveness of the drugs was recorded:

Patient	1	2	3	4	5	6	7	8	9	10	11	12
Covylice-1 (x)	58	52	48	30	48	20	32	50	38	12	36	12
Covidex (y)	90	72	60	38	70	35	33	64	48	24	50	18

Tasks:

(i) Plot a scatter diagram of y against x and comment on the relationship between the two drugs.

(ii) Draw a line of best fit and use it to find x when y = 68.

(iii) Calculate Spearman's rank correlation coefficient and test for significance at the 1% level.

ITEM TWELVE

(a) The management of Uganda Airlines conducted a survey of all employees to study their age distribution. The cumulative age data was as follows:

Age (years)	< 15	< 20	< 30	< 40	< 50	< 60	< 65	< 100
Cumulative Freq	0	17	39	69	87	92	98	98

Tasks:

(a)

(i) Calculate the mean and median age of the employees.

(ii) Determine the middle 70% age range.

(b) Represent the above information on a histogram and use it to estimate the modal age.

(c) You have been provided

Symbol	Value
f	m
$\Sigma f x$	177
$\Sigma f x^2$	5259
Standard Deviation	2.5

Find the value of m.

ITEM THIRTEEN

(a) A teacher wanted to check if performance in Mathematics was related to performance in Biology. The grades of 9 students were recorded:

Student	1	2	3	4	5	6	7	8	9
Math	A	E	F	A	B	B	C	D	B
Biology	C	O	E	C	C	B	A	F	D

Compute Spearman's rank correlation coefficient and test for significance at the 1% level.

(b)The following data shows the height distribution of students in a mathematics class:

Height (cm)	Frequency
20 – 29	9
30 – 34	12
35 – 44	27
45 – 49	13
50 – 54	25
55 – 59	18
60 – 74	30

Tasks:

(i) Draw a histogram and read off the modal height.

(ii) Calculate:

(a) the standard deviation,

(b) the number of students in the middle 60% height range.

ITEM FOURTEEN

(a) A school keeps a record of expenditure on different items for 2020 and 2021:

Item	2020	2021	Weight
Data	50,000	80,000	8
Eats	10,000	30,000	2
Airtime	45,000	100,000	5
Salary	51,000	90,000	10
Apps	10,000	25,000	1

Using 2020 as the base year, calculate the average weighted price index (to 5 significant figures).

(b)The table below gives the price relatives and weights for 4 school items:

Item	Books	Pens	Calculator	Sets
Weight	p	2p	q	6 + q
Price Rel.	110	140	130	118

Given that:

$p + 2p + q + (6 + q) = 40$,

Weighted average price index = 126.7,

Find p and q.

MECHANICS

ITEM FIFTEEN

(a) A coastal engineer is testing a device that ejects sensor capsules from the top of a cliff into the sea. A capsule is launched from the cliff top (height 98 m above sea level) with speed 49 m s^{-1} at an angle of 30° above the horizontal. The engineer needs to know how far from the foot of the cliff the capsule will land in the water (horizontal distance along the sea surface).

Task: Find the horizontal distance from the base of the cliff to the point where the capsule strikes the water. (State any working formulae you use, and give the final answer in metres.)

(b) Two laboratory technicians perform a timing experiment with vertically launched projectiles from the same ground point O. Particle P is projected vertically upwards from O with initial speed u m s^{-1}. When P reaches its highest point, a second particle Q is launched vertically upwards from O with speed 2u m s^{-1}. The two particles later collide at a point which is a distance x metres below the highest point reached by P (i.e. the collision point lies x m vertically beneath P's top).

Task: Using kinematics, prove that $u^2 = 32\,g\,x$. (Clearly state the times and displacements you equate, and any use of symmetry or standard formulae for uniformly accelerated motion.)

(c) An architect is checking whether a thrown object will clear a fixed vertical wall adjacent to a plaza. The object is projected from ground level at horizontal distance a from the foot of the wall, with speed V at angle α above the horizontal. The wall has height b. It is given that the object just clears the top of the wall (i.e. passes exactly over the top).

Tasks:

1. Find an expression for how far above the top of the wall the object passes (in terms of V, α, a, b and g).
2. If the object is projected so that it just clears the wall, show that the greatest possible height above the ground reached by the projectile (subject to the "just-clear" condition) is

$$\text{greatest height} = \frac{a^2 \tan^2 \alpha}{4(a \tan \alpha - b)}.$$

(You should begin from the projectile equations for horizontal and vertical displacement, eliminate the time variable to find the vertical co-ordinate when the projectile is at horizontal distance a, impose the "just clears" condition, and then optimise with respect to launch speed V where necessary. State any constraints on α, a and b that make the expression meaningful e.g. that a tan α > b so the trajectory can clear the wall.)

ITEM SIXTEEN

(a) An engineering student is testing a prototype electric scooter that starts from rest and accelerates uniformly. In a timed trial, the scooter covers a distance of 2p metres in q seconds, and in a separate timed run (starting again from rest with the same acceleration) it covers a distance of p − q metres in p seconds.

Show that with the same constant acceleration, the scooter would cover a distance of p + q metres in $q^2 - p^2$ seconds.

(b) Two children, Stephen and Frank, are standing 24 m apart on a straight flat path and start cycling towards each other at the same instant.

Stephen starts from rest at point A and accelerates uniformly with acceleration 2.0 m s^{-2}. Frank starts from the opposite point B and rides at a constant speed of 2.0 m s^{-1}.

Find how long after they start that the two children meet.

ITEM SEVENTEEN

At a coastal shipping dock, two workers are moving crates between two ramps that meet at a peak. The ramps are smooth and free of friction. One crate of mass m_1 is on the left ramp, which is inclined at an angle α to the horizontal, and another crate of mass m_2 is on the right ramp, inclined at an angle β. The crates are connected by a light, inextensible rope passing over a small, smooth pulley fixed at the top where the ramps meet.

When the workers let go, the heavier crate (m_1) begins to slide down its ramp, pulling the lighter crate (m_2) up the other ramp.

Show that the acceleration of the crates is given by:

$$a = \frac{m_1 g sin \propto -m_2 g sin\beta}{m_1 + m_2}$$

State any assumptions you make.

ITEM EIGHTEEN

(a) A dockyard is using a set of small cranes and spreader-bars to position heavy machine parts for installation on a ship. Several winch cables apply planar forces at various attachment points on the load. The engineer records the following forces acting in the plane at the given co-ordinates (origin is a fixed reference point on the deck):

Point (x, y) (m)	Force vector (N)
(2, 2)	$6\mathbf{i} + 5\mathbf{j}$
(5, 0)	$-10\mathbf{i} - 4\mathbf{j}$
(−4, −4)	$7\mathbf{i} - 7\mathbf{j}$
(0, −5)	$-8\mathbf{i} + 2\mathbf{j}$
(6, 0)	$5\mathbf{i} + 4\mathbf{j}$

Task: Show that this system of forces reduces to a pure couple (i.e. the net force is zero but there is a net moment). State the moment (magnitude and sense) of the couple about the origin. (Give your vector sum for forces and compute moments using the position vectors shown.)

(b) A mobile communications mast is stabilised by three guy ropes that radiate away from the mast base. Each rope is under a known tension and runs in a compass direction specified from the mast base. The tensions are: 10 N along S30°E,15 N along E60°N,20 N along North-West

Task: Resolve each force into rectangular components, find the vector sum, and hence determine the magnitude and direction (bearing or standard angle) of the resultant force acting on the mast.

ITEM NINETEEN

A rectangular support frame ABCD is bolted to a workshop floor. The rectangle is positioned so that AB is horizontal and AB = 3.0 m. The diagonal AC meets AB at angle CAB = 30° (i.e. angle between CA and BA at A is 30°). Three tensioning cables apply forces along the lines AC, AD and DB to adjust the frame:

10 N acting along AC

20 N acting along AD

20 N acting along DB

Task:

1. Using the geometry of the rectangle, resolve the three forces into components and compute the magnitude and direction of their resultant.
2. Hence determine the point where the resultant's line of action cuts the side AB (give the distance from A along AB to the intersection point).

(Include a labelled diagram of rectangle ABCD, show how you place axes, and show steps to locate the intersection of the resultant line of action with AB.)

ITEM TWENTY

A warehouse worker needs to tow a crate of mass 5 kg across a smooth concrete bay. The crate starts from rest and is pulled by a light rope attached to the top corner. The worker pulls with a constant force P, directed at 45° above the horizontal. During the first 5.0 s of motion the crate moves a distance of 10.0 m along the floor.

Assume the floor is smooth (no friction), $g = 9.8\ \text{m s}^{-2}$, and the rope remains taut and straight throughout the motion.

Tasks

(i) Find the acceleration of the crate.

(ii) Find the magnitude of the pull P.

(iii) Find the normal reaction between the crate and the floor.

Show clear working for each part and give answers with appropriate units.

ITEM TWENTY-ONE

A delivery driver places a 5 kg crate at the foot of a smooth loading ramp. The ramp is inclined to the horizontal at angle θ, where $\sin\theta = 3/5$. The driver then pushes the crate up the ramp with a constant horizontal force of magnitude 50 N. Assume the ramp and crate are smooth (no friction) and take $g = 9.8\ \text{m s}^{-2}$.

Tasks

(i) Find the normal reaction between the crate and the ramp.

(ii) Calculate the acceleration of the crate up the slope.

(iii) How far up the slope does the crate travel in the first 4.0 s after it is pushed?

(Show all working and give answers with appropriate units.)

ITEM TWENTY-TWO

A car of mass 1000 kg tows a caravan of mass 600 kg up a straight stretch of road which rises 1 m vertically for every 20 m along its length. The motion is opposed by constant frictional resistances of 200 N on the car and 100 N on the caravan. The combination accelerates up the slope at $1.2\ \text{m s}^{-2}$. Assume $g = 9.8\ \text{m s}^{-2}$ and that the tow-bar is light and the rope/connection transmits tension only along the line joining the car and caravan.

Tasks

(a) Find the total driving force produced by the car's engine (the forward force at the wheels) required to produce the stated motion.

(b) Find the tension in the tow-bar between the car and the caravan.

Explain your working clearly, resolve forces parallel and perpendicular to the slope where appropriate, and give answers with units.

ITEM TWENTY-THREE

A logistics company tests a prototype crate-release system on a loading ramp. A steel crate of mass 12 kg is released from rest at the top of a smooth, rigid ramp which is inclined at 50° to the horizontal. The ramp surface is not frictionless: the coefficient of friction between the crate and the ramp is 0.4.

Assume $g = 9.8\ \text{m s}^{-2}$.

Task: Calculate the acceleration of the crate as it slides down the ramp.

ITEM TWENTY-FOUR

A materials tester releases a 3 kg test block from rest on a rough ramp whose inclination θ satisfies $\sin\theta = 3/5$. After 2.5 s the block has reached a speed of $4.9\ \text{m s}^{-1}$ down the slope. Assume uniform acceleration and that the only forces along the slope are the component of weight and kinetic friction (with unknown coefficient μ). Take $g = 9.8\ \text{m s}^{-2}$.

Task: Determine the coefficient of friction μ between the block and the slope.

ITEM TWENTY-FIVE

At a coastal renewable energy test site, engineers are experimenting with a lightweight hexagonal solar panel frame labelled A, B, C, D, E, F in clockwise order. The hexagon is perfectly regular, each side measuring 2a meters, with its geometric center marked as point O.

During a storm simulation, wind pressure and cable tensions apply forces along the edges of the hexagon: a force of 4 N acts from A toward B, another of magnitude s N acts from B toward C, a third force of magnitude t N acts from C toward D, a force of 1 N acts from D toward E, a 7 N force acts from E toward F, and finally a 3 N force acts from F toward A.

Wind sensors detect that all these forces combine to produce a single resultant force of magnitude $2\sqrt{3}$ N acting in a direction perpendicular to side BC.

Your tasks are:

a) Determine the unknown forces s and t so that the system's resultant matches the recorded data.

b) Show that the sum of the moments of all these forces about the center O is $27a\sqrt{3}$ N·m. State whether this net moment tends to rotate the frame clockwise or anticlockwise.

c) The midpoint of BC is marked as M. Using a coordinate system where O is the origin, OM is the x-axis and OD is the y-axis, find the equation of the line of action of the resultant force.

ITEM TWENTY-SIX

At Atapara Secondary School, the Physics Club is conducting a practical experiment in the school lab. The club members are testing how forces can act together on a rectangular wooden frame, ABCD, which has dimensions AB = 4 m (horizontal) and BC = 3 m (vertical).

To make the experiment fun, the students attach ropes to different sides of the frame and pull with spring balances to measure forces:

James pulls with a 3 N force along AB from A toward B.

Maria pulls with a 5 N force along BC from B toward C.

Kelvin pulls with a 6 N force along CD from C toward D.

Sarah pulls with a 4 N force along DA from D toward A.

Emily pulls with a 7 N force along the diagonal AC from A toward C.

Given that AB is horizontal, the Physics Club is tasked to calculate:

a) The magnitude and direction of the resultant force acting on the frame.

b) The distance from point A along AB where the line of action of the resultant force cuts AB.

END OF TERM MESSAGE

Dear Beloved Students,

As we close this term, I want to reflect on the journey we've taken together over the past months. You have faced challenges, celebrated victories, and grown in ways that go beyond textbooks. I am truly proud of each of you.

Like I have always advised you in class, there is more to studies than only books. True education is about gaining knowledge you can apply outside the classroom; in your homes, your communities, and your everyday life. It's about learning how to think wisely, act responsibly, and live with purpose.

As you go on break, remain cautious and vigilant in all you do. Not every path will lead to a good destination. Avoid unnecessary vices such as dishonesty, laziness, bad company, gossip, disrespect, and substance abuse. These can destroy character and limit your potential.

Remember: you are the future of tomorrow. The habits and choices you make now will shape the kind of leaders you will become. Be vibrant, curious, and willing to try new, healthy, and productive things that will help you grow. Work hard both inside and outside of school; help your parents with chores, respect them deeply, and please greet them warmly for me.

Above all, remember God in the days of your youth, as Ecclesiastes 12:1 teaches: *"Remember your Creator in the days of your youth, before the days of trouble come…"* Keep Him at the centre of your life and let His wisdom guide your steps.

Believe in yourself. You have unique gifts and talents. No matter the obstacles, with determination, faith, and discipline, you can achieve great things. Let this holiday be a time of rest, reflection, and preparation for greater achievements in the coming term.

The world is waiting for your light, go and shine it brightly.

With pride and blessings,
Ayen Geoffrey Alexander