SECTION A (40 scores)

Item 1

Nambi is an old lady who designs mats in lungujja town, recently she designed a mat with a width of $25^{\frac{w}{2}}$ and length of 5^{w+1}. If the area of the mat is $125^{\frac{1}{4}}$,

(a) find the value of w

(b) What value of w will make the length 3

Item 2

Your school has participated in a mathematics context and they only have one task to handle in order to win the prize. The task goes if one root of the equation $ax^2 + bx + c = 0$ is the square of the other, show that $c(a-b)^3 = a(c-b)^3$. Help the school win the prize because everyone is counting on you.

Item 3

On a construction site, a ladder was inclined on a vertical wall, such that the ladder passes through points A(- 2 , 0) and B(0 ,4) on the ground and top of the wall respectively. If a point O where the horizontal ground from the ladder meets wall is taken to be the origin, determine the area and centroid of the triangle AOB formed

Item 4

2 and 3 are factors of 6, hence 6 can be written as 2×3. Using the knowledge of polynomials, express $x^3 - 3x^2 - 10x + 24$ as a product of its factors.

Item 5

In analysis of roots of quadratic equations, a discriminant can be used to predict the different types of roots. Apply the same idea to determine the possible values of p if the equation $x^2 - 8x = 8px - 64$ has repeated roots

Item 6

Mary is taking a calculus course but through graph sketching she was able to determine the gradient of the tangent to the curve $y = x^3 + 5x^2 - 8x + 7$ as 24. Guide Mary through calculus to obtain the coordinates where the tangent meets the curve.

Item 7

Three schools A, B and C participated in a football tournament where a unique system of awarding points was used. School A obtained 6 wins,2 draws and 6 losses ,B had 4 wins,7 draws and 3 losses , C got 2 wins , 2 draws and 10 losses. The total points obtained at the end of the tournament by teams A, B and C were 42, 44 and 26 respectively. Use these results to obtain the points awarded for a win, a draw and a loss

Item 8

Usually different mathematical statements can be written in different ways but critically mean the same thing, for example $\tan x$ and $\frac{\sin x}{\cos x}$,without using tables, calculators or any trigonometrical ratios of some angles ,prove if the same statement holds for expressions $\cos x$ and $\frac{\cos x}{\sin^2 x} - \cos x \cot^2 x$

SECTION B (60 scores)

Item 9

In the English Premier League, the prominent Manchester derby was held on 14th/09/2025 and the goals scored by Manchester city and Manchester united are represented by x and y respectively and are related by two equations $x + y = 3$ and $1 + \log_2(2x + y) = \frac{\log_3 12}{\log_3 2}$. Basing on analysis, it is also true that Manchester city is likely to end the season with total points given by the expression $17\left(K - \sqrt{3}\right)$ Where $K = \sqrt{28 + 10\sqrt{x}}$

Tasks

(a) Determine the number of goals scored by each of the two Manchester clubs
(b) How many points is Manchester city likely to end the season with?

Item 10

During the 130years anniversary celebration at Mengo Senior School ,the first lady was hosted and her tent was set up in triangular shape with vertices at coordinates $(3,2)$, $(1,4)$ and $(5,4)$. The triangular tent was surrounded by bullet proof glass shields forming a circular fence passing through the vertices of the tent. The first lady's seat was put at the centre of the circular fence and a van whose purpose was known to only the security team was parked near the tent. The team had only two available positions for parking the van ,either on a straight road given by the equation $2x = 12 + 5y$ or another one given by the coordinate $(7,8)$ but the condition was the van to be parked nearer to the seat of the first lady.

Tasks

(a) obtain the coordinate where her seat was placed
(b) Which position was the van parked?

Item 11

A security drone launched in Haiti has arms given by the parametric equations $x = 3\cos 2\theta$ and $y = 8\sin\theta\cos\theta$ where θ is the reflex angle of projection. The maximum height H in metres reached by the drone is given by the equation $H = x + y$

Tasks

(a) Show that $16x^2 + 9y^2 = 144$

(b) Find the maximum height reached by the drone and the corresponding reflex angle of projection

Item 12

Mr. Obua wrote a will showing distribution of his property worth Ugshs. 324 millions among his three children John, Peter and Andrew .The document just had an expression $\frac{3x+1}{x(x-1)^2}$ and that denominator with x for Andrew, denominator with $(x-1)^2$ for John, the remainder to charity, nothing to Peter who abandoned us the moment he got a family.

Tasks

Express into partial fractions and by substituting $x = 4$, help the lawyer determine the amount of money to give to Andrew, John and charity

Item 13

Kabale is one of the most hilly areas in Uganda, two roads are to be constructed on a hill represented by the equation $y = x^2 - 3$ at point $(1, -2)$ where one road must be perpendicular to another. However the engineers are puzzled about many things.

Tasks

As a mathematician use the knowledge of calculus to help the engineers;

(a) differentiate $y = x^2 - 3$ from first principles

(b) determine the equations representing the two roads

(c) obtain the point where the road that is perpendicular to the hill at point $(1, -2)$ meets it again.

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3:1/4

END

"Keep doing your best and add up the efforts, they will surely accumulate to success"

Item 1

(a) $A = L \times W$

$$5^{w+1} \times 25^{\frac{w}{2}} = 125^{1/4}$$

$$5^{w+1} \times 5^{w} = 5^{3/4} \quad B1$$

$$5^{w+1+w} = 5^{3/4}$$

$$2w+1 = 3/4 \quad B1$$

$$2w = -1/4$$

$$w = -1/8 \quad A1$$

(b) $5^{w+1} = 3$

$$\log 5^{w+1} = \log 3 \quad B1$$

$$(w+1)\log 5 = \log 3$$

$$w+1 = \frac{\log 3}{\log 5}$$

$$w = -0.3174 \quad A1$$

Item 2

Let the roots be α and α^2

$$\alpha^3 = c/a \quad B1 \qquad \alpha + \alpha^2 = -b/a \quad B1$$

$$\alpha^3 + \alpha^2 + \alpha = c/a - b/a \quad B1$$

$$\alpha(1+\alpha+\alpha^2) = \frac{c-b}{a}$$

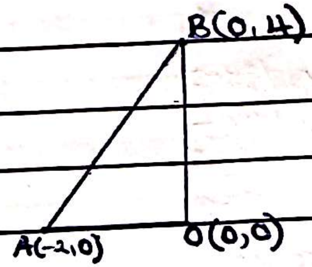
$$\alpha^3(1-b/a)^3 = \frac{(c-b)^3}{a^3} \quad B1$$

$$(a-b)^3 c/a = (c-b)^3 \quad B1$$

$$\therefore\ c(a-b)^3 = a(c-b)^3$$

Index Number..........

Item 3



base = 2units

height = 4units

$A = \frac{1}{2} \times 2 \times 4$ M1 B1

A = 4 square units A1

$$\text{centroid} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$ M1

$$= \left(\frac{-2+0+0}{3}, \frac{0+0+4}{3}\right)$$

Centroid is $\left(-\frac{2}{3}, \frac{4}{3}\right)$ A1

Item 4

Let $f(x) = x^3 - 3x^2 - 10x + 24$

if $x = 1$

$f(1) = 1 - 3 - 10 + 24$ B1

$f(1) = 12$, $x-1$ is not a factor

if $x = 2$

$f(2) = 8 - 12 - 20 + 24$

$f(2) = 0$ B1

$x-2$ is a factor

$$\begin{array}{r|l} & x^2 - x - 12 \\ \hline x-2 & x^3 - 3x^2 - 10x + 24 \\ & -(x^3 - 2x^2) \\ \hline & -x^2 - 10x + 24 \\ & -(-x^2 + 2x) \\ \hline & -12x + 24 \\ & -(-12x + 24) \\ \hline \end{array}$$

M1 $\Rightarrow f(x) = (x^2 - x - 12)(x-2)$

$= (x^2 - 4x + 3x - 12)(x-2)$ M1

$= [x(x-4) + 3(x-4)](x-2)$

$\therefore f(x) = (x-4)(x+3)(x-2)$ A1

Item 5

$x^2 - 8x = 8px - 64$

$x^2 - 8x - 8px + 64 = 0$

$x^2 - (8+8p)x + 64 = 0$ B1

for repeated roots, $b^2 = 4ac$ B1

$(-(8+8p))^2 = 4 \times 1 \times 64$

$64p^2 + 128p + 64 = 256$

$64p^2 + 128p - 192 = 0$ M1

$p = \dfrac{-128 \pm \sqrt{(128)^2 - (4 \times 64 \times -192)}}{2 \times 64}$

$\therefore p = 1$ or $p = -3$ A1 A1

Item 6

$y = x^3 + 5x^2 - 8x + 7$

$\dfrac{dy}{dx} = 3x^2 + 10x - 8$ B1

$3x^2 + 10x - 8 = 24$

$3x^2 + 10x - 32 = 0$

$x = \dfrac{-10 \pm \sqrt{10^2 - (4 \times 3 \times -32)}}{2 \times 3}$ M1

$x = 2$ or $x = -\frac{16}{3}$ (-5.333) B1

$y = 8 + 20 - 16 + 7$ $\qquad y = \dfrac{1085}{27}$ (40.185)

$= 19$

coordinates are $(2, 19)$ and $\left(-\frac{16}{3}, \frac{1085}{27}\right)$ A1 A1

Item 7

Let W denote a win

" D denote a Draw

" L denote a Loss

School A

$6W + 2D + 6L = 42$

School B

$4W + 7D + 3L = 44$

School C

$2W + 2D + 10L = 26$

B1

$7 \mid 6W + 2D + 6L = 42$

$2 \mid 4W + 7D + 3L = 44$

$42W + 14D + 42L = 294$

$- \quad 8W + 14D + 6L = 88$

$34W + 36L = 206$

$34(4+L) + 36L = 206$

$136 + 34L + 36L = 206$

$70L = 206 - 136$

$70L = 70$

$L = 1$ A1

$6W + 2D + 6L = 42$

$- \quad 2W + 2D + 10L = 26$

$4W - 4L = 16$

$W - L = 4$

$W = 4 + L$

$W = 4 + 1$

$W = 5$ A1

M1

$2W + 2D + 10L = 26$

$10 + 2D + 10 = 26$

$2D = 6$

$D = 3$ A1

5 points for a win, 3 for a draw and 1 for a loss

Item 8

$$\frac{\cos x}{\sin^2 x} - \cos x \cot^2 x = \frac{\cos x}{\sin^2 x} - \frac{\cos x \cdot \cos^2 x}{\sin^2 x} \quad B1$$

$$= \frac{\cos x}{\sin^2 x} - \frac{\cos^3 x}{\sin^2 x} \quad B1$$

$$= \frac{\cos x - \cos^3 x}{\sin^2 x} \quad B1$$

$$= \frac{\cos x\,(1 - \cos^2 x)}{\sin^2 x} \quad B1$$

$$= \frac{\cos x \ \sin^2 x}{\sin^2 x} \quad B1$$

$$= \cos x$$

$\therefore \frac{\cos x}{\sin^2 x} - \cos x \cot^2 x = \cos x$ and the same statement holds

Item 9

(a) $1 + \log_2(2x+y) = \dfrac{\log_3 12}{\log_3 2}$

$\log_2 2 + \log_2(2x+y) = \log_2 12$ B1 B1

$\log_2(4x+2y) = \log_2 12$ B1

$4x+2y=12$ B1

$2x+y=6$

$-\quad x+y=3$ M1

$x=3$ A1

$y=0$ A1

08

B) $K = \sqrt{28+10\sqrt{x}}$

Let $\sqrt{28+10\sqrt{3}} = \sqrt{a}+\sqrt{b}$ M1

$28+10\sqrt{3} = a+2\sqrt{ab}+b$

$a+b=28 \qquad 2\sqrt{ab}=10\sqrt{3}$

$a=28-b$ B1 $\qquad \sqrt{ab}=5\sqrt{3}$

$ab=75$ B1

$(28-b)b=75$

$b^2-28b+75=0$

07

$b^2-25b-3b+75=0$

$b(b-25)-3(b-25)=0$ M1

$b=3$ or $b=25$

$a=25$ or $a=3$ B1

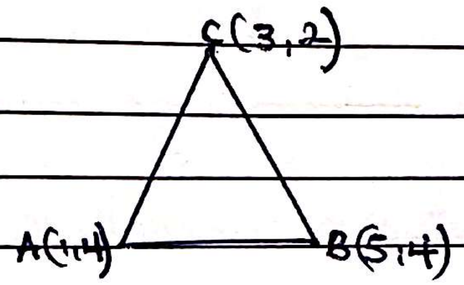
$K=5+\sqrt{3}$ B1

$17(K-\sqrt{3}) = 17(5+\sqrt{3}-\sqrt{3})$

85 points A1

Item 10

a)



Perpendicular bisector $\overline{AC}$

gradient $\overline{AC} = \frac{4-2}{1-3}$

$= -1$ B1

gradient of bisector $= 1$ B1

Midpoint $\overline{AC} = \left(\frac{3+1}{2}, \frac{4+2}{2}\right)$

$= (2, 3)$ B1

equation: $\frac{y-3}{x-2} = 1$

$y = x+1$ B1

Perpendicular bisector $\overline{BC}$

gradient $\overline{BC} = \frac{4-2}{5-3}$

$= 1$ B1

gradient of bisector $= -1$ B1

midpoint $\overline{BC} = \left(\frac{5+3}{2}, \frac{2+4}{2}\right)$

$= (4, 3)$ B1

Equation: $\frac{y-3}{x-4} = -1$

$y = -x+7$ B1

$\frac{10}{}$

$x+1 = -x+7$ M1

$2x = 6$ $x = 3$ and $y = 4$

The seat was located at $(3, 4)$ A1

b) Distance between $(3, 4)$ and $(7, 8)$

$D = \sqrt{(7-3)^2 + (8-4)^2}$ M1

$D = \sqrt{32}$

$D = 5.657$ units B1

Distance from $2x-5y-12=0$

$d = \left|\frac{(2\times3)+(-5\times4)-12}{\sqrt{2^2+(-5)^2}}\right|$ M1

$d = \left|\frac{-26}{\sqrt{29}}\right|$

$d = 4.828$ units B1

$\frac{05}{}$

The van was parked on the straight road given by $2x = 12 + 5y$ A1

Item 11

a) $x = 3\cos 2\theta$ $\quad$ $y = 8\sin\theta\cos\theta$

$y = 4\sin 2\theta$ B1

$\cos 2\theta = \frac{x}{3}$ B1 $\quad$ $\sin 2\theta = y/4$ B1

from $\cos^2 2\theta + \sin^2 2\theta = 1$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$ B1 B1 $\quad$ $\overline{05}$

$\therefore 16x^2 + 9y^2 = 144$

b) $H = 3\cos 2\theta + 4\sin 2\theta$

Let $3\cos 2\theta + 4\sin 2\theta = R\sin(2\theta + \alpha)$.

$3\cos 2\theta + 4\sin 2\theta = R\sin 2\theta\cos\alpha + R\cos 2\theta\sin\alpha$

$R\cos\alpha = 4$ B1 $\quad$ $R\sin\alpha = 3$ B1

$R^2 = 4^2 + 3^2$ $\quad$ $\tan\alpha = 3/4$

$R = 5$ B1

$\alpha = 36.87°$ B1

$\overline{10}$ $\quad$ $H = 5\sin(2\theta + 36.87°)$ B1

Maximum height $= 5$m A1

$\sin(2\theta + 36.87°) = 1$ B1

$2\theta + 36.87° = 90°, 450°, 810°$ B1

$2\theta = 53.13°, 413.13°, 773.13°$ B1

$\theta = 206.57°$

$\therefore$ Reflex angle of projection $= 206.57°$ A1

Item 12

Let $\frac{3x+1}{x(x-1)^2} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ M2

$\frac{3x+1}{x(x-1)^2} = \frac{A(x-1)^2 + Bx(x-1) + Cx}{x(x-1)^2}$ B1

$3x+1 \equiv A(x-1)^2 + Bx(x-1) + Cx$

Let $x = 0$

$1 = A$ B1

Let $x = 1$

$4 = C$ B1

Let $x = 2$

$7 = 1 + 2B + 8$

$2B = -2$

$B = -1$ B1

$\Rightarrow \frac{3x+1}{x(x-1)^2} = \frac{1}{x} - \frac{1}{x-1} + \frac{4}{(x-1)^2}$ A1A1A1

Andrew

$\frac{1}{4} \times 324$ B1

Ugshs 81millions A1

John

$\frac{4}{9} \times 324$ B1

ugshs. 144millions A1

charity

B1 $324 - (81 + 144)$

= Ugshs 99millions A1

Signature:..............................

Item 13

a) $y = x^2 - 3$

$y + \delta y = (x + \delta x)^2 - 3$

$\delta y = x^2 + 2x\delta x + (\delta x)^2 - 3 - (x^2 - 3)$ B1

$\delta y = 2x\delta x + (\delta x)^2$ B1

$\frac{\delta y}{\delta x} = 2x + \delta x$ B1

05

$\frac{dy}{dx} = \lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right)$ M1

$\frac{dy}{dx} = 2x$ A1

b) $dy/dx = 2x$

At $(1, -2)$

$\frac{dy}{dx} = 2$

$\frac{y+2}{x-1} = 2$ B1

$y + 2 = 2x - 2$

$y = 2x - 4$ A1

10

Road that is perpendicular

$m \times 2 = -1$

$m = -\frac{1}{2}$ B1

$\frac{y+2}{x-1} = -\frac{1}{2}$

$2y + 4 = -x + 1$

$2y = -x - 3$ A1

ⓒ $y = x^2 - 3$

$x = -3 - 2y$

$y = (-3 - 2y)^2 - 3$ M1

$y = 9 + 12y + 4y^2 - 3$

$4y^2 + 11y + 6 = 0$ B1

$y = \frac{-11 \pm \sqrt{11^2 - (4\times4\times6)}}{2\times4}$ M1

$y = -2$ or $y = -0.75$ B1

$x = -3 - (2x - 0.75)$

$x = -1.5$ B1

∴ The road meets the hill again at $(-1.5, -0.75)$ A1