

2/100

NAME SCORING GUIDE .....STREAM All.....

**MENGO SENIOR SCHOOL  
END OF TERM TWO ASSESSMENT 2025  
S.5 PURE MATHEMATICS (paper A)  
DURATION: 3 HOURS**

**Instructions:**

Answer **all** questions in section A and **only 4** in section B

Show your working clearly

Calculators and mathematical tables can be used

Each question in section A has a score of 5 and each in section B has a score of 15

**SECTION A (40 scores)**

**Item 1**

Timothy and Sharon are in a relationship but when he tried to ask her age, Sharon just wrote a simple note " add 18 to the value of  $x$  after solving the equation  $2^{2x} - 252(2^x) - 1024 = 0$ ". Two weeks have passed now and Timothy does not know Sharon's age. He is approaching you as a math student today, please rescue him.

**Item 2**

During installation of electric poles in a straight line ,the electrician remembers the two coordinates ( 1 , 11 ) and ( -3 , - 5 ) but in the third coordinate he doubts the  $y$ -coordinate so he has written ( 6 , w ). Help him obtain the value of w

**Item 3**

Real numbers have two square root values the same is with surds , verify this statement by finding the square root of  $7 + 4\sqrt{3}$

**Item 4**

During division of polynomials, it was found out that when  $ax^2 + 3x + b$  is divided by  $x - 1$  the remainder was 16 but when divided by  $x - 2$  the remainder is 34. Determine the values of a and b and the remainder when we divide the same polynomial by  $x + 3$

**Item 5**

John can differentiate  $y = 5x^2 - 13x + 9$  by rule or principle to obtain

$\frac{dy}{dx} = 10x - 13$  but he cannot do it from first principles, take him through to

obtain the same answer.

**Item 6**

We can confidently say that 4 is a perfect square of 2,  $a^2 + 2ab + b^2$  is a perfect square of  $a + b$ , what values of  $p$  and  $q$  make  $x^4 + 6x^3 + 13x^2 + px + q$  a perfect square?

#### Item 7

Daniel a S.3 student placed his calculator and obtained  $\sin 165^\circ$  as 0.258819045, now what troubles him is how this can be done without using tables or calculators and even the answer simplified but not in decimals. As an advanced mathematician guide Daniel.

#### Item 8

Root analysis is a major concept when studying quadratic equations. Without obtaining the roots themselves and stating a valid condition predict the type of roots in the following equations.

(i)  $2x^2 + 3x + 7 = 0$       (ii)  $x^2 - 7x = 12$       (iii)  $25x^2 + 4 = 20x$

### SECTION B (60 scores)

#### Item 9

Kiwesi is a S.5 geography student who analysed that the road from the school gate to his class is given by the equation  $3x - y = 9$  whereas that from his class to the staffroom is represented by the equation  $x + 2y + 4 = 0$ . He also found out that the school library is located at a coordinate  $(5, 1)$  and that the road joining the library to the head teacher's office is elevated at an angle of approximately  $45^\circ$  with the horizontal ground.

#### Tasks

As a mathematics student with the knowledge of geometry help Kiwesi to;

- determine the coordinate for the location of his class
- calculate the distance between the library and his class
- find the perpendicular distance of the library from the road joining his class to the staffroom
- obtain the equation that represents the road from the library to the head teacher's office

#### Item 10

Mr. Ongom and his wife usually make a monthly budget on the items to be used by all the members in their family. In the coming month of August, he wants to surprise the wife as he has already obtained an expression relating the loaves of bread and the pineapples as  $B = -3x^2 + 24x - 39$  where  $B$  represents the loaves of bread and  $x$  the number of pineapples. However the wife has also prepared something concerning the toothpaste, she is saying that the number of tubes of tooth paste raised to the power of 4 should be less than 4096. They also agreed in the previous months that the costs for a

loaf of bread and a pineapple can be expressed in terms of roots of quadratic equations whose sum is 6000 and product is 8000,000 keeping in mind that a loaf of bread is more expensive than a pineapple

### Tasks

- (a) Using the knowledge of roots of quadratic equations and completing squares, help them;
- Determine either the maximum or minimum number of loaves of bread and the corresponding number of pineapples
  - obtain the exact number of tubes of toothpaste to be used in August
  - remember the cost of a loaf of bread and a pineapple
  - know their total expenditure on bread and pineapples
- (b) With a reason, advise Mr. Ongom and the wife whether they should continue making the budgets or stop

### Item 11

Coffee is one of the main cash crops in Uganda, during the recent NRM party primaries in Masaka a contestant suggested that a land of  $946400 \text{ Km}^2$  can be used following the partial fractions of  $\frac{11x+12}{(2x+3)(x+2)(x-3)}$ . That the two positive partial fractions should be added and used for growing coffee and the remaining one has a negative sign meaning it is not for farming but house construction.

### Tasks

- Express the fraction above into its partial fractions
- Put  $x = 5$  and obtain the real fractions for growing coffee and house construction
- Determine the exact land in  $\text{Km}^2$  meant for growing coffee and house construction
- Obtain the remaining land in  $\text{Km}^2$  and with a reason suggest any activity that can be done on it

### Item 12

Alex and Francis are friends that always have daily arguments starting from the premier league teams they support that is, Chelsea and Arsenal respectively. Recently they were moving around school and entered a S.4 classroom where they found an expression on the chalkboard reading  $\sin 3A$ , immediately the argument started when Alex claimed that the handwriting looked familiar and Francis said it was a lie. They later moved to the next classroom of s.3 and found another expression on the white board reading  $3\sin A - 4\sin^3 A$ , another argument came in when Francis said that the two statements look different but mean the same

thing, “no way, it can not be , lets bet on it buddy”, were the statements of Alex.

As they were still arguing, Francis received a notification on his phone from BBC news showing that Iran was to launch a nuclear missile over Israel whose speed is given by the expression  $s = 3\cos 2\theta + \sin \theta$  where  $s$  is the speed in kilometres per hour( $Km/h$ ) and  $\theta$  the angle of projection

### Tasks

Using the knowledge of trigonometry help these two;

- without using calculators or tables ,settle the argument whether the statements mean the same thing or not
- determine the speed for the missile in  $Km/h$  if it is to cover 5Km in 300minutes
- calculate the possible angles of projecting the missile in the range  $0^\circ \leq \theta \leq 360^\circ$  if it is to move at the speed obtained in (b) above

### Item 13

RISU farm located in Wakiso is among the best producers of eggs and quality chicken in Uganda, its managing director Richard has already received orders from three clients for the next month as illustrated below

Event	Trays of eggs	Number of cocks	Number of hens	Amount(Ugshs)
Robert's birthday	5	3	6	405,000
Kizza's wedding	30	53	19	3,495,000
Joan's graduation	12	9	5	771,000

Another client Henry submitted an order on only cocks for his “kuhingira ceremony “, and he just wrote a mathematical statement to the director reading

$$\log_2 x - \log_x 8 = 2$$

Note that RISU farm sells a whole chicken not just part

### Tasks

- Make analysis to enable the clients know the costs of one tray of eggs, a cock and a hen
- Help director Richard to know the exact number of cocks that Henry wanted
- Compute the amount that Henry will pay to the director

**END**

***“No matter how tough the beginning was, there is always a comeback”***

S.5 PURE MATHEMATICS SCORE GUIDE

1)  $2^{2x} - 252(2^x) - 1024 = 0$   
 $(2^x)^2 - 252(2^x) - 1024 = 0$  ✓

Let  $2^x$  be  $m$

$m^2 - 252m - 1024 = 0$

$m = \frac{252 \pm \sqrt{(-252)^2 - 4(1024)}}{2 \times 1}$  ✓

$m_1 = -4, m_2 = 256$  ✓

⇒ Either

$2^x = -4$  (Invalid)

OR

$2^x = 256$

$2^x = 2^8$

∴  $x = 8$  ✓

Sharon's age

$= (18 + 8)$  year + 2 weeks ✓

∴ Sharon is now 26 years and 2 weeks old

05

Also consider 26 years

2)  $(1, 11), (-3, -5)$  and  $(6, w)$

$\frac{w+5}{6+3} = \frac{11+5}{1+3}$  ✓ ✓

$\frac{w+5}{9} = 4$  ✓

$w+5 = 36$  ✓

∴  $w = 31$  ✓

05

3) let  $\sqrt{7+4\sqrt{3}} = \pm(\sqrt{x} + \sqrt{y})$

$7+4\sqrt{3} = x+y+2\sqrt{xy}$  ✓

$x+y = 7$

$2\sqrt{xy} = 4\sqrt{3}, \Rightarrow x = \frac{12}{y}$  ✓

$\frac{12}{y} + y = 7$

$y$

$y^2 - 7y + 12 = 0$  ✓

$(y-3)(y-4) = 0$

$y = 3, y = 4$  ✓

$x = \frac{12}{3}, x = 3$

∴  $\sqrt{7+4\sqrt{3}} = \pm(\sqrt{3} + 2)$  ✓

05

4) let  $f(x) = ax^2 + 3x + b$

$f(1) = a + 3 + b = 16$

$a + b = 13 \dots \dots \textcircled{i} \checkmark$

$f(2) = 4a + 6 + b = 34$

$4a + b = 28 \dots \dots \textcircled{ii} \checkmark$

$\textcircled{ii} - \textcircled{i}$

$3a = 15$

$a = 5 \checkmark$

$b = 8$

$\Rightarrow f(x) = 5x^2 + 3x + 8$

when  $f(x)$  is divided by  $x + 3$

$f(-3) = 5(-3)^2 + 3(-3) + 8 \checkmark$

$f(-3) = 44$

$\therefore 44$  is the remainder  $\checkmark$

$\textcircled{05}$

5)  $y = 5x^2 - 13x + 9$

let  $\Delta y$  and  $\Delta x$  be small changes in  $y$  and  $x$  respectively.

$y + \Delta y = 5(x + \Delta x)^2 - 13(x + \Delta x) + 9 \checkmark$

$y + \Delta y = 5x^2 + 10x\Delta x + 5(\Delta x)^2 - 13x - 13\Delta x + 9 \checkmark$

$\Delta y = 5x^2 + 10x\Delta x + 5(\Delta x)^2 - 13x - 13\Delta x + 9 - 5x^2 + 13x - 9$

$\Delta y = 10x\Delta x + 5(\Delta x)^2 - 13\Delta x \checkmark$

$\frac{\Delta y}{\Delta x} = 10x + 5\Delta x - 13 \checkmark$

$\Delta x$

As  $\Delta x \rightarrow 0, \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$

$\textcircled{05}$

$\frac{dy}{dx} = 10x - 13 \checkmark$

6) let  $(x^2 + ax + b)^2 = x^4 + 6x^3 + 13x^2 + px + q \checkmark$

$x^4 + ax^3 + bx^2 + ax^3 + a^2x^2 + 2abx + b^2 =$

$x^4 + 6x^3 + 13x^2 + px + q$

$2ax^3 + (2b + a^2)x^2 + 2abx + b^2 = 6x^3 + 13x^2 + px + q$

$\Rightarrow 2a = 6, \quad a = 3$

$2b + a^2 = 13, \quad b = 2$

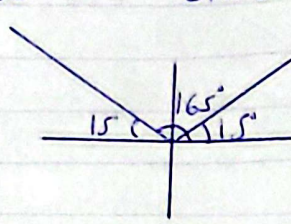
$2ab = p, \quad p = 12$

$b^2 = q, \quad q = 4$

$\therefore p = 12 \checkmark$  and  $q = 4 \checkmark$

$\textcircled{05}$

$$7) \sin 165^\circ = \sin 15^\circ = \sin(45^\circ - 30^\circ) \checkmark$$



$$\begin{aligned} \sin 165^\circ &= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \checkmark \\ &= \left(\frac{\sqrt{2}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right) \left(\frac{1}{2}\right) \checkmark \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \checkmark \end{aligned}$$

$$\sin 165^\circ = \frac{1}{4} (\sqrt{6} - \sqrt{2}) \checkmark$$

(05)

$$8) 2x^2 + 3x + 7 = 0, \quad x^2 - 7x = 12, \quad 25x^2 + 4 = 20x.$$

Using  $b^2 - 4ac$ ,

For  $2x^2 + 3x + 7 = 0$ ,

$$b^2 - 4ac = (3)^2 - 4(2)(7) = -47. \checkmark$$

Since  $b^2 - 4ac = -47$ ,  $\therefore$  the equation  $2x^2 + 3x + 7 = 0$  has complex roots.  $\checkmark$

For  $x^2 - 7x = 12$ ,

$$b^2 - 4ac = (-7)^2 - 4(1)(-12) = 97.$$

Since  $b^2 - 4ac = 97$  for  $x^2 - 7x = 12$ , therefore it has two real distinct roots.  $\checkmark$

For  $25x^2 + 4 = 20x$ ,

$$b^2 - 4ac = (-20)^2 - 4(25)(4) = 0 \checkmark$$

Since  $b^2 - 4ac = 0$ , the equation  $25x^2 + 4 = 20x$  has two equal real roots.  $\checkmark$

(05)

## SECTION B.

9) c)  $3x - y = 9$  ,  $x + 2y + 4 = 0$ .

$$y = 3x - 9 \quad \checkmark$$

$$\Rightarrow x + 2(3x - 9) + 4 = 0 \quad \checkmark$$

$$x + 6x - 18 + 4 = 0$$

$$7x = 14$$

$$x = 2 \quad \checkmark$$

$$y = 3(2) - 9 = -3 \quad \checkmark$$

$\therefore (2, -3)$  are the co-ordinates  $\checkmark$  for the location of the class.

b) Library  $(5, 1)$  Class  $(2, -3)$ .

$$\text{Distance} = \sqrt{(5-2)^2 + (1+3)^2} \text{ units} \quad \checkmark$$

$$= \sqrt{9 + 16} \text{ units} \quad \checkmark$$

$$\text{Distance} = 5 \text{ units} \quad \checkmark$$

c) let the distance be  $d$ .

$$d = \left| \frac{5(1) + 1(2) + 4}{\sqrt{1 + 4}} \right| \quad \checkmark$$

$$= \frac{10}{\sqrt{5}} \text{ units} \quad \checkmark$$

$$d = \frac{10\sqrt{5}}{5} \text{ units or } 4.9193 \text{ or } 4.92 \text{ units} \quad \checkmark$$

d)  $y = mx + c$ .

$$m = \tan 45^\circ = 1 \quad \checkmark$$

$$\frac{y-1}{x-5} = 1 \quad \checkmark$$

$$y-1 = x-5$$

$$y = x-4 \quad \checkmark$$

$$y = x - 4 \quad \checkmark$$

Item 10

a)  $B = -3x^2 + 24x - 39$   
 $B = -3(x^2 - 8x + 13)$  ✓

$$B = -3(x^2 - 8x + 16 + 13 - 16)$$

$$B = -3((x-4)^2 - 3)$$
 ✓

$$B = 9 - 3(x-4)^2$$
 ✓

∴ 9 is the maximum number of loaves of bread.

$$x-4 = 0, \quad x = 4$$
 ✓

4 pineapples is the corresponding number with 9 loaves of bread.

(05)

b) Let  $y$  be the number of tubes of toothpaste

$$y^4 < 4096$$
 ✓

$$y^4 - 4096 < 0$$
 ✓

Critical Values.

$$y^4 - 4096 = 0$$

$$(y^2)^2 - (64)^2 = 0$$

$$(y^2 - 64)(y^2 + 64) = 0$$

$$(y - 8)(y + 8)(y^2 + 64) = 0$$
 ✓

$$\therefore y = \pm 8, \quad y^2 = -64.$$

⇒ Analysis table.

	$y < -8$	$-8 < y < 8$	$y > 8$	
$y^4 - 4096$	+	-	+	✓

∴  $-8 < y < 8$  is the solution.

∴  $\{0, 1, 2, 3, 4, 5, 6, 7\}$  is the exact number of tubes of toothpaste to be used in August.

(05)

c) Let  $m$  and  $n$  be the cost of a loaf of bread and a pineapple respectively.

$$mn = 8,000,000$$
 ✓

$$m+n = 6000$$
 ✓

∴ The factors are 4000 and 2000 ✓

$$\Rightarrow m = 4000 \quad \text{and} \quad n = 2000$$

1v) Total Expenditure.

$$= (4000 \times 9) + 4(2000)$$

$$= 44000$$

(04)

b) Advise

Reason

(01)

Item II

$$a) \frac{11x+12}{(2x+3)(x+2)(x-3)} \equiv \frac{A}{2x+3} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$11x+12 \equiv A(x+2)(x-3) + B(2x+3)(x-3) + C(2x+3)(x+2)$$

let  $x = 3$ ,

$$11(3) + 12 = C(6+3)(5)$$

$$\frac{45}{45} = \frac{45C}{45}$$

$$\therefore C = 1$$

let  $x = -2$

$$11(-2) + 12 = B(-4+3)(-2-3)$$

$$\frac{-10}{5} = \frac{5B}{5}$$

$$\therefore B = -2$$

let  $x = -\frac{3}{2}$

$$11\left(-\frac{3}{2}\right) + 12 = A\left(-\frac{3}{2} + 2\right)\left(-\frac{3}{2} - 3\right)$$

$$\frac{-4.5}{-2.25} = \frac{-2.25A}{-2.25}$$

$$A = 2$$

$$\therefore \frac{11x+12}{(2x+3)(x+2)(x-3)} \equiv \frac{2}{2x+3} - \frac{2}{x+2} + \frac{1}{x-3}$$

b) If  $x = 5$ ,

$$\frac{2}{2x+3} = \frac{2}{13}, \quad \frac{-2}{x+2} = \frac{-2}{7} \quad \text{and} \quad \frac{1}{x-3} = \frac{1}{2}$$

Fraction for growing coffee.

$$= \frac{2}{13} + \frac{1}{2}$$

$$= \frac{17}{26} \quad \checkmark$$

(05)

Fraction for House construction

$$= \frac{2}{7} \quad \checkmark$$

c) Land for Growing coffee.

$$= \frac{17}{26} \times 946400 \text{ km}^2$$

$$= 618,800 \text{ km}^2 \quad \checkmark$$

Land for House Construction

$$= \frac{2}{7} \times 946400 \text{ km}^2$$

$$= 270,400 \text{ km}^2 \quad \checkmark$$

(02)

d) Remaining land.

$$= (946400 - 618800 - 270,400) \text{ km}^2 \quad \checkmark$$

$$= 57,200 \text{ km}^2$$

Activity - - -  $\checkmark$

Reason - - -  $\checkmark$

(02)

### Item 12

a) Required to prove that  $\sin 3A = 3\sin A - 4\sin^3 A$ .

From L.H.S

$$\text{L.H.S} = \sin 3A$$

$$= \sin(2A + A) \quad \checkmark$$

$$= \sin 2A \cos A + \sin A \cos 2A \quad \checkmark$$

$$= 2\cos A \sin A \cos A + \sin A(1 - 2\sin^2 A) \quad \checkmark$$

$$= 2\sin A \cos^2 A + \sin A - 2\sin^3 A \quad \checkmark$$

$$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A \quad \checkmark$$

$$= 3\sin A - 4\sin^3 A$$

(05)

$\therefore$  Since  $\sin 3A = 3\sin A - 4\sin^3 A$ , the two statements mean the same thing.

b) Distance = 5 km  
 Time = 300 min  
 =  $\frac{300}{60}$  hrs

Speed =  $\frac{5 \text{ km}}{5 \text{ hrs}}$  ✓

Speed = 1 km/h ✓  
 ∴ 1 km/h is the speed of the missile.

03

c)  $S = 3 \cos 2\alpha + \sin \alpha$

$3 \cos 2\alpha + \sin \alpha = 1$  ✓

$3(1 - 2\sin^2 \alpha) + \sin \alpha = 1$

$3 - 6\sin^2 \alpha + \sin \alpha = 1$

$6\sin^2 \alpha - \sin \alpha - 2 = 0$  ✓

$6\sin^2 \alpha - 4\sin \alpha + 3\sin \alpha - 2 = 0$

$2\sin \alpha (3\sin \alpha - 2) + 1(3\sin \alpha - 2) = 0$  ✓

$(3\sin \alpha - 2)(2\sin \alpha + 1) = 0$  ✓

Either

$\sin \alpha = \frac{2}{3}$  ✓

$\alpha = \sin^{-1}(\frac{2}{3})$  ✓

$\alpha = 41.81^\circ, 138.19^\circ$

$\alpha = \{41.81^\circ, 138.19^\circ, 210^\circ, 330^\circ\}$  ✓

OR

$\sin \alpha = -\frac{1}{2}$

$\alpha = \sin^{-1}(-\frac{1}{2})$  ✓

$\alpha = 210^\circ, 330^\circ$

07

### Item 13

a) Let  $x$ ,  $y$  and  $z$  represent the unit costs of the one tray of eggs, a cock and a hen respectively

$5x + 3y + 6z = 405,000$  --- (i) ✓

$30x + 53y + 19z = 3,495,000$  --- (ii) ✓

$12x + 9y + 5z = 771,000$  --- (iii) ✓

$-6 \mid 5x + 3y + 6z = 405,000$

$30x + 53y + 19z = 3,495,000$

~~$38y - 11z = 1,470,000$  --- (iv) ✓~~

$35y - 17z = 1,065,000$  --- (v) ✓

$$\begin{array}{r}
 12 \mid 5x + 3y + 6z = 405,000 \\
 -5 \mid 12x + 9y + 5z = 771,000 \\
 \hline
 \phantom{12 \mid} -9y + 47z = 1,005,000 \dots \textcircled{v} \\
 9 \mid 35y - 17z = 1,065,000 \\
 35 \mid -9y + 47z = 1,005,000 \\
 \hline
 \phantom{9 \mid} 1492z = 44,760,000 \checkmark \\
 \phantom{9 \mid} 1492z = 44,760,000 \\
 \phantom{9 \mid} 1492 \phantom{z} = 30,000 \checkmark
 \end{array}$$

$$\begin{aligned}
 y &= 47(30,000) - 1,005,000 \\
 &= 45000 \checkmark
 \end{aligned}$$

$$x = 405,000 - 3(45000) - 6(30,000)$$

$$x = 18000 \checkmark$$

$\therefore$  U<sub>g</sub> Sh. 18000, U<sub>g</sub> Sh. 45000 and U<sub>g</sub> Sh. 30,000 are the costs of one tray of eggs, a cock and a hen respectively.

$$b) \log_2 x - \log_2 8 = 2$$

$$\log_2 x - \frac{3}{\log_2 x} = 2 \checkmark$$

$$\text{let } m = \log_2 x$$

$$m - \frac{3}{m} = 2 \checkmark$$

$$m^2 - 2m - 3 = 0$$

$$(m-3)(m+1) = 0$$

$$m = 3, \quad m = -1$$

$$\log_2 x = 3, \quad x = 8 \checkmark$$

$$\log_2 x = -1, \quad x = \frac{1}{2} \neq 1$$

$\therefore x = 8 \checkmark \Rightarrow$  Henry bought 8 cocks

c) Amount for 8 coats

$$= 8 \times \text{UG Sh. } 45000 \checkmark$$

$$= \text{UG Sh. } 360,000 \checkmark$$

$\therefore$  Henry will pay UG Sh. 360,000 to the director. (02)