Section A (40 Marks)

*Answer **all** the questions in this section.*

Qn 1: A group of ten students are to sit around a circular table. Among these students, there are two students who should not sit side by side. Find the possible number of different ways in which the group of students can sit so as to achieve the given condition/restriction. **[5 Marks]**

Qn 2: Show that the equation of the line through the points $(1, 2, 1)$ and $(4, -2, 2)$ is given as $\frac{x-1}{3} = \frac{y-2}{-4} = z - 1$. **[5 Marks]**

Qn 3: Differentiate $\frac{1}{2x^2-3}$ from first principles; with respect x.

[5 Marks]

Qn 4: (i). Find the locus in the complex plane given by $|z - i| = 2$.

(ii). By shading the unwanted region, show in a diagram the region given by $|z - i| < 2$.

[5 Marks]

Qn 5: Solve the equation $2\sin^2 x + \sin^2 2x = 2$ for $-180° \leq x \leq 180°$. **[5 Marks]**

Qn 6: The height, h, of a right-angled triangle increases at a constant rate of 0.05 m per hour while keeping the base constant at 5 metres. Find the rate of change of the angle opposite the height, θ when $\theta = \frac{\pi}{3}$. **[5 Marks]**

Qn 7: Show that the equation $y^2 - 4y = 4x$ represents a parabola and hence determine the coordinates of its vertex and focus.

[5 Marks]

Qn 8: The rate of change of the temperature, θ°C of a mug of coffee is given by $\frac{d\theta}{dt} = \frac{1}{25}t - k$; where t is the elapsed time, in minutes, after the coffee is poured into the mug and k is a constant. Initially, the temperature of the coffee is 100°C. 10 minutes later, the temperature has fallen to 82°C. Express θ in terms of t.

0.05 m/h

[5 Marks]

$\frac{2 \times t^2}{25}$

$\frac{d\theta}{dt} = \frac{1}{25}t$

Section B (60 Marks)

Answer any **five** *questions from this section.*
All *questions carry equal marks.*

Question 9:

(a). Solve for m: $4^{2m+1} - 2(4^{m+2}) + 48 = 0$.

$(4^m)^2 \cdot 4^1 - 2(4^m) \cdot 4^2 + 48$

[5 Marks]

(b). If $(x-5)^2$ is a factor of $x^3 - 2ax^2 + 3bx - 200$, find the values of a and b.

$4m - 2m$

[7 Marks]

Question 10:

The points A and B have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively.
Point C is along OA produced such that $AC = 2OA$.
Point D is along OB produced such that $BD = \boldsymbol{b}$.
Point E is such that $DE = 3\boldsymbol{a}$.
Point F divide AB in the ratio $2:3$.

(a). Express each of the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(i). DC.

(ii). CF.

[6 Marks]

(b). Show that O, F and E are collinear. **[6 Marks]**

Question 11:

(a). Express $\frac{x^4-7x^3+20x^2-15x-50}{(x-1)(x-2)(x-4)}$ in partial fractions.

Hence evaluate $\int_5^7 \frac{x^4-7x^3+20x^2-15x-50}{(x-1)(x-2)(x-4)}\,dx$, correct to 3 decimal places.

[12 Marks]

Question 12:

(a). (i). Use binomial theorem to expand $(1+4x)^{\frac{1}{4}}$ up to term in x^3.

(ii). State the range of values of x within which the expansion is valid.

(iii). Using the expansion in (i) above, find $82^{\frac{1}{4}}$ and give your answer to 4 decimal places. **[7 Marks]**

(b). Find the coefficient of the term x^6 in the expansion of $\left(-3x + \frac{2}{x}\right)^{12}$.

[5 Marks]

Question 13:

(a). An arc AB subtends an angle of $\frac{2\pi}{3}$ radians at the centre of a circle of radius, R.
Show that $\frac{\text{Length of arc } AB}{\text{Length of chord } AB} = \frac{2\pi}{3\sqrt{3}}$.

[5 Marks]

(b). Given that ABC is a triangle, prove that $\frac{b^2-a^2}{2c^2} = \frac{1-\tan A \cot B}{2(1+\tan A \cot B)}$.

[7 Marks]

Question 14:

(a). Show that $\int_{-\frac{1}{2}}^{1} \frac{3x-2}{2x^2+2x+5} dx$ **[7 Marks]**

(b). Find the area enclosed by the curve $y = \operatorname{cosec}\left(\frac{x}{2}\right)$, the x-axis, and the lines $x - \pi = 0$ and $3x - 4\pi = 0$. **[5 Marks]**

Question 15:

A triangle ABC has vertices $A(0,-2)$, $B(4,0)$ and $C(h,k)$. Part of the line with equation $4x - 3y = 6$ forms side AC while part of the line with equation $3x + 4y = 12$ forms side CB of the triangle.

Determine

(a). the value of h and k.

(b). the shortest distance of vertex C from the line segment joining vertices A and B.

(c). the equations of the bisectors of the angles between the lines which were used to form sides AC and CB.

[12 Marks]

Question 16:

If x is real and $y = \frac{5x^2+8x+4}{x^2+x}$,

(i). Prove that y cannot lie between -4 and 4.

(ii). Find the coordinates of the turning points on the graph of $y = \frac{5x^2+8x+4}{x^2+x}$ and the coordinates of the point where the graph crosses its horizontal asymptote.

(iii). Sketch the graph, showing all the necessary points, given that it has no intercepts.

[12 Marks]

END