

**INSTRUCTIONS:**

1. Attempt four items in all.
2. Show your working clearly.
3. Make clear conclusions based on the solutions obtained.

**ITEM 1**

Mr. Mwasame is studying the relationship between the population of two bacterial species X and Y in a controlled environment. The population of this bacteria denoted as x and y follows the logarithmic relationships;

- i) The growth rate equation for species X is

$$\log_4(6 - x) = \log_2 x$$

**Task**

As a student of pure mathematics, help Mr. Mwasame to determine the value of x.

- ii) The interaction equation between X and Y are

$$\log_{10} x - \log_{10} y = \log_{10} 25 \text{ and}$$

$$\log_{10} x + \log_{10} y = 1$$

**Task**

Help Mr. Mwasame to determine the population values X and Y that satisfy these equations.

**ITEM 2**

- a) Kira motors is manufacturing electric vehicles. The engineers need to calculate exact dimensions for specific components to ensure proper fitment. The chief engineer has given you measurements for a triangular mounting bracket. For a battery bracket involving surds. The diagonal support brace of the bracket needs to be exactly  $7(\sqrt{50} + 3\sqrt{98} - 5\sqrt{72})$  long.

- b) Two surds of the triangle are;

$A = (6 + 3\sqrt{5})$  and  $b = (9 - 2\sqrt{5})$  cm long while the third side is d cm long.

*fold*

- c) The engineer was tools that you are a good mathematician with deep understanding of surds. Logarithms and indices. He wishes to solve for the unknown variable x from these models.

Model one  $\sqrt{x + 5} + \sqrt{x + 2} = \sqrt{6x + 40}$

Model two  $2^{2(x+1)} - 5(2^x) + 1 = 0$

Model three  $\log_3 x + 6 \log_x 3 = 5$

### Task

Help the engineer;

- Express the diagonal support length in the form  $\propto \sqrt{2}$ . Hence state the value of  $\alpha$ .
- Find the length of the third side of the triangle, if the perimeter of the entire triangle is 25cm. Hence find the value of  $\frac{a}{b} + \frac{a}{d}$  in a simplified form.
- Solve for  $x$  in each of the three models to make more fits for the best electronic vehicles.

### ITEM 3

MTN Uganda has been following its customers over the past time years. The number has been growing following a geometrical progression. The company had 850,000 customers in 2020 and has experienced a constant annual growth. You are a data analyst at MTN Uganda tasked to analyse this growth to help the company plan for the future. In 2023 the number of customers had grown to 1,309,500.

### Task

- Determine the annual growth rate of the MTN customers. Write your answer as a percentage correct to 2 decimal places.
- Predict the number of customers in 2025.
- How many years will it take for the number of customers to reach 5 million?

### ITEM 4

An engineer is testing the stresses tolerance of a new composite beam. The beam is deflection  $D$  in millimetres under the load is modelled by the polynomial  $D(x) = x^3 - 6x^2 + 11x - 6$ , where  $x$  is the load in tons. The beam fails if deflection exceeds 10mm. the engineer needs to find the load values where deflection is zero to understand the beams fundamental behaviours before testing its limits.

### Task

Show that  $x=1$  is a root of the polynomial  $D(x)$ .

Factorise the polynomial  $D(x)$  completely.

Based on the roots, between what load values does the deflection change direction.

### ITEM 5

Draw a circle of a given radius  $r$  having its centre at the origin of the cartesian plane. A point  $P(x,y)$  lies on the circumference of the circle in the first quadrant of the plane. If the distance of point  $P(x,y)$  from the origin is  $r$ , and the perpendicular distance from the  $y$ -axis is  $x$  and the perpendicular distance from the  $x$ -axis is  $y$ .

#### Task

a) Using a pythagoras theorem, show that

i)  $\cos^2 \theta + \sin^2 \theta = 1$

ii)  $1 + \tan^2 \theta = \sec^2 \theta$

iii)  $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$

b) If  $a \sin \theta = p - b \cos \theta$  and  $b \sin \theta = q + a \cos \theta$  show that  $p^2 + q^2 = a^2 + b^2$ .

c) Solve the equation

$$3 \tan^2 \theta + 5 = 7 \sec \theta$$

**END**