PURE MATHEMATICS

REVISION ITEMS

SET 2

DEADLINE: 3 DAYS FROM TODAY.

Section A.

Attempt ALL ITEMS in this section.

Item 1.

During preparation for the annual Mathematics Challenge at a secondary school, the organizing committee decided to reward participants using a structured token system. The number of tokens awarded to a participant follows an arithmetic progression. It was recorded that the 10th participant received 29 tokens, while the 15th participant received 44 tokens. The committee plans to reward the first 60 participants using the same pattern.

Task: Find the number of tokens awarded to the first participant. Hence determine the total number of tokens required to reward the first 60 participants.

Item 2.

An engineer designing a parabolic metallic arch for a modern bridge models the shape of the arch using the equation $y = (x + 1)(2x + 3)$. For structural reinforcement, a supporting beam is to be fixed at the point (2,21). To determine the direction of the support and the line perpendicular to it for stability analysis, the engineer needs the equations of the tangent and the normal at that point.

Task: Help this engineer determine the equation of the tangent and the normal to the curve at the given point.

Item 3.

A software developer at a financial technology company models a profit function using the polynomial $2x^3 + 6x^2 + qx - 5$**.** During system testing, it was discovered that whenever the input value is $x = -2$, the output becomes zero. This confirms that $x + 2$ is a factor of the polynomial. For further system calibration, the developer now wants to know what value remains when the same expression is divided by $2x - 1$.

Task: Help this developer determine what remains when the expression is divided by $2x - 1$.

Item 4.

During a military navigation training exercise in Gulu, Lieutenant Okumu and Sergeant Achan are analysing two angles of elevation used in a coordinated signalling system. The first angle α and the second angle β are both acute. From their calculations, they established

that $\sin\alpha = \frac{7}{25}$ and $\cos\beta = \frac{5}{13}$. For accurate alignment of their signalling equipment, they need to determine the combined effect of the two angles.

Task: Without using tables or a calculator, help them determine;

i) $\sin(\alpha + \beta)$
ii) $\tan(\alpha + \beta)$

Item 5.

A civil engineer, Eng. Tumwine, is designing a three-component structural support system for a high-rise building in Kampala. The magnitudes of the three internal forces acting along the supports are represented by x, y, and z. After analysing equilibrium conditions at a joint, he obtained the following system of equations: $x - 2y - 2z = 0$, $2x + 3y + z = 1$ and $3x - y - 3z = 3$. To ensure the structure remains stable, he must determine the exact values of the three forces.

Task: Help Eng. Tumwine determine the values of x, y, and z.

Item 6.

At a publishing company, a special bonus scheme is introduced for editors working on a long-term mathematics textbook project. The bonuses are paid in a structured pattern that follows an arithmetic progression. The first editor in the sequence receives 2 units of bonus, and each subsequent editor receives an additional $\frac{2}{3}$ units more than the previous one. At the end of the project, management observes an interesting fact: the total bonus paid to the last four editors is 72 units more than the total bonus paid to the first four editors.

Task: Find the number of editors, n, involved in the project.

Item 7.

During a mathematics discussion, Namusoke and Ocen were analysing a trigonometric expression that appeared in their lecture notes. They encountered $\frac{\cos A+\cos B}{\sin A+\sin B}$ and their teacher asserted that it is equivalent to $\cot\left(\frac{A+B}{2}\right)$**.** However, a disagreement arose as Ocen was not convinced that the two expressions are indeed identical.

Task: Help them resolve this disagreement by establishing whether the two expressions are equivalent.

Item 8.

A senior six student, Kato, was revising logarithmic equations in preparation for his final examinations. While solving one of the problems in his revision booklet, he encountered the equation $\log_3 x + \log_x 3 = \frac{10}{3}$**.** He attempted several approaches but failed to obtain a solution and is now uncertain about the correct method to apply.

Task: Help Kato determine the value of x.

Item 9.

An engineering student, Wanyama, was analysing oscillatory motion in a physics experiment. The displacement of a particle was modelled by the function $y = x \sin x$. To estimate the total displacement over the interval from 0 to π, he was instructed to use the trapezium rule with equal strips of width $\frac{\pi}{6}$. He was also required to present the final result correct to 4 significant figures. However, he was unsure how to organise the computation systematically.

Task: Help Wanyama determine the required approximation.

Item 10.

During a revision session at King's College Budo, a student defined a function by $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$. He was required to show that its derivative simplifies to $\frac{1}{1-\sin x}$. However, he encountered difficulty in applying the appropriate differentiation techniques and simplifications.

Task: Show that the stated derivative is correct.

Item 11.

A technology company is testing a new digital reward system where performance credits grow in a geometric pattern. The number of credits awarded in successive stages forms a geometric progression. During analysis, the data team observes that:

- The difference between the credits awarded at the 5th stage and the 2nd stage is 156.
- The difference between the credits awarded at the 7th stage and the 4th stage is 1404.

The company now wants to determine the growth rate of the reward system.

Task: Find the possible values of the common ratio of the geometric progression.

Item 12.

At a telecommunications company, an engineer is testing signal cables for a large installation project. One cable measuring 10 metres is to be cut into ten pieces whose lengths form a geometric progression, so that each piece is a constant multiple of the previous one. For efficiency in installation, it is required that the longest piece be 8 times as long as the shortest piece. The engineer now needs to determine the exact size of one of the intermediate sections for proper labelling.

Task: Calculate, to the nearest centimetre, the length of the third piece.

Section B.

Attempt ALL ITEMS from this section.

Item 13.

During a mathematics seminar at Mbarara High School, two teachers, Mr. Mugisha and Ms. Atwine, were discussing multiple-angle trigonometric identities. Mr. Mugisha stated that if $t = \tan\theta$ then the expression for $\tan 4\theta$ can be written as $\tan 4\theta = \frac{4t(1-t^2)}{t^4-6t^2+1}$. Ms. Atwine agreed but insisted that the result must be derived systematically from known identities.

Later in the same discussion, they encountered the equation $\sin x + \sin 5x = \sin 2x + \sin 4x$ where $0° < x < 180°$, and they wanted to determine all possible values of x within the given interval.

Task: Help them justify the stated identity and determine the possible values of x.

Item 14.

During advanced algebra revision at St. Mary's College Kisubi, two students, Aine and Okello, were analysing a quadratic equation whose roots are expressed in terms of squares of unknown positive numbers. They discovered that α^2 and β^2 are the roots of the equation $x^2 - 21x + 4 = 0$ where α and β are both positive real numbers. For further study, they needed to determine the vale of certain constants, but they were unsure how to proceed systematically from the given quadratic equation.

Tasks:

Help Aine and Okello determine;

a) The value of;
   i) $\alpha - \beta$
   ii) $\frac{1}{\alpha} + \frac{1}{\beta}$
b) The equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

Item 15.

A laboratory technician in Entebbe, Dr. Namara, was recording two experimental measurements, A and B. Due to instrument limitations, the true values were rounded off to a and b with rounding errors e_1 and e_2 respectively. During analysis, it was established that when two measured quantities are multiplied, the absolute relative error in the product AB is given by $\frac{|a||e_2|+|b||e_1|}{ab}$. Later in the same experiment, the recorded measurements were A = 6.43 and B = 37.2, each rounded to the number of decimal places shown. She was uncertain about the correct procedure for handling propagated errors.

Tasks:

Help Dr. Namara;

a) Show that the absolute error in AB is given by the identity as stated.
b) Determine the maximum possible errors in A and B,

c) Calculate the absolute error in the product AB,
d) Establish the limits within which the product AB lies, correct to 4 decimal places.

Item 16.

An agricultural economist in Lira, Mr. Odongo, was modelling crop yield using the equation $3x^3 + x - 5 = 0$**.** During analysis, he suspected that the equation had a real root within the interval $1 < x < \frac{3}{2}$, but he needed mathematical justification before proceeding further. After confirming the existence of the root in that interval, he intended to do some estimations. However, he was uncertain about the correct step-by-step procedure to follow.

Tasks:

Help Mr. Odongo;

a) Show that the real root of the given equation lie within the interval stated.
b) Use linear interpolation to obtain an approximate value of the root, and
c) Apply Newton Raphson's formula to refine the approximation by performing two iterations, giving the final answer correct to two decimal places.

Item 17.

During a national mathematics contest training camp in Jinja, three finalists; Achan, Kato and Ssemanda were given a set of advanced trigonometry challenges. They were first required to analyse the expression $6\sin x - 3\cos x$ in order to determine its greatest possible value. They were then presented with the expression $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$ which was claimed to be equal to $\tan 56°$, and they were asked to justify the claim. Finally, in a triangle ABC, they were required to establish that $\sin B + \sin C - \sin A = \frac{1}{4}\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$**.** Although they attempted the problems, they could not complete the arguments satisfactorily.

Tasks:

Help them;

a) Determine the required maximum value.
b) Establish the stated trigonometric relationship.
c) Prove the given result in triangle ABC.

Item 18.

During a calculus lesson at Ntare School, Ms. Kemigisa introduced her class to differentiation of composite and quotient trigonometric functions. She defined two functions: $f(x) = sin^2 x \cos 2x$ and $g(x) = \frac{\sin x}{x}$. The students were required to apply appropriate differentiation techniques to analyse the behaviour of these functions. However, several learners were unsure which rules should be applied at each stage.

Tasks:

Help them;

a) Differentiate $f(x)$ with respect to x.
b) Differentiate $g(x)$ with respect to x.

Item 19.

During preparation for a national mathematics quiz competition, three finalists; Namusoke, Okello and Tumuhimbise were given rational algebraic models arising from different applied contexts. The first model represented the response of a mechanical system and was given by $\frac{6}{(x+3)(x-3)}$. The second model described a simplified electrical transfer function: $\frac{x}{(2+x)(2-x)}$. The third model arose from an economic growth adjustment equation: $\frac{x-1}{3x^2-11x+10}$. To analyse the behaviour of each model separately, they were required to decompose the expressions into simpler rational components.

Task:

Help them express all the three model equations into simpler rational components.

Item 20.

A cultural events organiser is designing a rotating stage platform for performances along the shores of Lake Victoria. The brightness of a decorative floodlight projected onto the stage depends on the rotation angle θ (in degrees) of the stage platform. Due to the combined effect of two light beams, the brightness level is modelled by $B(\theta) = 7\cos 2\theta + 6\sin 2\theta$.

From technical studies carried out by engineers, any expression of the form $a\cos 2\theta + b\sin 2\theta$ can be written as $R\cos(2\theta - \alpha)$ where R is a constant and α is an acute angle. For safety and mechanical control, the rotation angle of the stage must satisfy $0° \leq \theta \leq 180°$. For the performance to proceed comfortably, the brightness level must be exactly 5 units, while the organisers are also interested in knowing the highest possible brightness the system can produce.

Tasks:

a) Determine whether the given model can be written in the form $R\cos(2\theta - \alpha)$ and find the values of R and α.

b) Find the possible rotation angles of the stage for which the brightness level is 5 units.

c) Determine:

   i) The maximum possible brightness level.

   ii) The rotation angle(s) at which this maximum brightness occurs.

Item 21.

At a carpentry workshop in Mbarara, two different decorative wooden panels are being designed for the reception area of a hotel. The edges of the panels are shaped using computer-controlled cutting machines, and each edge follows a mathematical model. The first panel has its edge modelled by $y = x^3 - x^2 - 5x + 6$ while the second panel has its edge

modelled by $y = x^4 + 2x^3$. Before production begins, the designer needs to identify the exact points where each curve changes direction, in order to smoothen sharp bends and ensure structural stability.

Task: For each curve, find the coordinates of the turning points and distinguish between their nature.

Item 22.

At a roadside sign-post workshop in Lira, a metal fabricator is designing two different arch frames for advertising boards. To ensure accuracy before cutting the metal sheets, the shapes of the upper edges are first modelled using mathematical curves on a coordinate grid. The first arch is modelled by $y = 4x - x^2$ while the second arch is modelled by $f(x) = x^2 + 4x + 3$**.** The fabricator needs clear sketches of both curves in order to identify their key features before fabrication begins.

Task: Sketch each of the following curves, clearly showing all important features:

Item 23.

During preparations for the annual cultural gala at a secondary school in Gulu, a group of students designed a rotating spotlight to shine on performers as they move on stage. The brightness of the light at any instant depends on the angle of rotation. In their design notes, the students discovered that when the rotation angle is A (where A is acute), the brightness after three rotations can be expressed as $\sin 3A = 3\sin A - 4sin^3 A$. Before presenting their project to the judges, they must confirm that this relationship is mathematically correct.

At the same time, the school carpenter who helped mount the rotating mirror wants to fix it at an angle θ, where $0 < \theta < 2\pi$, such that the brightness after three rotations is exactly the same as after one rotation. From the design condition, this happens when $\sin 3\theta = \sin\theta$. He needs to determine all the possible angles that satisfy this condition before tightening the fittings.

Tasks:

a) Help these students by clearly showing them that the two equations are equal if on given conditions.
b) Advise the school carpenter on the angles he should tilt the mirror to obtain the required level of brightness.

Item 24.

A research team is comparing two different growth models used in forecasting production output. The first model increases steadily each week and follows an arithmetic progression with a common difference of 3 units. The second model grows exponentially and follows a geometric progression with a common ratio of 2. To analyse the difference between the two models, a new sequence is formed by subtracting each term of the arithmetic progression from the corresponding term of the geometric progression. It is observed that:

- The third term of the new sequence is 4.
- The sixth term of the new sequence is 79.

The team now wants to determine the starting values of the two original models.

Task:

a) Find the first term of the arithmetic progression.
b) Find the first term of the geometric progression.

Item 25.

At a savings and investment seminar, a financial advisor presents two related income plans to a group of young entrepreneurs. Under the first plan, a client deposits money monthly in a pattern that increases steadily and forms an arithmetic progression. The first deposit is 9 thousand shillings. After analysing the structure of the plan, the advisor notices an interesting relationship: the amounts deposited in the 1st month, the 4th month and the 8th month themselves form a geometric progression. To help the entrepreneurs understand how the plans compare over time, the advisor asks them to determine the characteristics of both patterns and compare their early totals.

Task:

a) Find the common difference of the arithmetic progression.
b) Find the common ratio of the geometric progression formed by the 1st, 4th and 8th deposits.
c) Find the difference between the sums of the first 6 terms of the arithmetic progression and the geometric progression.

END

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