SENIOR FIVE HeLP MATHEMATICS SEMINAR TO BE HELD ON SATURDAY 21ST AUGUST .2021 AT 10:00AM

INSTRUCTIONS TO STUDENTS AND TEACHERS:

Dear students and teachers we would like to welcome you to participate in the forthcoming mathematics seminar for senior five at 10:00a.m as they revise their mathematics concepts. The seminar is organized by teachers under the Holistic eLearning programme and seminar questions can be downloaded plus all the reading materials for the week on our zero rated platform (No data is needed to access the platform) https://help.sc.ug .This is a free seminar and no one should charge you any fees. The process to be followed by both the teachers and students is suggested below:

1. Teachers share the Seminar questions with their students and ask for volunteers to discuss any of the questions.
2. Teachers talk to the parents to allow the children participate as presenters in the seminar on Saturday **21st August from** $10:00-1:00$**pm.** Other students will just be participants.
3. If your student is going to present then as the teacher(s) prepare her/him by looking through the calculations made by the student. Then encourage the student to write out the solution neatly in black pen including any graph. Then they scan or take a picture and send to the teacher. They can also type out the solution in a word or PowerPoint document and share with the teacher. .
4. The teacher could now train the student on how to present in zoom as far as sharing a screen and using the whiteboard. Alternatively the students' presentation will be loaded on the computer screen and they explain to us their solution.
5. When we receive all the solutions before the seminar day we shall build an online course for the students' work that all other students will access and download the solutions at www.help.sc.ug. This is our home as an initiative by teachers.
6. The teacher or student will hand in the solutions to Ronald Ddungu (0701433878(W)) or Kaziba Stephen (0787698238(W)) by Thursday **19th August 2021.**The process here is very important

Holistic eLearning Platform is inviting you to a scheduled Zoom meeting.
Topic: SENIOR FIVE MATHEMATICS SEMINAR
Time:
SENIOR FIVE: AUGUST 21, 2021 10:00 AM
Join Zoom Meeting
https://us02web.zoom.us/j/3143746044?pwd=MGlKcUpDc3FnRTZUSnBjdEZFNFRPUT09
Meeting ID:314 374 6044
Passcode: HeLP

S 5 PURE MATHEMATICS SEMINAR QUESTIONS 2021

1. Solve the simultaneous equations: $$\begin{aligned} x + 3y - 3z &= -4 \\ 3x - y + 2z &= 1 \\ -2x + y + z &= 7 \end{aligned}$$

2. Solve the equations: $\frac{x-y}{2} = \frac{3y-z}{3} = \frac{2x-z}{5}$ and $3x + 2y - 2z = 10$.

3. Solve: $\frac{24}{x^2+5x} + 10 = -5x - x^2$

4. Use the substitution $y = (x^2 - 2x)$ to solve the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$.

5. Express $1 - 5x - 2x^2$ in the form $a - b(x+c)^2$, hence, deduce the maximum value of the expression and state the value of x.

6. Solve the equation: $\sqrt{(y+6)} - \sqrt{(y+3)} = \sqrt{(2y+5)}$

7. Rationalise: $\frac{1+\sqrt{3}}{(\sqrt{3}-1)^2}$

8. Solve the equation: $\log_3 n - 2\log_n 3 = 1$

9. Given that α and β are the roots of the equation $2x^2 - 3x + 4 = 0$, form an equation whose roots are $\frac{1}{5\alpha+\beta}$ and $\frac{1}{5\beta+\alpha}$.

10. When a polynomial $f(x)$ is divided by $(x-2)$ the reminder is -2, and when it is divided by $(x-3)$ the remainder is 3. Find the remainder when $f(x)$ is divided by $(x-2)(x-3)$.

11. Determine x if $(2x+1), (x^2+x+1), (3x^2-3x+3)$ are consecutive terms of an A.P.

12. An A.P has a common difference of 3 and the G.P has a common ratio of 2. A third sequence is formed by subtracting the terms of the A.P from the corresponding terms of the G.P. If the third term of the new sequence is 4 and the sixth term is 79, find the original A.P and G.P.

13. Find the coefficient of x^2 in the expansion of $(2-3x+x^2)(1+2x)^4$.

14. Find the value of n for which the coefficient of x, x^2 and x^3 in the binomial expansion of $(1+x)^n$ are in an Arithmetic progression.

15. Find the equation of the tangent at $(2,\ 1)$ to the curve $y^2+3xy=2x^2-1$.

16. Find the equation of the tangent at the point $(1,\ -1)$ to the curve $y=2-4x^2+x^3$. What are the coordinates of the point where the tangent meets the curve again?

17. Differentiate: $y=\dfrac{2+x}{\sqrt{2-x}}$ simplifying your answer.

18. Differentiate from first principles: $y=x^2+5$

19. The profit y generated from the sale is given by the function $y=600x+15x^2-x^3$. Calculate the value of x which will give maximum profit and determine the maximum profit.

20. A closed right circular cylinder has a volume of $54\pi\ cm^3$. Find the value of the radius and the height which will make the surface area a minimum.

21. Solve the equation $3\sec^2\frac{x}{2} = \tan\frac{x}{2} + 5$ for $0^0 \leq x \leq 360^0$.

22. Solve the equation: $4\sin^2 x + 8\cot^2 x = 5\operatorname{cosec}^2 x$ for $0 \leq x \leq 360^o$.

23. Without using tables or calculators, show that $\tan(-15^o) = -2 + \sqrt{3}$.

24. Given that $\sin(x+\alpha) = 2\cos(x-\alpha)$, prove that $\cot x = \dfrac{\cot\alpha - 2}{2\cot\alpha - 1}$.

STATISTICS

1. (a) Find the lower and upper quartiles of the set of numbers $10, 12, 13, 15, 19, 19, 24, 26, 26$

   (b) The table below shows the heights(cm) of 400 students in a certain school

Height(cm)	Number of students
$100 \le x \le 110$	27
$110 \le x \le 120$	85
$120 \le x \le 130$	215
$130 \le x \le 140$	320
$140 \le x \le 150$	370
$150 \le x \le 160$	395
$160 \le x \le 170$	400

   (i) Calculate the mean

   (ii) Draw a cummulative frequency curve and From the curve estimate the

   (a) median

   (b) Quartile deviation

   (c) 10^{th}percentile and 90^{th}percentile

2. (a) In an experiment to compare two methods of rearing real calves eight pairs of identical twins were used, one twin of each pair being allocated at random of each method of rearing. At the end of the experiments the calves were slaughtered and sample joints were cooked and scored for palatability with the following results:

Twin pair	1	2	3	4	5	6	7	8
Method A	27	37	31	38	29	35	41	37
Method B	23	28	30	32	27	29	36	31

   Do the methods differ on the palatability score?

   (b) Two locally developed antiviral drugs to cure COVID-19 were used to treat 12 symptomatic people in rotation of days and the results for the effectiveness of the herbal drugs was recorded in percentage as follows.

Covylice-1(x)	58	52	48	30	48	20	32	50	38	12	36	12
Covidex(y)	90	72	60	38	70	35	33	64	48	24	50	18

   (i) Plot a scatter diagram for the data .Comment on the relationship between the two drugs

   (ii) Draw a line of best fit for the scatter diagram ,hence find x when $y = 68$

(iii) Calculate the rank correlation coefficient for the effectiveness of the drugs .Comment on your result at 1% level of significance

3. The cumulative distribution of the ages (in years) of the employees of Uganda airlines is given in the table below:

Age	< 15	< 20	< 30	< 40	< 50	< 60	< 65	< 100
Cumulative frequency	0	17	39	69	87	92	98	98

(a) Find the:

(i) Mean and median age.

(ii) Middle 70% age range.

(b) Represent the above information on a histogram and use it to estimate the modal age.

(c) A certain frequency distribution with standard deviation 2.5 has the following results :$\sum f = m$,$\sum fx = 177$ and $\sum fx^2 = 5259$ .Find the value of m

4. (a) The information below shows the grades scored by a group of students in biology and mathematics examinations

Student	1	2	3	4	5	6	7	8	9
Math	A	E	F	A	B	B	C	D	B
Biology	C	0	E	C	C	B	A	F	D

Compute the rank correlation coefficient for the perfomance between the two subjects .Comment on your result at 1% level of significance

(b) The table below shows the distribution of heights of 134 students in a Maths Class.

Heights	$20-29$	$30-34$	$35-44$	$45-49$	$50-54$	$55-59$	$60-74$
No of students	9	12	27	13	25	18	30

(i) Draw a Histogram and read off the modal height.

(ii) Calculate

(a) the standard deviation

(b) the number of students with in the middle 60% range

5. (a) The table below shows the HeLP expenditure on teachers in 2020 and 2021.

Item	**2020**	**2021**	**Weight**
Data	50,000	80,000	8
Eats	10,000	30,000	2
Airtime	45,000	100,000	5
Salary	51,000	90,000	10
Apps	10,000	25,000	1

Using 2020 as the base year ,Calculate the average weighted price index correct it to 5 significant figures

(b) The price relative for four school items with their respective weights are as follows

Items	Books	Pens	Calculator	Sets
Weights	p	$2p$	q	$6+q$
Price Relatives	110	140	130	118

Given that the sum of the weights is 40 and the weighted average price index for the items is 126.7, find the values of p and q

PROBABLITY

6. (a) Events A and B which are not mutually exclusive are such that P(A)=$\frac{19}{30}$,P(B)=$\frac{3}{5}$ and P$(A \cup B)'$=$\frac{1}{10}$.Find the

(i) P$(A \cap B)$

(ii) P$(A \cap B')$

(b) The probability that Mawejje will play in the CECCAFA challenge champion for his team ,Uganda cranes is 0.8.The probability that Uganda cranes wins the tournament when Mawejje is playing is 0.75 ,otherwise it is 0.5.Find the probability that

(i) Uganda cranes wins the tournament

(ii) Mawejje is playing if they lose the tournament

(c) A four-man team is to be selected from three women and 4 men, What is the probability that

(i) the women will form the majority

(ii) at least 3 men will be on the committee?

(d) Two tetrahedral dice, with faces labeled 1, 2, 3 and 4 are thrown and the number on which each lands is noted. The score is the sum of the two numbers. Find the probability that:

(a) the score is even, given that at least one die lands on three.

(b) at least one die lands on three given that the score is even.

7. (a) A r.v X is uniformly distributed over the interval $[\alpha, \beta]$ .Given that E(X)=4 and Var(X)=3,find

   (i) the values of α and β (ii) $P(X > 5)$

   (b) A continuous r.v N has the following p.d.f f(n)

$$f(n) = \begin{cases} x + yn & ;0 \le n \le 1 \\ 0 & ;\text{otherwise} \end{cases}$$

   Given that F(0.5)=$\frac{3}{5}$,find the :

   (i) values of x and y (ii) P($N \le 0.25$)

   (c) X is a continous random variable whose cdf is

$$F(x) = \begin{cases} \lambda x^3 & ;0 \le x \le 3 \\ 1 & ;x \ge 3 \end{cases}$$

   Find

   (i) the value of the constant λ

   (ii) $P(x > 1/x < 2)$

8. (a) A biased coin is tossed six times ,the coin is such that the ratio of the tail to the head is 2 : 1 ,determine the probability of getting :

   (i) At least 4 heads

   (ii) Between 2 and 4 tails

   (b) The discrete random variable X can take values 0, 1, 2 and 3 only.
   Given $P(X \le 2) = 0.9$, $P(X \le 1) = 0.5$ and $E(X) = 1.4$, find:

   (i) $P(x = 1)$

   (ii) $P(x = 0)$

   (c) A random variable X has a cummulative distribution function defined by

$$\begin{cases} 0 & ;x \le 0 \\ \frac{x^2}{3} & ;0 \le x \le a \\ ax + b & ;1 \le x \le 2 \\ \frac{1}{6}(6x - x^2 - 3) & ;a \le x \le b \\ 1 & ;x \ge b \end{cases}$$

   Where a and b are constants .Determine

(i) the values of a and b

(ii) P$(0.5 < X < 2.3)$

(iii) pdf f(x) hence the mean (E(x))

9. (a) In an examination 30% of the candidates fail and 10% achieve distinction. Last year the pass mark (out of 200) was 84 and the minimum mark required for a distinction was 154. Assuming that the marks of the candidates were normally distributed, estimate the mean mark and the standard deviation.

   (b) A random sample of size n is to be drawn from a normal population with unknown mean μ and known standard deviation 1.8. Find the smallest value of n which gives a 90% confidence interval for μ for width less than 1.

10. A battery in a certain laptop has mean life time of 160 hours with standard deviation of 30 hours ,assuming the battery life is normally distributed .Calculate the:

   (a) Probability that the battery life lies between 150 hours and 180 hours

   (b) Range symmetrical about the mean with in which 75% of the battery lives life

NUMERICAL METHODS

11. (a) Find the range within which the exact value of z lies, given that

$$Z = \frac{1}{x} + \frac{1}{y} + xy, \qquad x = 4.165 \pm 0.001, \qquad y = 6.72 \pm 0.01$$

   (b) Use the trapezium rule with 11 ordinates to find the approximate value of $\int_1^2 x \log_{10} x dx$ to 4 d.p

   (c) Find the exact value of $\int_1^2 x \log_{10} x dx$ .Hence calculate the error in (b) above. How can this error be reduced when using the trapezium rule.

12. By constructing a table of values for $f(x) = 3xe^x - 1$ in the range $0.1 \leq x \leq 1.1$, using intervals of 0.2, obtain;

   (i) the value of f (1.13) using Linear extrapolation.

   (ii) the root of f(x) correct to 3d.p using Newton-Raphson's formula.

13. (a) Show that the iterative formula for solving the equation $4x - \sec^2 x$ is $x_{n+1} = \frac{1}{4}\sec^2 x_n$. Starting with $x_0 = 0.2$, find the solution of the equation to 4 significant figures.

   (b) The numbers x,y and z were estimated with errors δx,δy and δz respectively .Show that the maximum relative error in $\frac{xy^2}{z}$ is

$$|\frac{\delta x}{x}| + |2\frac{\delta y}{y}| + |\frac{\delta z}{z}|$$

   .State the assumptions made

14. (a) Show that the Newton raphson iterative formula for solving the equation $2x^2-6x-3 = 0$ is $X_{n+1} = \frac{2X_n^2+3}{4X_n-6}$ for n = 1, 2, 3 $\cdots$

(b) Show also that the positive root for $2x^2 - 6x - 3= 0$ lies between X =3 and X=4, takingX_0=3 as the first approximation, find the root correct to two decimal places.

15. (a) Use the trapezium rule with three strips to estimate the integral $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1+\cos x}}$,correct to three decimal places

(b) Calculate the percentage error in your estimation in (a) above

MECHANICS

16. (a) A particle is projected out of the sea with a velocity of $49ms^{-1}$ from a top of acliff 98m high at an angle of 30^0 with the horizontal. Find how far from the bottom of the cliff a particle reach the water.

(b) A particle **P** is projected vertically upwards with a speed of ums^{-1} from a point **O** on the ground,while at its highest point a second perticle is projected vertically upwards with a speed $2ums^{-1}$ from point **O** .The particles collide at apoint which is at a distance xm from the highest point of **P**.prove that :$u^2 = 32gx$

(c) A ball is projected from a point on the ground distance **a** from the foot of avertical wall of height **b**, the velocity of projection **V** at an angle α to the horizontal. Find how high above the wall the ball passes it, if the ball just clears the wall, prove that the greatest height reached is

$$\frac{\alpha^2 \tan^2 \alpha}{4(a \tan \alpha - b)}$$

17. (a) When a motorist is driving with velocity $6i - 8j$ the wind appears to come from the direction i. When he doubles his velocity the wind appears to come from the direction $i + j$. Find the true velocity of the wind.

(b) A ship A is travelling on a course of 060^0 at a speed of $30\sqrt{3}$ kmh^{-1} and a ship B is travelling on a course of 030^0 at 20kmh^{-1}. At noon B is 260km due east of A. Find the time when A and B are closest together.

18. (a) A body initially at rest undergoes uniform acceleration. It covers a distance $2p$ in q seconds; and a distance $p - q$ in p seconds. Show that it covers a distance $p + q$ in $\sqrt{q^2 - p^2}$ seconds

(b) At the same instant, two children who are standing $24m$ apart begin to cycle directly towards each other. Stephen starts from rest at a point A riding with a constant acceleration of 2 ms$^-$2 and Frank rides with a constant speed of 2 ms$^-$1.Find how long it is before they meet

(c) Consider two particles of masses m_1 and m_2 resting on smooth faces of a double inclined plane and are connected by a light inextensible string passing over a small smooth pulley at the vertex of the plane.if the faces of the plane are inclined at angles α and β to the horizontal respectively.Show that the acceleration

$$a = \frac{m_1 g \sin\alpha - m_2 g \sin\beta}{m_1 + m_2} \qquad \text{Assume } m_1 > m_2$$

.

19. (a) Forces of $6i+5j, -10i-4j, 7i-7j, -8i+2j, 5i+4j$N act at the points $(2,2), (5,0), (-4,-4), (0,-5)$ $(6,0)$ rspectively.Show that the system reduces to a couple and state the moment of the couple

(b) Forces of magnitude 10 N, 15 N and 20 N act away from a common point in the directions $S30^0E$, E 60^0N and North-West respectively. Find the resultant force.

(c) ABCD is a rectangle with $\overline{AB} = 3m$,$C\hat{A}B = 30^0$ .Forces of 10 N, 20 N and 20 N act along AC ,AD and DB respectively. Calculate the magnitude and direction of the resultant force hence find where its line of action cuts AB

20. (a) A light elastic string has natural length 2m and modulus of elasticity 200N. The ends of the strings are attached to two fixed points P and Q which are on the same horizontal level 3m apart. An object is attached to the midpoint of the string and hangs in equilibrium at a point $0.5m$ below PQ. Calculate to 2 significant figures:

(i) The mass of the object.

(ii) The elastic potential energy stored in the string in this position

(b) A force of magnitude 26N acting in the direction $3i - 4j + 12k$ causes a particle to undergo a displacement $6i - 3j + 6k$. Find the work done by the force in moving the particle

(c) A particle moves along a curve so that its position vector at time t is $r = (3t-2)i + t^4 j$ metres. If one of the forces, acting on the particle is $F = 5t^2 i - jN$, find the power of F in terms of t.Hence find the work done by F in the interval $0 \le t \le 2$.

END