CIRCULAR MOTION AND SAFETY (S.6 PHYSICS)

Learning Outcomes

By the end of this topic, learners should be able to:

- (U) Understand forces in circular motion
- (S) Derive and apply expressions for angular quantities
- (GS) Work collaboratively in investigations
- (V/A) Appreciate safety applications in real life

Circular Motion

Circular motion is the motion of a body along a circular path with constant or varying speed.

Examples in daily life

- A stone tied to a string and whirled around
- A car turning around a bend (navigating roundabouts)
- Blades of a fan
- The Earth/Planets revolving around the Sun
- Passengers on a Ferris wheel
- Water in a washing machine spin cycle
- Ceiling fan blades.

Types

- Uniform circular motion → constant speed
- Non-uniform circular motion → changing speed

Uniform Circular Motion: Moving in a circle at a constant speed. Although speed is constant, the velocity is constantly changing direction, meaning the object is **accelerating**.

Angular Quantities

(a) Angular Displacement (θ): Angle swept by a radius in radians.

(b) Angular Velocity (ω): Rate of change of angular displacement.

$$\omega = \frac{\theta}{t}$$

Where ω = angular velocity (in rads^{-1})

θ = angle (in radians)

t = time (in s)

Also,

$$\omega = \frac{2\pi}{T}$$

Where T is the time for one complete revolution/circle

(c) Angular Acceleration (α): Rate of change of angular velocity.

$$a = \frac{\Delta\omega}{t}$$

Where a = angular acceleration (in rad s $^{-2}$)

$\Delta\omega$ = change in angular velocity

$$a = \frac{v^2}{r} = r\omega^2$$

Linear and Angular Relationships

$$v = r\omega$$

Centripetal Force

This is the force that keeps a body moving in a circular path. It acts **towards the centre** of the circle.
A body moving in a circle experiences a force directed towards the centre.

$$F = \frac{mv^2}{r} = mr\omega^2$$

Where F = centripetal force (in N)
m = mass (in kg)
v = linear velocity (in ms^{-1})

Nature of Forces in Circular Motion

- The force is always directed **towards the centre**
- It changes the **direction** of motion, not necessarily the speed
- Without this force, the object would move in a straight line (tangential path)

Forces Providing Centripetal Force

Depending on the situation, centripetal force may be provided by:

- Tension → rotating string
- Friction → vehicles on curved roads
- Gravitational force → satellites
- Normal reaction → objects in circular tracks

Horizontal Curve (Road): Static friction (f_s) between the tires and the road provides F_c.
Maximum speed $v_{max} = \sqrt{\mu_s r g}$
If the required F_c exceeds maximum friction, the car skids.

Banking of Roads (Safety Application)

The horizontal component of the Normal Force ($N\sin\theta$) provides the centripetal force, reducing reliance on friction for safety.

To prevent skidding, roads are inclined.
Condition for no friction (ideal banking):

$$\tan\theta = \frac{v^2}{rg}$$

Implication:

- Higher speed → greater banking angle required
- Reduces dependence on friction → improves safety

Vertical Circle (Top): $T + mg = \frac{mv^2}{r}$ (Minimum speed needed, $v_{min} = \sqrt{gr}$).

Vertical Circle (Bottom): $T - mg = \frac{mv^2}{r}$ (Tension is maximum).

Applications to Safety (Very Important)

(a) Vehicles on Curved Roads

- Cars need friction to provide centripetal force
- If speed is too high → insufficient centripetal force →car may skid off the road

Safety measure:

- Roads are **banked (tilted)** to help provide centripetal force

(b) Seat Belts in Cars

- When a car turns suddenly, your body tends to move in a straight line
- Seat belts provide the force needed to keep you moving in the circular path

(c) Spinning Objects

- Loose objects may fly off due to lack of centripetal force
- Example: water leaving a rotating bucket

(d) Riding a Bicycle or Motorcycle

- Riders lean towards the centre to maintain balance
- This helps provide the required centripetal force

(e) Amusement Park Rides

- Riders experience strong centripetal forces
- Safety harnesses prevent them from being thrown outward

(f) Centrifuges

- Used in laboratories (e.g., blood separation)
- High speeds require strong materials for safety

(g) Satellites

- Gravity provides centripetal force
- Wrong velocity → satellite escapes or crashes

Worked Example (UACE Standard)

1. A car of mass 1200 kg moves at 25 m/s around a bend of radius 50 m.
   Find:
   (a) centripetal acceleration
   (b) centripetal force

2. A car travels at 15m/s around a curve of radius 100m. What is the ideal banking angle?

Key Takeaways

- Circular motion requires a centre-seeking force
- Increasing speed increases risk
- Safety measures are based on physics principles

- Real-life applications are essential for understanding

ITEM 1: Road Safety and Circular Motion

During a study tour to Fort Portal, a school bus approaches a curved section of road on a hillside. The driver notices that some passengers are not wearing seat belts and luggage has been placed on the overhead rack. As the bus negotiates the bend at high speed, it slightly skids outward before regaining control.
Learners later discuss the incident in their physics class.

Tasks

(a) Explain why the bus tends to move outward while taking the bend.
(b) State the force responsible for keeping the bus moving along the curved path and explain its direction.
(c) Derive an expression for the centripetal force acting on the bus in terms of mass (m), velocity (v), and radius (r).
(d) If the bus has a mass of 4000 kg, moves at 20 m/s, and the radius of the curve is 50 m, calculate:
   (i) the centripetal acceleration
   (ii) the centripetal force acting on the bus
(e) Explain the importance of:
   (i) wearing seat belts
   (ii) proper placement of luggage when the bus is negotiating a curve.
(f) Suggest two ways in which road design can improve safety on curved roads.

ITEM 2: Laboratory Centrifuge and Rotational Motion

At a school laboratory, learners use a centrifuge machine to separate components of a liquid mixture. The machine spins test tubes in a circular path at high speed. One learner forgets to properly secure a test tube, and it flies off when the centrifuge starts rotating.
The class is asked to investigate the physics behind this incident.

Tasks

(a) Explain why the test tube moved away from the circular path when not properly secured
(b) Define the following terms:
   (i) angular velocity
   (ii) angular acceleration
(c) Show that the centripetal acceleration of a body moving in a circle is given by: $a = \frac{v^2}{r}$
(d) A test tube of mass 0.2 kg is rotating in a circle of radius 0.5 m at a speed of 10 m/s. Calculate the centripetal force acting on it.
(e) Explain how increasing the speed of rotation affects:
   (i) centripetal force
   (ii) safety of the apparatus
(f) State three safety precautions that should be taken when using a centrifuge.

CONICAL PENDULUM (S.6 PHYSICS)

Learning Outcomes

By the end of the lesson, learners should be able to:

- (U) Explain the motion of a conical pendulum
- (S) Derive expressions for tension, velocity, and period
- (V/A) Apply concepts to real-life situations and safety

Definition

A conical pendulum is a system where a small mass (bob) attached to a string moves in a horizontal circular path, while the string traces out a cone.

A conical pendulum consists of a mass (m) attached to a string of length (l), suspended from a fixed point and revolving in a horizontal circle at a constant angular speed (ω).

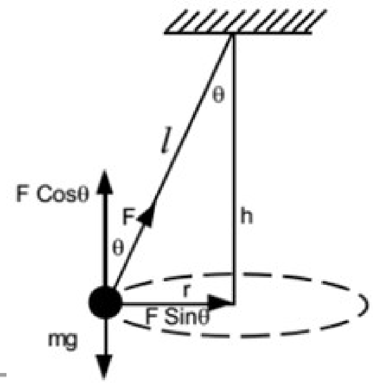
Description of Motion

- The bob moves with constant speed in a horizontal circle
- The string makes an angle θ with the vertical
- The radius of the circle is:

$$r = l \sin \theta$$

Where:

- l = length of string
- θ = angle with vertical



Forces Acting on the Bob

Two main forces:

- Tension (T) in the string
- Weight (mg) acting downward

Resolve tension into components:

- Vertical component: $T \cos \theta$
- Horizontal component: $T \sin \theta$

Derivation of Key Expressions

(a) Vertical Balance

No vertical acceleration:

$T \cos \theta$ = mg

(b) Horizontal Motion (Centripetal Force)

$$T \sin \theta = \frac{mv^2}{r}$$

(c) Dividing the Equations gives $\tan \theta = \frac{v^2}{rg}$

(d) Velocity of the Bob

$$v = \sqrt{rg \tan \theta}$$

(e) Angular Velocity $\omega = \frac{v}{r}$

(f) Time Period (T) = $\frac{2\pi r}{v}$

Substitute for (v) gives $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$.

The vertical height of the suspension point above the circle = $l\cos\theta$

Key Observations

- Period depends on length and angle, not mass
- As θ increases:
  - Speed increases
  - Tension increases
- Motion is uniform circular motion

Real-Life Applications

(a) Banked Roads and Racing Tracks: The physics of a car turning on a banked curve is analogous to a conical pendulum, where the normal force provides the tension component.

(b) Satellite Motion
Gravitational force acts like tension

(c) Amusement Park Rides: "Swing rides" or "Chair-o-planes" operate exactly as conical pendulums, where the angle of the swing increases with speed.

(d) Swinging Objects (e.g., tether balls) Demonstrate conical motion

(e) Flyball Governors (Centrifugal Governor): Used in 18th-19th century steam engines to regulate speed. As speed increased, the bob rose (decreasing θ), closing the steam valve.

(f) Laboratory Equipment: Used in experiments to measure gravitational acceleration and in rotational equipment, such as to rotate telescope mirrors.

Safety Implications

- Higher speed → higher tension → risk of breaking
- Strong materials needed in rotating systems
- Proper design reduces accidents

Worked Example

A conical pendulum has a string length of 1.5 m and makes an angle of 30° with the vertical.
Find the speed of the bob.

Key Summary

- Conical pendulum = circular motion + force balance
- Tension provides centripetal force
- Motion depends on angle and length
- Concepts apply directly to real-life safety systems

CONDITIONS FOR NON-SKIDDING ON BANKED AND HORIZONTAL ROADS

Learning Outcomes

By the end of the lesson, learners should be able to:

- (U) Explain conditions for non-skidding on roads
- (U) Define and interpret the angle of banking
- (S) Derive expressions relating speed, radius, and banking angle
- (V/A) Apply concepts to road safety

Introduction

When a vehicle moves along a curved path, it requires a centripetal force to keep it in motion.
If this force is insufficient → skidding occurs. Motion on a Horizontal (Flat) Road

Forces Acting

- Weight (mg) (downwards)
- Normal reaction (R) (upwards)
- Friction (f) → provides centripetal force

Condition for Circular Motion

Friction provides centripetal force:

$$f = \frac{mv^2}{r}$$

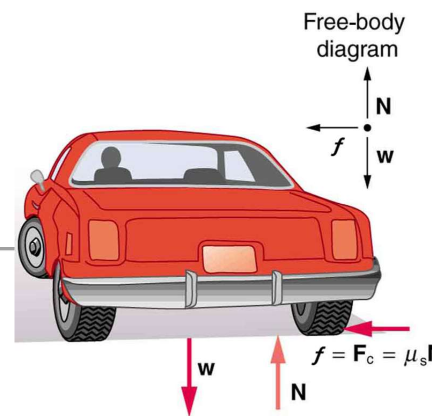
Maximum friction:

$$f_{max} = \mu R = \mu mg$$

Condition for Non-Skidding

$$\frac{mv^2}{r} \leq \mu mg$$

Simplifying: $v \leq \sqrt{\mu r g}$



Interpretation

- Higher speed → higher risk of skidding
- Larger radius → safer turns
- Rougher surface (larger μ) → more grip

Motion on a Banked Road

Definition

A banked road is tilted at an angle θ to the horizontal to reduce reliance on friction.
Raising the outer edge of a road above the inner edge is called **banking**. It tilts the normal force, providing a horizontal component that assists in providing centripetal force, reducing reliance on friction

Angle of Banking (θ): The angle at which the outer edge of a curved road is raised above the inner edge.

Forces on a Banked Road

- Weight (mg) (downwards)
- Normal reaction (R) (perpendicular to surface)
- Friction (may act up or down the slope)

Resolve forces:

- Horizontal → provides centripetal force
- Vertical → balances weight

Ideal Banking (No Friction Required)

In ideal design, the horizontal component of the normal reaction (R) provides the necessary centripetal force without needing friction.

Derivation

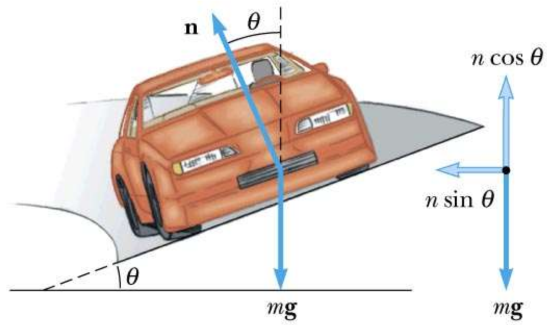
Vertical balance:

$$R\cos\theta = mg$$

Horizontal component provides centripetal force:

$$R\sin\theta = \frac{mv^2}{r}$$

Dividing: $\tan\theta = \frac{v^2}{rg}$



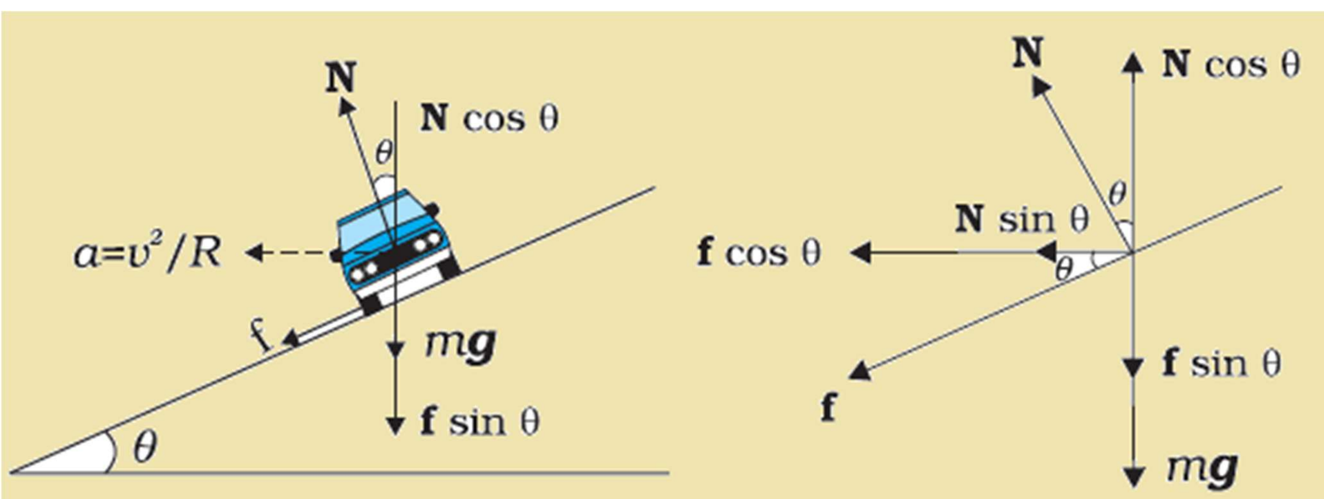
Expression for Safe Speed

$$v = \sqrt{rg\tan\theta}$$

Interpretation of Banked Roads

- Designed for a particular speed
- No friction needed at that speed
- Reduces chances of skidding

When Friction is Present



If a car travels faster than the optimal speed ($V > V_0$), it tends to skid *up* the slope. Friction acts *down* the slope.

Max Speed Formula, $v_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}$

- At lower speeds → vehicle may slide inward
- At higher speeds → vehicle may skid outward

- Friction adjusts to maintain circular motion

1. The maximum safe speed is independent of the mass of the vehicle.
2. **Safety Enhancement:** Banking increases maximum speed and allows safe passage even when friction is low (ice/rain).

Safety Applications

(a) Road Design

- Banking helps vehicles turn safely at higher speeds
- Used on highways and racetracks

(b) Speed Limits

- Based on radius and road conditions
- Prevent exceeding safe centripetal force

(c) Tyre Quality

- Good tyres increase friction → reduce skidding

(d) Wet or Slippery Roads

- Reduced friction → higher risk
- Drivers must reduce speed

Worked Example

A car moves on a flat road of radius 40 m. Coefficient of friction = 0.5.
Find the maximum speed to avoid skidding.

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12. Key Summary

- Circular motion requires centripetal force
- On flat roads → provided by friction
- On banked roads → provided by road inclination
- Proper design and speed control ensure road safety

SCENARIO ITEM 1: Circular Motion & Road Safety (Horizontal Road)

During a rainy day in Kampala, a taxi driver is negotiating a sharp bend at high speed. Suddenly, the vehicle begins to skid outward, nearly causing an accident. The passengers panic and question why the car could not stay on the road.

Tasks

a) Explain the forces acting on the car as it moves along the circular path.
b) Identify which force provides the centripetal force on a horizontal road.
c) Derive the condition for non-skidding of a vehicle on a flat (horizontal) road.
d) Explain how rain increases the chances of skidding.
e) Suggest two safety measures the driver should take to avoid skidding.

SCENARIO ITEM 2: Conical Pendulum in Real Life

At an amusement park, a rotating swing ride lifts children into the air such that each seat moves in a horizontal circle while suspended by a chain at an angle. One student notices that the chain does not remain vertical and wonders why.

Tasks

a) Describe the forces acting on the seat and rider.

b) Resolve the tension into vertical and horizontal components.

c) Derive expressions for the motion of a conical pendulum

d) Explain how increasing speed affects the angle of the chain.

f) State one real-life application of conical pendulum motion.

SCENARIO ITEM 3: Banked Roads & Safety

Engineers are designing a curved section of a highway to reduce accidents caused by vehicles skidding off the road. They decide to bank the road at a certain angle so that vehicles can safely navigate the curve even at higher speeds.

Tasks

a) Explain what is meant by "angle of banking."

b) Describe the forces acting on a car on a banked road.

c) Derive the condition for safe motion without reliance on friction

d) Explain why banking improves safety compared to a flat road.

e) State what happens if a vehicle moves:

- too fast
- too slow

  on a banked road.

f) Suggest two real-life examples where banking is used besides roads.