TRIGONOMETRY PRACTICE PROBLEMS:

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INTRODUCTION AND USE OF TRIG IDENTITIES:

1. Solve each of the following trig equations in the range; $-360^0 \leq x \text{ or } y \leq 360^0$
   a). $4\sin^2\left(\frac{1}{2}x\right) = 3$
   b). $5 + 2\tan\left(\frac{\pi}{3} - 3x\right) = 3$
   c). $27cosx = \sec^2 x$
   d). $2\cot(x - 30^0) = 3$
   e). $6\cos^2 x = 4 - sinx$
   f). $\frac{3+\sin^2 2x}{-2+cos2x} = 3cos2x$
   g). $\frac{5+\cos(4y-80)}{3} = 1.5$
   h). $\frac{3+\sin^2 x}{-2+cosx} = 3cosx$

2. Solve the following for; $0^0 \leq \theta \text{ or } x \text{ or } y \leq 360^0$
   a). $2cosx = 3tanx$
   b). $\frac{4}{tan^2 3\theta} + 2 = \frac{7}{sin3\theta}$
   c). $tan^4\theta = 6 + tan^2\theta$

d). $cos^2 2y + 5\,sin^2 2y = 4$

e). $\frac{cot\theta}{(cosec\theta - 1)} - \frac{(-1 + cosec\theta)}{cot\theta} = 4$

f). $\frac{4}{2sec\theta - 2sin\theta + 1} = cot\theta$

g). $2 + 4\,cos^2\theta = 7sin\theta cos\theta$

h). $\frac{1 - cos\theta}{sin\theta} = \sqrt{3}sin\theta$

i). $sin\theta\,tan^2\theta\,(3 + 2sin\theta) + tan^2\theta = 0$

j). $\left(-2sin3\theta + \sqrt{3}\right)\left(\sqrt{3} + 2\,cos\left(\frac{1}{2}\theta\right)\right) = 0$

k). $3tan\theta + 2cos\theta) = 0$

l). $6cos\theta = 5tan\theta$

m). $3tan\theta sin\theta - 1 = cos\theta$

n). $4tan\theta sin\theta cos\theta + 4tan\theta cos\theta = -1$

o). $\frac{4}{tan^2(3\theta)} + 2 = \frac{7}{sin(3\theta)}$

p). $\frac{5cos2\theta + sin2\theta}{3sin2\theta} = 7$

q). $5tany + secy = -5$

r). $sec^2(2x) - 3\,tan(2y) = -1$

s). $\frac{4\,sin^2 x}{cosecx} + \frac{3}{cosec^2 x secx} = sin^2 x$

3. Given that; $f(x) = x^3 - x^2 - 3x + 3$
   a) Show that; $(x - 1)$ is a factor of $f(x)$
   b) Factorize completely $f(x)$
   c) Hence, solve the trig equation: $\tan^3\theta - \tan^2\theta - 3tan\theta = -3$ for $0^0 \leq \theta \leq 360^0$
4. Given that; $h(x) = -\frac{1}{2}x + x^3 + 2 - 4x^2$.
   a) Factorize completely $h(x)$
   b) Hence, solve the equation: $\cos^3\emptyset + 2 = 4cos\emptyset\left(\frac{1}{8} + cos\emptyset\right)$ for $-\pi < \emptyset < \pi$
5. Given that; $\frac{1}{2}\sin^4\emptyset + \frac{1}{3}\cos^4\emptyset = \frac{1}{5}$. Show that; $\tan^2\emptyset = \frac{2}{3}$
6. Prove the following identities:

i). $\frac{sinA}{1 + cosA} + \frac{1 + cosA}{sinA} = 2cosecA$

ii). $\frac{cosB}{1 - sinB} + \frac{1 - sinB}{cosB} = 2secB$

iii). $\frac{cosycoty}{1 - siny} - 1 = cosecy$

iv). $\frac{sinA}{cotA + cosecA} 1 - cosA$

v). $tan2A = (sec2A + 1)\sqrt{(\sec^2 A - 1)}$

vi). $\sec^2 p\,(\cot^2 p - \cos^2 p) = \cot^2 p$

vii). $(1 - sinm)(1 + cosecm) = cosmcotm$

viii). $secy - \frac{\sin^2 y}{cosy} = cosy$

ix). $(1 - cosx)(1 + secx) = sinxtanx$

x). $\frac{tanA - cotB}{tanB - cotA} = tanAcotB$

xi). $\frac{sinA}{(1 - sinA)} - \frac{sinA}{(1 + sinA)} = 2\tan^2 A$

xii). $\frac{1}{(-1 + cosecA)} + \frac{1}{(1 + cosecA)} = 2secAtanA$

xiii). $\left(\frac{1 + sinx}{cosx}\right)^2 + \left(\frac{1 - sinx}{cosx}\right)^2 = 2(1 + 2\tan^2 x)$

xiv). $\frac{1 - cos2A + sin2A}{1 + cos2A + sin2A} = tanA$

xv). $\frac{2tanx}{tanx + sinx} = \sec^2\left(\frac{1}{2}x\right)$

xvi). $\frac{1 + sinA}{1 - sinA} = (secA + tanA)^2$

xvii). $cosec\theta - cot\theta = \tan\left(\frac{1}{2}\theta\right)$

xviii). $tanxsinx + cosx = secx$

xix). $sinx - sinxcos^2x = \sin^3 x$

xx). $\cos^2 x = \frac{cosecxcosx}{tanx + cotx}$

xxi). $\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

xxii). $\frac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x$

xxiii). $\frac{1 - sinx}{cosx} = \frac{cosx}{1 + sinx}$

xxiv). $1 - 2\cos^2 x = \frac{\tan^2 x - 1}{1 + \tan^2 x}$

xxv). $\tan^2 r = cosec^2 rtan^2 r - 1$

xxvi). $secy + tany = \frac{cosy}{1 - siny}$

xxvii). $(sint - cost)^2 + (sint + cost)^2 = 2$

xxviii). $\frac{\sin^2 x + 4sinx + 3}{\cos^2 x} = \frac{3 + sinx}{1 - sinx}$

xxix). $\frac{cosx}{1 - sinx} - tanx = secx$

xxx). $(1 + cotx - cosecx)(1 + tanx + secx) = 2$

xxxi). $\tan^2 p + 1 + tanpsecp = \frac{1 + sinp}{\cos^2 p}$

xxxii). $tanx + \frac{cosx}{1 + sinx} = \frac{1}{cosx}$

xxxiii). $\frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$

xxxiv). $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

xxxv). $(sinx - tanx)(cosx - cotx) = (sinx - 1)(cosx - 1)$

xxxvi). $(sinx + cosx)(tanx + cotx) = secx + cosecx$

xxxvii). $\frac{\sin^3 x+\cos^3 x}{sinx+cosx} = 1 - sinxcosx$

xxxviii). $\frac{cosx+1}{\sin^3 x} = \frac{cosecx}{1-cosx}$

xxxix). $cosec^4x - \cot^4 x = cosec^2x + \cot^2 x$

xl). $\frac{\sin^2 x}{\cos^2 x+3cosx+2} = \frac{1-cosx}{2+cosx}$

xli). $\frac{tanx+tany}{cotx+coty} = tanxtany$

xlii). $\frac{1++tany}{1-tany} = \frac{1+2sinxcosx}{\cos^2 x-\sin^2 x}$

xliii). $\sqrt{\frac{1-sin\emptyset}{1+sin\emptyset}} = sec\emptyset - tan\emptyset$

xliv). $\frac{\sin^2 \emptyset-1}{tan\emptyset sin\emptyset-tan\emptyset} = cos\emptyset + cot\emptyset$

xlv). $(acosA + bsinA)^2 + (asinA - bcosA)^2 = a^2 + b^2$

xlvi). $\frac{tan\theta+sec\theta-1}{tan\theta-sec\theta+1} = sec\theta + tan\theta$

xlvii). $\sqrt{\frac{sec\emptyset-1}{sec\emptyset+1}} + \sqrt{\frac{sec\emptyset+1}{sec\emptyset-1}} = 2cosec\emptyset$

xlviii). $(cosecA - sinA)(secA - \cos A) = \frac{1}{tanA+cotA}$

xlix). $\frac{\tan^3 x}{1+\tan^2 x} + \frac{\cot^3 x}{1+\cot^2 x} = 2cosec2x - sin2x$

l). $\frac{cosA-sinA+1}{cosA+sinA-1} = cosecA + cotA$

li). $\frac{sinA+cosA}{sinA-cosA} + \frac{sinA-cosA}{sinA+cosA} = -2sec2A$

lii). $2(\sin^6 \emptyset + \cos^6 \emptyset) - 3(\sin^4 \emptyset + \cos^4 \emptyset) = -1$

liii). $(\sin^8 \theta - \cos^8 \theta) = (\sin^2 \theta - \cos^2 \theta)(1 - 2\sin^2 \theta \cos^2 \theta)$

liv). $\sin^6 x + \cos^6 x + 3\sin^2 x \cos^2 x = 1$

lv). $\sqrt{(\sec^2 \emptyset + cosec^2\emptyset)} = tan\emptyset + cot\emptyset = 2cosec2\emptyset$

7. Given that; $secA + tanA = \frac{2}{3}$, find the value of; $sinA$.
   Hence, identify the quadrant where A lies.
8. Express; $\frac{1+tan30^0}{1-tan30^0}$, in the form; $p + m\sqrt{r}$, where p, m and r are constants.
9. Given that; $4siny - \frac{cosy}{2} = \frac{4}{siny} - \frac{1}{2cosy}$.
   Show that; $tany = 2$
10. Eliminate θ or α or $\emptyset$ (**or any both**) from each of following parametric equations:
    a). $x = 4sin\theta,\ y = 3cos\theta$
    b). $x = sec\theta + tan\theta,\ y = sec\theta - tan\theta$
    c). $xcos\theta + ysin\theta = c,\ xcos\theta - ysin\theta = d$

d). $m = cosec\theta - sin\theta,\ n = sec\theta - cos\theta$
e). $dcos^3\theta + 3dcos\theta \sin^2\theta = r,\ dsin^3\theta + 3dsin\theta \cos^2\theta = p$
f). $\sin(\alpha + \emptyset) = 2a,\ \sin(\alpha - \emptyset) = 2b$
g). $x = a\sin^3\emptyset,\ p = dcos^3\emptyset$
h). $x = 2sec\emptyset,\ y = cos4\emptyset$
i). $p = h + acos\emptyset, q = k + bsin\emptyset$

11. If; $sin\emptyset + cos\emptyset = m$ and $sec\emptyset + cosec\emptyset = n$, prove that; $n(m^2 - 1) = 2m$
12. If; $5x = sinA$ and $y = 2cosA$, show that; $100x^2 + y^2 = 4$.
13. If; $acos\theta + bsin\theta = m$ and $asin\theta - bcos\theta = n$, prove that; $a^2 + b^2 = m^2 + n^2$
14. If; $asin\emptyset + bcos\emptyset = c$, then, prove that; $acos\emptyset - bsin\emptyset = \pm\sqrt{(a^2 + b^2 - c^2)}$
15. If; $tanA + sinA = s$ and $tanA - sinA = t$, then prove that; $s^2 - t^2 = 4\sqrt{(st)}$
16. If; $x = asecA + btanA$ and $y = atanA + bsecA$, then, prove that; $x^2 - y^2 = a^2 - b^2$
17. Show that eliminating; x and y from the equations; $sinx + siny = a,\ cosx + cosy = b$ and $tanx + tany = c$ gives; $\frac{8ab}{(a^2+b^2)^2 - 4a^2} = c.$
18. **If;** $a = xcos\emptyset + ysin\emptyset$ and $b = xsin\emptyset - ycos\emptyset$, prove that; $tan\emptyset = \frac{bx+ay}{ax-by}$
19. Given; $cosec\theta - sin\theta = m$ and $sec\theta - cos\theta = n$.
    Prove that; $(m^2n)^{\frac{2}{3}} + (n^2m)^{\frac{2}{3}} = 1$
20. If; $3sin\emptyset + 4cos\emptyset = 5$, then, prove that; $3cos\emptyset - 4sin\emptyset = 0$
21. Given; $\boldsymbol{acos^3\theta + 3\,asin^2\theta\,cos\theta = m}$ and $\boldsymbol{asin^3\theta + 3asin\theta\,cos^2\theta = n.}$
    Prove that; $\boldsymbol{(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} = 2a^{\frac{2}{3}}}$
22. If; $\boldsymbol{sec\theta = x + \frac{1}{4x}}$, prove that; $\boldsymbol{sec\theta + tan\theta = 2x}$ **or** $\boldsymbol{\frac{1}{2x}}$.
23. Given; $\boldsymbol{7cot^2x + 6cotx = 1}$ and $\boldsymbol{6tan\theta = 8 + sec^2\theta.}$
    Show that; $\boldsymbol{tan(x+\theta) = -\frac{1}{2}}$.
24. If; $tan\theta + cot\theta = 5$, find the exact numerical value of each of the following;
    a). $\tan^2\theta + \cot^2\theta$
    b). $\tan^3\theta + \cot^3\theta$
    c). $tan\theta - cot\theta$
    d). $\tan^2\theta - \cot^2\theta$
25. The acute angles; x and y satisfy the trig equations below;

    $$\boldsymbol{2tanx = 1} \text{ and } \boldsymbol{\sin(x+y) = \frac{7}{\sqrt{50}}}.$$

    Show that the possible values of; $tany$ are; 3 and $\frac{13}{9}$.
26. Given that; $\boldsymbol{\frac{sin(x-\emptyset)}{\cos(x-\emptyset) - 2tan\emptyset\,\sin(x-\emptyset)} = tanx.}$
    Show that; $\boldsymbol{tanx = 2tan\emptyset.}$
27. Given that; $\boldsymbol{2cos\theta + sin\theta = 1}$.

Show that the possible values of; $\boldsymbol{7cos\theta + 6sin\theta}$ are; 2 and 6.

28. Given that the second, third and fourth terms of a Geometric series are; $\boldsymbol{cos\theta}$, $\boldsymbol{\sqrt{2}\ sin\theta}$ and $\boldsymbol{\sqrt{3}\ tan\theta}$ where; $0 \leq \theta \leq \frac{\pi}{2}$.

    Show that the sum of the first six terms is; $\boldsymbol{\frac{43}{12}(6 + \sqrt{6})}$.

29. Given; $\frac{sin2\theta}{1+sin\theta} = 1 - sin\emptyset$ and $\theta + \emptyset = \pi$

    a). Show that; $\boldsymbol{tan\theta = \frac{1}{2}}$

    b). Prove that; $\boldsymbol{\tan(3\theta + 5\emptyset) = -\frac{4}{3}}$

30. Given that; $\boldsymbol{4sin\theta + cos\theta = 2}$ for $0^0 \leq \theta < 360^0$.

    Show that; $\boldsymbol{cos\theta = \frac{2\pm4\sqrt{13}}{17}}$.

31. Given; $\boldsymbol{\frac{x}{a}sin\theta - \frac{y}{b}cos\theta = 1}$ and $\boldsymbol{\frac{x}{a}cos\theta + \frac{y}{b}sin\theta = 1}$.

    Prove that; $\boldsymbol{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

32. If; $cos\emptyset + sin\emptyset = \sqrt{2}cos\emptyset$, prove that; $cos\emptyset - sin\emptyset = \sqrt{2}sin\emptyset$

33. If; $cosec\theta - sin\theta = a^3$ and $sec\theta - cos\theta = b^3$, prove that; $a^2b^2(a^2 + b^2) = 1$.

34. If; $sin\emptyset + \sin^2\emptyset = 1$, prove that; $\cos^2\emptyset + \cos^4\emptyset = 1$

35. Prove that; $\frac{cot(90^0-A)}{tanA} + \frac{cosec(90^0-A)sinA}{tan(90^0-A)} = sec^2 A$.

36. Given; $x = psec\theta cos\beta$, $y = qsec\theta sin\beta$ and $z = rtan\theta$.

    Show that; $\frac{x^2}{p^2} + \frac{y^2}{q^2} - \frac{z^2}{r^2} = 1$.

37. If; $A + B = 45^0$, prove that; $(1 + tanA)(1 + tanB) = 2$

    Hence, deduce that; $tan22\frac{1}{2}^0 = -1 + \sqrt{2}$.

38. a) If; $cosecA - cotA = q$, then, show that; $\frac{q^2-1}{q^2+1} + cosA = 0$

    b) Solve the equation; $\boldsymbol{3\tan^3\theta - 3tan^2\theta = tan\theta - 1}$, for $0 \leq \theta \leq 2\pi$.

    c) If; $t = tan\theta$ and $k = sec2\theta + tan2\theta$, prove that; $t = \frac{k-1}{k+1}$

USE OF COMPOUND ANGLE FORMULAE AND FACTOR FORMULAE:

39. Prove out the following;

    a). $\sin\left(x + \frac{\pi}{4}\right) = \cos\left(x - \frac{\pi}{4}\right)$

    b). $\cos\left(x + \frac{\pi}{3}\right) + \sqrt{3}\sin\left(x + \frac{\pi}{3}\right) = 2cosx$

    c). $\frac{\sin(x+y)}{cosxcosy} = tanx + tany$

    d). $\tan\left(x + \frac{\pi}{4}\right)\tan\left(x - \frac{\pi}{4}\right) = -1$

e). $\sin\left(x+\frac{\pi}{3}\right)-\sqrt{3}\cos\left(x+\frac{\pi}{3}\right)=\frac{sin2x}{cosx}$

f). $\tan(x+60^0)\tan(x-60^0)=\frac{\tan^2 x-3}{1-3tan^2x}$

g). $\sin(x+y)\sin(x-y)=\cos^2 y-cos^2x$

h). $\cot(x+y)=\frac{cotxcoty-1}{cotx+coty}$

i). $\cos(x+y)\cos(x-y)=\cos^2 x-sin^2y$

j). $sinR-sinD=2\cos\left(\frac{R+D}{2}\right)\sin\left(\frac{R-D}{2}\right)$

k). $cosx+sinxtan2x=\frac{cosx}{cos2x}$

l). $\sin^2\left(\emptyset+\frac{\pi}{4}\right)+sin^2\left(\emptyset-\frac{\pi}{4}\right)=1$

40. If; $\sin(\emptyset+\beta)=2sin\emptyset$, show clearly that; $tan\emptyset=\frac{sin\beta}{2-cos\beta}$

41. In each of the following, show that;

a). $tan75^0=2+\sqrt{3}$

b). $cosec15^0=\sqrt{2}(1+\sqrt{3})$

c). $tan105^0=-2-\sqrt{3}$

d). $sec75^0=\sqrt{2}+\sqrt{6}$

e). $cot75^0=2-\sqrt{3}$

f). $sin15^0=\frac{\sqrt{6}-\sqrt{2}}{4}$

g). $cos105^0=\frac{1}{4}(\sqrt{2}-\sqrt{6})$

h). $tan15^0=2-\sqrt{3}$

i). $cos165^0=-sin75^0$

42. Given; $sinA=\frac{12}{13}$ and $cosB=\frac{4}{5}$

If A is Obtuse and B is acute, show clearly that; $\sin(A+B)=\frac{33}{65}$

43. Given; $sin\theta=\frac{8}{17}$ and $cos\emptyset=\frac{5}{13}$.

If θ is obtuse and $\emptyset$ is acute, show clearly that; $\cos(\theta+\emptyset)=-\frac{171}{221}$

44. Given; $sinR=\frac{8}{17}$ and $tanT=\frac{4}{3}$.

If R is obtuse and T is reflex, show clearly that; $\cos(R-T)=\frac{13}{85}$

45. Given; $sinA=\frac{1}{3}$ and $cosB=\frac{1}{2}$.

If; A is obtuse and B is reflex,

a). Show clearly that; $\sin(A+B)=\frac{1-2\sqrt{6}}{6}$

b). find the exact value of;

i). $\cot(2A+2B)$

ii). $\sin\frac{1}{2}(A-B)$

iii). $\cos\frac{1}{2}(A+B)$

iv). $\tan(A+B)$

v). $\sec(A+B)$

46. Given; $sinx = \frac{12}{13}$ and $cosy = \frac{15}{17}$.

If x is obtuse and y is acute, show clearly that; $\sin(x-y) = \frac{220}{221}$

47. Solve each of the following trig equations:

a). $\sin(y-48^0) = \cos(y+12^0)$, $0^0 < y < 360^0$

b). $2\cos\left(\emptyset + \frac{\pi}{2}\right) + \sin\left(\emptyset + \frac{\pi}{3}\right) = 0$, $0 \le \emptyset < 2\pi$

c). $\sin\left(\theta + \frac{\pi}{4}\right) = sin\theta$, $0 \le \theta < 2\pi$

d). $3\cos(x+30^0) = \sin(x-60^0)$, $0^0 < x \le 360$

e). $10\sin^2 3x + 10sin3xcos3x - \cos^2 3x = 2$, $0^0 \le x \le 120^0$.

48. The function, $f(\emptyset)$ is defined by; $f(\emptyset) = \frac{1}{k+cos\emptyset+2sin\emptyset}$, $0 \le \emptyset \le 2\pi$ and k is a constant.

If the maximum value of; $f(\emptyset)$ is; $\frac{(3+\sqrt{5})}{4}$, find the value of k.

49. a) Express; $8\cos\frac{1}{2}x + 5\sin\frac{1}{2}x$in the form; $Rcos\left(\frac{1}{2}x - \emptyset\right)$, where R and $\emptyset$ are constants with $R > 0$ and $0^0 < \emptyset < 90^0$

b) Write down the range of the function; $f(x) = 12 - 8\cos\frac{1}{2}x - 5\sin\frac{1}{2}x$

c) Hence, solve the equation; $8\cos\frac{1}{2}x = 6 - 5\sin\frac{1}{2}x$ for; $0^0 \le x < 360^0$.

50. Solve; $\frac{tan2x+tan32^0}{1-tan2xtan32^0} = 5$ for; $-90^0 < x < 90^0$

51. a) Show that; $\tan(3A-45^0) = \frac{tan3A-1}{1+tan3A}$

b) Hence, solve; $(1+tan3A)\tan(A+28^0) = tan3A - 1$ for; $0^0 < A < 180^0$

52. Mr. Jk models the height of sea waters, H metres, on a particular day by the equation;

$\boldsymbol{H = 6 + 3\sin\left(\frac{4\pi}{25}\right)t - \frac{3}{2}\cos\left(\frac{4\pi}{25}\right)t,\ 0 \le t < 12}$

Where t hours is the number of hours after Mid – day.

a). Calculate the maximum height of sea waters in a day as predicted by Mr. JK's model. Also, at what time will sea water have this maximum height?

b) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

53. Show that; $\frac{sin2\emptyset}{1+cos2\emptyset} = tan\emptyset$.

Hence, find, for $-180^0 \leq \emptyset < 180^0$, all the solutions of; $\frac{sin2\emptyset}{1+cos2\emptyset} = 1$

54. a) Express; $2cos3x - 3sin3x$ in the form; $Dcos(3x+\alpha)$, where D and α are constants, $D > 0$ and $0^0 < \alpha < 90^0$
    b) Given; $f(x) = e^{2x}cos3x$
    Show that; $f'(x)$ can be written in the form; $f'(x) = De^{2x}\cos(3x+\alpha)$, where; D and α are the constants found in part (a).
    c) Henec, or otherwise, find the smallest positive value of x for which the curve with equation; $y = f(x)$ has a turning point.
55. Express; $14\cos^2 x - 48sinxcosx$ in the form; $Rcos(2x+y)$ where; where R and y are constants, $R > 0$ and $0^0 < y < 90^0$
    Hence, find the maximum and minimum values of the function;
    $$\boldsymbol{f(x) = 10 + 48sinxcosx - 14cos^2 x}$$
    And the corresponding smallest positive x −value for which each occurs.
56. Given that; $\mathbf{2\cos(x+50^0) = \sin(x+40^0)}$
    a) Show, without using a calculator, that; $tanx^0 = \frac{1}{3}tan40^0$
    b) Hence, solve, for $0^0 \leq \theta \leq 360^0$, $\boldsymbol{2\,cos(2\theta+50^0) = sin(2\theta+40^0)}$
57. Given that; $H(\theta) = 4 + 5(2sin3\theta - 4cos3\theta)^2$
    a). Find the maximum and minimum value of $H(\theta)$
    b). Hence, find the smallest positive value of θ for which each value in part (a) above occurs.
58. Prove that; $sec2y + tan2y = \frac{cosy+siny}{cosy-siny}$
    Hence, solve; $sec2y + tan2y = \frac{1}{2}$ for; $0 \leq y \leq 2\pi$
59. a) Prove that; $\frac{sin4r+cosr}{cos4r+sinr} = sec3r + tan3r$
    b) Prove that; $\frac{3sin\theta+sin2\theta}{1+3cos\theta+cos2\theta} = tan\theta.$
    Hence, solve the equation; $\frac{3sin\theta+sin2\theta}{1+3cos\theta+cos2\theta} = \frac{1}{\cos^2\theta} = 2$ for; $0^0 \leq \theta \leq 360^0$.
60. Show that; $cos4y + 4cos2y = 8\cos^4 y - 3$
61. Using; $t = tan\theta$,
    a). write down $tan2\theta$ in terms of t.
    Hence, prove the identities;
    i). $cot\theta - tan\theta = 2cot2\theta$
    ii). $cot2\theta + tan\theta = cosec2\theta$
    b) Show that; $\tan(\theta+45^0) + \tan(\theta-45^0) = \frac{4t}{1-t^2}$.
62. Prove the following identities:
    a). $\frac{sin2A}{1+cos2A} = tanA$

b). $\frac{sin2A+cos2A+1}{sin2A-cos2A+1} = cotA$
c). $cotx - cosec2x = cot2x$
d). $\frac{sin3A+sinA}{2sin2A} = cosA$
e). $\frac{cos3\emptyset-sin3\emptyset}{1-2sin2\emptyset} = cos\emptyset + sin\emptyset$
f). $\frac{sin4A}{sinA} = 8\cos^3 A - 4cosA$
g). $cos\theta - cos3\theta = 4\sin^2\theta\, cos\theta$
h). $sin6x + sin4x - sin2x = 4cos3xsin2xcosx$
i). $\frac{2sin4A+sin6A+sin2A}{2sin4A-sin6A-sin2A} = \cot^2 A$
j). $\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)-\cos(A-B)} = -cotB$
k). $\frac{\cos(\emptyset+30^0)+\cos(\emptyset+60^0)}{\sin(\emptyset+30^0)+\sin(\emptyset+60^0)} = \frac{1-tan\emptyset}{1+tan\emptyset}$
l). $\frac{cos2\emptyset}{cos\emptyset-sin\emptyset} = cos\emptyset + sin\emptyset$
m). $\frac{cos2x}{sinx} + \frac{sin2x}{cosx} = cosecx$
n). $\frac{1}{cos\theta-sin\theta} - \frac{1}{cos\theta+sin\theta} = 2sin\theta sec2\theta$
o). $sin2y = \frac{2tany}{1+\tan^2 y}$
p). $\sqrt{(2+2cos2\theta)} = 2cos\theta$
q). $cos2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$
r). $\frac{\sec^2 m}{1-\tan^2 m} = sec2m$
s). $\frac{tan2\emptyset-sin2\emptyset}{tan2\emptyset} = 2\sin^2\emptyset$
t). $(3sin\theta + 5cos\theta)^2 = 17 + 8cos2\theta + 15sin2\theta$
u). $cosec2\theta - cot2\theta = tan\theta$
v). $coty - tany = 2cot2y$
w). $\frac{sin2x+sinx}{cos2x+cosx+1} = tanx$
x). $2 - 2tan\emptyset - \frac{2tan\emptyset}{tan2\emptyset} = (1-tan\emptyset)^2$
y). $\frac{2tanx}{tanx+sinx} = \sec^2\left(\frac{x}{2}\right)$
z). $cot2x = \frac{(\cot^2 x-1)}{2cotx}$

63. Prove out the following identities:
a). $4cosec^2 2x - cosec^2 x = \sec^2 x$
b). $2\cos^4\theta + \frac{1}{2}\sin^2\theta - 1 = cos2\theta$

c). $\frac{cos2\emptyset}{\sqrt{(1+sin2\emptyset)}} = cos\emptyset - sin\emptyset$

d). $8\cos^4\left(\frac{1}{2}\theta\right) = cos2\theta + 4cos\theta + 3$

e). $\sin^4\theta + \cos^4\theta = \frac{1}{4}(3 + cos4\theta)$

f). $\frac{sec\theta+tan\theta-1}{tan\theta-sec\theta+1} = \frac{cos\theta}{1-sin\theta} = sec\theta + tan\theta$

g). $sin3\emptyset + sin2\emptyset - sin\emptyset = 4sin\emptyset\cos\frac{\emptyset}{2}\cos\frac{3\emptyset}{2}$

h). $cotA = \frac{1}{2}\left(\cot\frac{A}{2} - \tan\frac{A}{2}\right)$

i). $\cot(A + 15^0) - \tan(A - 15^0) = \frac{4cos2A}{1+2sin2A}$

j). $\frac{\sin(n+1)A+2\sin(nA)+\sin(n-1)A}{\cos(n-1)A-\cos(n+1)A} = \cot\frac{A}{2}$

k). $\frac{1+cos\theta+sin\theta}{1+cos\theta-sin\theta} = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

l). $\frac{1-cosA+cosB-\cos(A+B)}{1+cosA-cosB-\cos(A+B)} = \tan\frac{A}{2}\cot\frac{B}{2}$

m). $\cot(A + B) = \frac{cotAcotB-1}{cotA+cotB}$

n). $\frac{sin\emptyset sin\theta}{cos\theta+cos\emptyset} = \frac{2\tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\emptyset}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)\tan^2\left(\frac{\emptyset}{2}\right)}$.

o). $\frac{\cot(90^0-A)}{tanA} + \frac{cosec(90^0-A)sinA}{\tan(90^0-A)} = -\sec^2 A$

p). $\frac{sin5x-sin7x+sin8x-sin4x}{cos4x-cos5x-cos8x+cos7x} = cot6x$

q). $\frac{sin\theta cos2\theta+sin3\theta cos6\theta}{sin\theta sin2\theta+sin3\theta sin6\theta} = cot5\theta$

r). $\frac{sin2A+cos2A+1}{sin2A+cos2A-1} = \frac{\tan(45^0+A)}{tanA}$

s). $sin^2 A - sin^2 B = sin(A + B)\,sin(A - B)$

t). $\frac{sin3Asin6A+sinAsin2A}{sin3Acos6A+sinAcos2A} = tan5A$

64. Solve each of the following trigonometric equations:

a). $sin\theta + \frac{1}{4}sec\theta = 0,\quad 0 \le \theta \le \pi$

b). $sin4y = sin2y,\quad 0^0 \le y \le 180^0$

c). $\frac{sin\emptyset}{cos\emptyset} + \frac{cos\emptyset}{sin\emptyset} = 4,\quad 0^0 < \emptyset < 360^0$

d). $2cosp = 2tanpsinp + secp,\quad -360^0 \le p \le 360^0$

e). $5\sin^2 2x - 3sin2xcos2x - 14\cos^2 2x = 0,\quad 0^0 \le x \le 90^0$

65. Using; t −formula, solve; $4cos\theta + 3sin\theta = 5$, for; $0 \le \theta \le 2\pi$.

66. In each of the following, show that;

a) $\tan^{-1}\left(\frac{2}{3}\right)+\tan^{-1}\left(\frac{5}{12}\right)=\tan^{-1}\left(\frac{3}{2}\right)$
b) $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{2}{11}\right)=\tan^{-1}\left(\frac{3}{4}\right)$.
c) $\sin(2\,tan^{-1}(x))=\frac{2x}{1+x^2}$
d) $\sin^{-}(x)+\cos^{-1}(x)=\frac{\pi}{2}$
e) $tan^{-1}\left(\frac{1}{3}\right)+tan^{-1}\left(\frac{4}{3}\right)=tan^{-1}(3)$
f) $tan^{-1}\left(\frac{1}{2}\right)+tan^{-1}\left(\frac{1}{3}\right)=\frac{1}{4}\pi$
g) $cos^{-1}\left(5^{-\frac{1}{2}}\right)+cos^{-1}\left(10^{-\frac{1}{2}}\right)=\frac{3}{4}\pi$
h) $2\,sin^{-1}\left(\frac{2}{3}\right)=cos^{-1}\left(\frac{1}{9}\right)$
i) $2\,tan^{-1}\left(\frac{3}{2}\right)+tan^{-1}\left(\frac{12}{5}\right)=\pi$
j) $tan^{-1}x+tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{\pi}{4}$
k) $tan^{-1}\left(\frac{4}{3}\right)+tan^{-1}(2)-tan^{-1}(3)=\frac{\pi}{4}$
l) $4\,cot^{-1}(2)+tan^{-1}\left(\frac{24}{7}\right)=\pi$
m) $tan^{-1}\left\{\frac{\sqrt{1+sinx}-\sqrt{1-sinx}}{\sqrt{1+sinx}+\sqrt{1-sinx}}\right\}=\frac{1}{2}x$
n) $2\,tan^{-1}\left(\frac{1}{5}\right)+cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)+tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$
o) $tan[tan^{-1}(3)-tan^{-1}(2)]=\frac{1}{7}$
p) $tan\left[tan^{-1}\frac{1}{3}+tan^{-1}\frac{1}{4}\right]=\frac{7}{11}$
q) $2\,cos^{-1}\left(\frac{4}{5}\right)=cos^{-1}\left(\frac{7}{25}\right)$
r) $sin[2\,sin^{-1}(x)+cos^{-1}(x)]=(1-x^2)^{\frac{1}{2}}$
s) $tan^{-1}\left[\sqrt{\frac{1-x}{1+x}}\right]=\frac{1}{2}cos^{-1}(x)$
t) $tan\left[\frac{1}{2}sin^{-1}(x)\right]=\frac{1-\sqrt{1-x^2}}{x}$
u) $tan^{-1}\left(\frac{w}{m}\right)-tan^{-1}\left(\frac{w-m}{w+m}\right)=\frac{\pi}{4}$
v) $sin^{-1}(1-x)-2\,sin^{-1}(x)=\frac{\pi}{2}$
w) $2\sin^{-1}(x)=\sin^{-1}\left(2x\sqrt{(1-x^2)}\right)$
x) $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$
y) $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{8}{19}\right)=\frac{\pi}{4}$

67. Find the exact value(s) of x in;

a). $\sin^{-1}(x) = \cos^{-1}(2x)$
b). $2\tan^{-1}\left(\frac{3}{x}\right) = \tan^{-1}\left(\frac{6x}{25}\right)$
c). $\tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{1}{1+x}\right) = \frac{\pi}{4}$
d). $\sin\left[\sin^{-1}\left(\frac{1}{4}\right) + \cos^{-1}x\right] = 1$
e). $\tan^{-1}\left(\frac{x-5}{x-1}\right) + \tan^{-1}\left(\frac{x-4}{x-3}\right) = \frac{\pi}{4}$
f). $\sin^{-1}(x) + \cos^{-1}\left(\frac{3}{5}\right) = 2\tan^{-1}\left(\frac{3}{4}\right)$. **Ans:** $x = \frac{44}{125}$
g). $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}(x)$. **Ans:** $x = \frac{1}{\sqrt{3}}$
h). $\sin^{-1}(2x) + \cos^{-1}(x) = \frac{5\pi}{6}$. **Ans:** $x = \frac{1}{2}$
i). $\sin^{-1}\left(\frac{x}{-1+x}\right) + 2\tan^{-1}\left(\frac{1}{1+x}\right) = \frac{\pi}{2}$. **Ans:** $x = 0$
j). $2\tan^{-1}(x-2) + \sin^{-1}\left(\frac{1-x}{1+x}\right) = \frac{\pi}{2}$. **Ans:** $x = 4$
k). $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$. **Ans:** $x = \sqrt{3}$
l). $\tan^{-1}\left[x\cos\left(2\sin^{-1}\left(\frac{1}{x}\right)\right)\right] = \frac{1}{4}\pi$. **Ans:** $x = -1, x = 2$
m). $\tan^{-1}(x) + \sin^{-1}(x) = \tan^{-1}(2x)$. **Ans:** $x = 0$
n). $\cot^{-1}\left(\frac{3x^2+1}{x}\right) = \cot^{-1}\left(\frac{1-3x^2}{x}\right) - \tan^{-1}(6x)$. **Ans:** $x = \pm\frac{1}{\sqrt{3}}$
o). $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

68. Prove that; $sin3A = 3sinA - 4\sin^3 A$
Hence, or otherwise, solve the equation; $\sin^{-1}x = 3\sin^{-1}\left(\frac{1}{3}\right)$
69. Solve the following simultaneous equations;
$$\mathbf{\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}(8)}$$
$$\mathbf{x + y = 2}$$
70. Given that; $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, show that; $xy + yz + zx = 1$
71. Prove that, if $|x| \le 1$, then; $\mathbf{\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right) = \frac{1}{2}\cos^{-1}(x)}$.
72. Prove that; $\frac{1+\cot^2 x}{cotxcosecx} = secx$
Hence, solve the equation; $\frac{4(1+\cot^2 x)}{cotxcosecx} = \tan^2 x + 5$, for; $0 \le x \le 2\pi$
73. Prove that; $tan3\beta = \frac{3t-t^3}{1-3t^2}$, where $t = tan\beta$.
Hence, solve the equation; $\mathbf{1 - 3t^2 = 3t - t^3}$, correct your answers to 3 dps.
74. Prove that; $\cos(x + 30^0) + \cos(x - 30^0) = \sqrt{3}cosx$.

Hence, show that; $cos75^0 + cos15^0 = \frac{1}{2}\sqrt{6}$

75. The acute angles; θ and $\emptyset$ satisfy the relationships;
$\boldsymbol{7\cot^2\theta + 6cot\theta = 1}$ and $\boldsymbol{6tan\emptyset = 8 + \sec^2\emptyset}$
    a). Determine the value of $tan\theta$ and the value of $tan\emptyset$
    b). Show that; $\tan(\theta + \emptyset) = -\frac{1}{2}$
76. Prove that; $cos3\emptyset = 4\cos^3\emptyset - 3cos\emptyset$
Hence, solve the equation; $8\cos^3\emptyset + 1 = 6cos\emptyset$ for; $0 < \emptyset < 2\pi$
77. The temperature of the water T^oC in a tropical fish tank is modelled by the equation;
$$\boldsymbol{T = 32 + \sqrt{(3)}\sin(15t)^0 + \cos(15t)^0}$$
Where t is the time in hours measured since midnight.
    a). State the maximum temp of the water in the tank, and the time when this maximum temperature occurs.
    b). Show that the temp of the water in the tank reaches 30.5^0C at; **13:14hrs** and **18:46hrs.**
78. Give that; $f(\theta) = \frac{1}{\sin^2\theta + 3sin\theta cos\theta + 5\cos^2\theta}$.
Show that; $\frac{2}{11} \leq f(\theta) \leq 2$
79. Solve; $\boldsymbol{cos\emptyset = sin3\emptyset}$**,** for; $-360^0 \leq \emptyset \leq 360^0$
80. Using; $t = tan\emptyset$, show that; $tan3\emptyset = \frac{t(3-t^2)}{1-3t^2}$
81. Given that; $x + y + z = \pi$, prove that;
$\boldsymbol{\cot\frac{x}{2} + \cot\frac{y}{2} + \cot\frac{z}{2} = \cot\frac{x}{2}\cot\frac{y}{2}\cot\frac{z}{2}}$
82. If; $\boldsymbol{msin\theta = nsin(\theta + 2\alpha)}$**,** then, prove that**;** $\boldsymbol{\tan(\theta + \alpha)\, cot\alpha = \frac{m+n}{m-n}}$
83. If; $\boldsymbol{\cos(\theta + \emptyset) = mcos(\theta - \emptyset)}$**,** then, prove that; $\boldsymbol{tan\theta = \frac{1-m}{1+m}cot\emptyset}$
84. If; $sec\theta + tan\theta = p$, show that; $\frac{p^2-1}{p^2+1} = sin\theta$
85. a) If; $3\sin(A - \alpha) = \cos(A + \alpha)$, show that; $cotA = \frac{1+3cot\alpha}{3+cot\alpha}$.
    Hence, determine the exact value of; $\tan(A + \alpha)$ when; $cot\alpha = -\frac{1}{2}$.
    b) Solve; $cosx = \sin\left(\frac{1}{2}x\right)$ for; $0^0 \leq \frac{x}{2} \leq 90^0$

86. a) Using; $t = tanA$ or otherwise, solve; $sin4A = sin2A$ for; $0^0 < A < 360^0$
    b) Given that; $\theta = sin\alpha + cos\beta$ and $\emptyset = cos\alpha - sin\beta$, show that;
$$\boldsymbol{\frac{\theta^2 - \emptyset^2}{2\theta\emptyset} = \tan(\alpha + \beta)}$$
87. a) Prove that; $\boldsymbol{sin4\emptyset = \frac{4tan\emptyset(1-\tan^2\emptyset)}{(1+tan^2\emptyset)}}$

b) Solve the equation; $\boldsymbol{tan^{-1}(1+x)+tan^{-1}(1-x)=\frac{\pi}{4}}$.

88. Show that; $-\sqrt{5}\le cosx+2sinx\le\sqrt{5}$
89. Prove that; $\frac{\sin(A+B)}{cos(A-B)}+1=\frac{(1+cotA)(1+tanB)}{cotA+tanB}$
90. Using t −formula, prove that; $1+sec2\theta=tan2\theta cot\theta$
91. Given that; $y=\frac{sinx-2sin2x+sin3x}{sinx+2sin2x+sin3x}$
    a). Prove that; $y+tan^2\left(\frac{x}{2}\right)=0$
    b). Hence, express the exact value of; $tan^2 15^0$ in the form; $m+r\sqrt{t}$ where m, r and t are integers.
    c). Find the value(s) of x in the range; $0^0\le x\le 360^0$ for which; $2y+\sec^2\left(\frac{x}{2}\right)=0$ holds.
92. Prove that; $\frac{cos11^0+sin11^0}{cos11^0-sin11^0}=tan56^0$
93. Given that; $p=2cosx+3cos2x$ and $q=2sinx+3sin2x$.
    a). Find the least and greatest values of; p^2+q^2
    b). Given that; $p^2+q^2=19$, find x for; $0^0\le\frac{x}{2}\le 45^0$
    Hence, show that; $pq=-\frac{5\sqrt{3}}{4}$
94. Prove that; $\boldsymbol{\sin(10^0)\sin(30^0)\sin(50^0)\sin(70^0)=\frac{1}{16}}$.
95. Show that; $\boldsymbol{\tan^{-1}\left(\frac{cos66^0-sin48^0}{cos48^0+sin66^0}\right)=-12^0}$.
96. Given; $8\sin(x-\beta)=3\sin(x+\beta)$.
    Prove that; $5tanx=11tan\beta$.
97. If; $\tan(x+y)=2\tan(x-y)$, show that; $tanxtany=3$
98. Given that; $\boldsymbol{\sin(x+y)\sin(x-y)=\frac{5}{36}}$ and $\boldsymbol{cosx+cosy=\frac{5}{6}}$.
    Show that; $\boldsymbol{\cos(x-y)=\frac{1+2\sqrt{6}}{6}}$
99. a) Solve the equation; $sin2x+sin3x+sin5x=0$, for; $0^0<x<360^0$
    b) Show that; $\frac{\cos(45^0+\theta)}{\cos(45^0-\theta)}=\frac{1-tan\theta}{1+tan\theta}$.
    Hence, solve; $4\cos(45^0+\theta)=cos(45^0-\theta)$, for; $0^0<\theta<90^0$
100. a) Given that; $2A+B=135^0$, show that; $\frac{\tan^2 A-2tanA-1}{1-2tanA-\tan^2 A}=tanB$
    b) If; $\emptyset$ is acute angle and $tan\emptyset=\frac{4}{3}$, show that; $\boldsymbol{4\,sin(\theta+\emptyset)+3\,cos(\theta+\emptyset)=5cos\theta}$.
    Hence, solve for θ in the equation; $\boldsymbol{4\,sin(\theta+\emptyset)+3\,cos(\theta+\emptyset)=\frac{1}{4}\sqrt{(300)}}$, for; $-180^0\le\theta\le 180^0$
101. Given that; $sin45^0=\frac{1}{\sqrt{2}}$, show without using a calculator or tables, that;

$$\boldsymbol{sin292\frac{1}{2}^0 = -\frac{1}{2}\sqrt{2+\sqrt{2}}}$$

102. If; $cos\alpha - cos\beta = \frac{2}{5}$ and $sin\alpha - sin\beta = \frac{5}{6}$, find the value of;

a). $\sin\frac{1}{2}(\alpha + \beta)$

b). $\cos(\alpha + \beta)$

103. Show that; $sin3\theta = 3sin\theta - 4\sin^3\theta$. Hence, solve the equation; $8x^3 - 6x = 1$.

104. Prove that; $(sin2\alpha - sin2\beta)\tan(\alpha + \beta) = 2(sin^2\alpha - sin^2\beta)$.

105. Given that; $\sin(x + \beta) = 2\cos(x - \beta)$, prove that; $tanx = \frac{2 - tan\beta}{1 - 2tan\beta}$

SOLUTIONS OF A TRIANGLE:

106. a) In a triangle ABC, show that; $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$

b) Hence, if; $a = 4cm$, $b = 6cm$ and $C = 127.2^0$, solve this triangle.

107. a) In a triangle ABC, prove that; $\frac{bc}{ab+ac} = \frac{cosec(B+C)}{cosecB+cosecC}$

b) Express; $4sinx - 3cosx$, in the form; $Rsin(x - D)$.

Hence, find the maximum and minimum values of the function; $\frac{1}{6+4sinx-3cosx}$, stating clearly the corresponding smallest positive values of x.

108. Prove that, in any triangle, ABC, $(a + b + c)\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 2c\cot\frac{C}{2}$

109. In a triangle, ABC, prove that;

a). $asin(B - C) + bsin(C - A) + csin(A - B) = 0$

b). $[cbcosA + cacosB + abcosC] = \frac{1}{2}(a^2 + b^2 + c^2)$

c). $cosA + cosB - cosC = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

d). $tanA + tanB + tanC = tanAtanBtanC$

e). $sinB + sinC - sinA = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

f). $sin^3A + sin^3B + sin^3C = 3cos\frac{A}{2}cos\frac{B}{2}cos\frac{C}{2} + cos\frac{3A}{2}cos\frac{3B}{2}cos\frac{3C}{2}$

g). $sinA + sinB + sinC = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

h). $sin2A + sin2B + sin2C = 4sinAsinBsinC$

i). $\cos^2 2A + \cos^2 2B + \cos^2 2C = 1 + 2cos2Acos2Bcos2C$

j). $sin^2\frac{A}{2} + sin^2\frac{B}{2} + sin^2\frac{C}{2} = 1 - 2\,sin\frac{A}{2}sin\frac{C}{2}sin\frac{B}{2}$

k). $cosA + cosB + cosC = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

l). $cos2A + cos2C + cos2B = -1 - 4cosAcosBcosC$

m). $\frac{sinB}{sinC} = \frac{c - acosB}{b - acosC}$

n). $tan(B-C)+tan(C-A)+tan(A-B)=tan(A-B)\,tan(B-C)\,tan(C-A)$
o). $\frac{a-c+b}{a+b+c}=\tan\frac{A}{2}\tan\frac{B}{2}$
p). $\frac{b^2-c^2}{a^2}=\frac{\sin(B-C)}{\sin(A+B)}$
q). $\frac{1}{a}cos^2\frac{A}{2}+\frac{1}{b}cos^2\frac{B}{2}+\frac{1}{c}cos^2\frac{C}{2}=\frac{(a+b+c)^2}{4abc}$.
r). $3tanC=tanB$, *if;* $\boldsymbol{tan\frac{A}{2}=sec^2\frac{A}{2}sin(B-C)}$
s). $cot\frac{A}{2}cot\frac{B}{2}cot\frac{C}{2}=cot\frac{A}{2}+cot\frac{B}{2}+cot\frac{C}{2}$
t). $tan\frac{A}{2}tan\frac{B}{2}+tan\frac{B}{2}tan\frac{C}{2}+tan\frac{A}{2}tan\frac{C}{2}=1$
u). $sin^2A+sin^2B+sin^2C=2+2cosAcosBcosC$
v). $\sin\frac{1}{2}(B-C)=\left(\frac{b-c}{a}\right)\cos\frac{1}{2}A$

110. In a triangle, PQR, prove that;
a). $p^2=r^2+q^2-2rqcosP$
b). Hence, solve the triangle in which; $q=5cm$, $r=8cm$ and $P=60^0$.

111. a) In a triangle, ABC, $\overline{AB}=10cm$, $\overline{BC}=17cm$ and $\overline{AC}=21cm$, solve the triangle ABC.
b) Solve; $sin3x+sin7x=sin5x$, for; $0^0\le x\le 90^0$.

112. Three points; A, B and C are in a straight line on a horizontal ground with B between A and C.A vertical pole at A is supported by two wires attached to its top and the points; B and C. The wire at C makes an angle of 30^0 with the ground and the wire at B makes an angle of 50^0 with the ground. If $\overline{BC}=4cm$, find the;
a). Length of the two wires.
b). Length of the pole.

END

"TAKE THIS ACADEMIC EXCELLENCE SO PERSONAL"