THE UGANDA INTER SCHOOL VIRTUAL A LEVEL MATHEMATICS SEMINAR 2025.

Saturday 05^{th} July 2025 (9:00 a.m)

INSTRUCTIONS TO STUDENTS AND TEACHERS:

Dear students and teachers we would like to welcome you to participate in the forthcoming Mathematics seminar for senior six students. This is in preparation for the forthcoming final exams(UNEB) and the Mock Examinations. **This is a free seminar and no one should charge you any fees.**The process to be followed by both the teachers and students is suggested below:

1. Teachers share the Seminar questions with their students and ask for volunteers to discuss any of the questions. Questions should be pinned up and learners write down all the questions in their books.
2. Teachers talk to the school administrators to allow the children participate as presenters in the seminar on Saturday **05th July from 09:00am - 2:00 pm.** Other students will just be participants.
3. The student together with the teachers select atleast two best done presentations and the students to represent the school.The solutions and pictures/videos should be uploaded on padlet `https://bit.ly/S4MATHSEMINAR2023` and sent on email kazibastephen42@gmail.com
4. Hold a mock presentation where all your discussants present to the rest of the class.After that release the rest of the class and record your best presenter in a very quiet environment but with good light.Record each part of the question separately .
5. The teacher could now train the student on how to present on zoom as far as sharing a screen and using the whiteboard. Alternatively the students' presentation will be loaded on the computer screen and they explain to us their solution.

SEMINAR DETAILS

S.6 virtual Mathematics seminar 2025.

Time: 05 JULY 2025, 09:00 AM

Join Zoom Meeting
Meeting ID:99061580873
Passcode: HeLP2025

P425/1	P425/2
1. Analysis (6 questions) (a) Differentiation (b) Integration (c) Differential equations 2. Vectors (2 questions) (a) Vectors in 2-D (b) Vectors in 3-D (c) Ratio theorem (d) Lines and their properties (e) Planes and their properties 3. Trigonometry (2 questions) 4. Geometry (2 questions) (a) Coordinate geometry of lines and triangles (b) Locus and circles (c) Parabola,ellipse ,hyperbola 5. Algebra (4 questions) (a) Surds,indices and logarithms (b) Quadratics (c) Polynomials (d) Simultaneous equations (e) Inequalities (f) Partial fractions (g) Complex numbers (h) Permutation and combinations	1. Mechanics (6 questions) (a) Kinematics (b) Dynamics (c) Statics 2. Numerical analysis (4 questions) (a) Location of the roots of an equation (b) Trapezium rule of numerical integration (c) Newton Raphson method (d) Errors (e) Flow charts 3. Statistics and probability(6 questions) (a) Mean ,mode,median (b) Index numbers (c) Correlation coefficient (d) Scatter diagram (e) Discrete probability distributions (f) Continuous probability distributions (g) Distributions i. Uniform distribution ii. Normal distribution iii. Binomial distribution iv. Normal approximation to binomial distribution (h) Estimations

PURE MATHEMATICS (P425/1)

TOPICS	SECTION A	SECTION B
ALGEBRA	2	2
ANALYSIS	3	3
TRIGONOMETRY	1	1
VECTORS	1	1
GEOMETRY	1	1

ALGEBRA

1. Solve the equation for x if

   (a)
   $$4|x-1| \leq |3x+2|$$

   (b)
   $$7(4^x - 2^{x+1}) = 8(8^{x-1} - 1)$$

   (c)
   $$\log_2(x+1) - 4\log_{(x+1)} 2 = 3$$

2. The expansion of $(ax-2)^4(1+\frac{b}{x})^3$ is written in descending powers of x.The first 3 terms of this expansion are $81x^4 + 999x^3 + cx^2$.It is given that a, b and c are positive integers.Find the values of a, b and c

3. (a) The first three terms of an arithemetic progression are $\ln p$,$\ln p^4$ and $\ln p^7$,where p is a positive constant. The sum of n terms of this progression is $4845 \ln p$.Find the value of n

   (b) The first three terms of a geometric progression are q^{3x}, q^x and q^{-x}, where q is a positive integer.Find the nth term of this progression giving your answer in the form $q^{(a+bn)}x$

4. (a) Given that x and y are real numbers ,find the values of x and the values of y
   $$(x+iy)^2 = 7 - (6\sqrt{2})i$$

   (b) i. Show that $z-3$ is a factor of $2z^3 - 4z^2 - 5z - 3$

   ii. Solve $2z^3 - 4z^2 - 5z - 3 = 0$

5. (a) Solve for n in ${}^nC_3 +^n C_2 = 4n$

   (b) Solve the simultaneous equation
   $$e^{3x+4y} = 2e^{2x-y}$$
   $$e^{2x+y} = 8e^{x+6y}$$

6. (a) Show that $1+i$ is a root of the equation $z^4 + 3z^2 - 6z + 10 = 0$.Hence find other roots

   (b) Given that the complex number z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z} - 2z + 2\bar{z} = 5 - 4i$.Find the possible values of z

7. (a) Solve the simultaneous equations

$$m^2 - 4mn + n^2 = 1$$
$$m^2 + n^2 - \frac{17m}{4} = 0$$

(b) Find the range of values of x for which $\frac{2x+1}{x+2} > \frac{1}{2}$

8. Solve the simultaneous equations :

(a)

$$\log(m+n) = 0$$
$$2\log m = \log(5+n)$$

(b)

$$2a - b + 4c = 26$$
$$3a - 2c - b = 0$$
$$a + 3b - c = 5$$

9. (a) A student deposits UGX 50,000 with a bank every start of a three month period.Their understanding is that the bank gives her a compound interest of 3% every three month period. How long will it take her to accumulate UGX 3 million if there is no withdrawal.

(b) Show that

$$\sqrt[3]{\frac{1-x}{1+x}} \approx 1 - \frac{2}{3}x + \frac{2}{9}x^2$$

10. (a) Prove by induction that $3^{2n} + 5^n - 2$ is always a multiple of 4 for all integers ,$n \geq 1$.

(b) Given that $\sum_1^n r^3 = \frac{1}{4}n^2(n+1)^2$,find the series sum $1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3$.

11. (a) A team of 10 players is to be chosen from 15 players.

i. Find the number of different teams that can be chosen if there are no restrictions.

ii. The 15 players include 3 sisters who must not be separated. Find the number of different teams that can be chosen

(b) A 6 digit number is to be formed using the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9.The 6 digit number cannot start with 0 and all six digits must be different.Find how many 6 digit numbers can be formed if the 6 digit number is even

12. (a) Express $\frac{6x^2-24x+15}{(x+1)(x-2)^2}$ in partial fractions.

(b) Hence obtain the expansion of $\frac{6x^2-24x+15}{(x+1)(x-2)^2}$ in ascending powers of x ,up to and including the term in x^2

13. (a) Solve the equation $\sqrt{x+7} - \sqrt{x-1} = 2$

(b) Express $\frac{z+1}{z-i}$ in the form $a + bi$ where a and b are the real and imaginary variables respectively. Hence if $Arg(a+bi) = -\frac{\pi}{4}$,show that the locus of z is a circle

14. Skecth the curve

$$y = \frac{2(x-2)(x+2)}{2x-5}$$

.

TRIGONOMETRY

15. (a) Express $3\cos 2\theta + 4\sin 2\theta$ in the form $R\sin(2\theta+\alpha)$,where $R > 0$ and $0^0 < \alpha < 180^0$.Give the value of R and the value of α to 2 d.ps.

    (b) Hence;

    i. State the maximum value of $3\cos 2\theta + 4\sin 2\theta$

    ii. Solve the equation $3\cos 2\theta + 4\sin 2\theta = 4$ for $0^0 < \theta < 180^0$

16. (a) Show that $\frac{\sin\theta\tan^2\theta}{1+\tan^2\theta}$ can be written as $\sin^3\theta$.

    (b) Hence solve the equation

    $$\frac{\sin 3x \tan^2 3x}{1+\tan^2 3x} = \frac{1}{8}$$

    for $-180^0 \leq x \leq 180^0$

17. (a) Simplify $\tan 75^0$ giving your answer in the form $a + b\sqrt{3}$. Hence show that $\tan 105^0 + \cot 105^0 = -4$.

    (b) Given that $a = \tan\theta - \sin\theta$ and $b = \tan\theta + \sin\theta$.Prove that

    $$(a^2 - b^2)^2 = 16ab$$

18. Show that

    (a)

    $$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$$

    (b)

    $$\sin(270-\theta) + \sin(270+\theta) = -2\cos\theta$$

    (c)

    $$\tan(x + \frac{\pi}{4}) = \frac{\cos x + \sin x}{\cos x - \sin x}$$

    (d)

    $$\frac{\sin\beta + \sin 3\beta + \sin 5\beta}{\cos\beta + \cos 3\beta + \cos 5\beta} = \tan 3\beta$$

    (e)

    $$\frac{\tan B + \sec B - 1}{\tan B - \sec B + 1} = \frac{1+\sin B}{\cos B}$$

19. If A,B ,C are angles of a triangle ,prove that

(a)

$$1-\sin^2\frac{A}{2}-\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=2\sin\frac{A}{2}\sin\frac{B}{C}\sin\frac{C}{2}$$

(b)

$$\sin\frac{B}{2}=\cos\left(\frac{A+C}{2}\right)$$

(c)

$$\cos A+\cos B+\cos C-1=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

(d)

$$\frac{\cos A+\cos B}{\sin A+\sin B}=\tan\frac{C}{2}$$

(e)

$$\cos 2A+\cos 2B+\cos 2C=1-\cos A\cos B\cos C$$

20. (a) Solve $\cos\theta+\sqrt{3}\sin\theta=2$ for $0^0\leq\theta\leq 360^0$

(b) Solve for β if $\cos\beta+\cos 5\beta+\cos 3\beta=0$ for $0^0\leq\beta\leq\pi$

(c) If P,Q and R are angles of a triangle prove that :

$$\frac{1}{p}\cos^2\left(\frac{P}{2}\right)+\frac{1}{q}\cos^2\left(\frac{Q}{2}\right)+\frac{1}{r}\cos^2\left(\frac{R}{2}\right)=\frac{(p+q+r)^2}{4pqr}$$

VECTORS

21. (a) A plane is given by $r=\begin{pmatrix}3\\1\\-2\end{pmatrix}+\alpha\begin{pmatrix}2\\-3\\1\end{pmatrix}+\beta\begin{pmatrix}-1\\2\\3\end{pmatrix}$ for scalars α and β.Determine its cartesian equation

(b) Find the coordinates of the foot of the perpendicular dropped from $(-5,-7,25)$ to this plane.

22. The lines π and ϕ are given by $3\lambda i+(1-2\lambda)j+(3+\lambda)k$ and KL respectively,where $K(-1,-6,6)$ and $L(7,4,2)$.

(a) Find the vector equation of ϕ

(b) Show that π and ϕ intersect at right angles

(c) Determine the equation of the line that perpendicularly cuts both lines

23. (a) Calculate the angle between the line $r=(4-2\alpha)i+(1-\alpha)j-3\alpha k$ and the plane $\mathbf{p}.\begin{pmatrix}2\\3\\1\end{pmatrix}=-12$

(b) Calculate the area of the triangle with vertices P$(2,-1,3)$,Q$(-3,2,1)$, and R$(1,3,-2)$

24. (a) Given the vectors $8i-2j+5k$ and $i+2j+pk$ are perpendicular,find the value of the constant p.

(b) The line L_1 passes through the point $(-3, 1, 5)$ and is parallel to the vector $7i - j - k$.

i. Write down a vector equation of the line L_1

ii. The line L_2 has vector equation $r = i - 2j + 2k + \mu(i + 8j - 3k)$.Show that L_1 and L_2 do not intersect

25. The origin **O** and the points A,B and C are such that **OABC** is a rectangle. With respect to **O** ,the position vectors of the points A and B are $-4i + pj - 6k$ and $-10i - 2j - 10k$.

(a) Find the value of the positive constant p

(b) Find a vector equation of the line **AC**

(c) Show that the line AC and the line ,L, with vector equation

$$r = 3i + 7j + k + \mu(-4i - 4j - 3k)$$

intersect and find the position vector of the point of intersection.

(d) Find the acute angle between the lines AC and L

GEOMETRY

26. The circles with equations $x^2 + (y - 4)^2 = 16$ have $(x - 4)^2 + (y - 1)^2 = 1$ have centres at A and B respectively.

(a) Show that these circles touch each other externally

(b) Find the equation of the common tangent at their point of touch

(c) The two circles also touch the x- axis at the origin ,**O** and point $P(4, 0)$ respectively .Calculate the area of the shape 0ABP

27. (a) The line $y = mx+c$ meets the curve $x^2+y^2 = r^2$ at only point.Show that $c^2 = r^2(1+m^2)$.

(b) Find the possible equations of the lines drawn from $(-1, 13)$ and touching the curve $x^2 + y^2 = 10$

28. The equation of the chord PQ of a curve is $pqy + x = 2(p + q)$ for parameters **p** and **q** at points **P** and **Q** respectively.

(a) Deduce the equation of the tangent to this curve at a point with parameter ,t.

(b) The normal at $T(2t, \frac{2}{t})$ meets the curve again at point R.Find the coordinates of R in terms of parameter ,t.

29. (a) $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are points on the parabola $y^2 = 4ax$.If the chord passes through the focus ,show that $pq = -1$.If M is the midpoint of PQ ,deduce that the locus of M is $y = 2a(x - a)$

(b) Show that the equation $y^2 = 9(x + y)$ represents a parabola;hence determine its focus and directrix

30. (a) A tangent from the point $Q(q^2, 2q)$ touches the curve $y^2 = 4x$ .Find

i. the equation of the tangent

ii. the equation of line L parallel to the normal at $(q^2, 2q)$ and passes through $(1, 0)$

iii. The point of intersection , P of the line L and the tangent

ANALYSIS

31. The equation of a curve is $y = \frac{e^{-3x+2}}{x+1}$ where $x < -1$.

    (a) Show that $\frac{dy}{dx} = \frac{e^{-3x+2}}{(x+1)^2}(Cx + D)$ where C and D are integers to be found.

    (b) Hence show that there is only one stationary point on the curve and find its exact coordinates.

32. (a) On the same axes,sketch the curves $f(x) = 2x^2 + 4x - 6$ and $g(x) = \frac{1}{2x^2+4x-6}$.

    (b) Find the volume of the solid generated when the region between the curve $f(x) = 2x^2 + 4x - 6$,lines $x = -1, x = 0$ and $y = 0$ is rotated through 360^0 about the line $y = 0$

33. (a) A cone with a semi-vertical angle of 30^0 collects very soft ice-cream from a vendor's machine at a rate of 9 cm^3 per second. Calculate the rate at which the surface area of the ice-cream increases when the ice cream reaches a depth of 10 cm.

    (b) A cylinder is inscribed in a hemisphere of radius ,R .Its length lies along the diameter of the hemisphere .Show that the maximum volume of this cylinder is $\frac{\sqrt{3}}{9}\pi R^3$

34. In a community of 1000 people, a rumour spreads at a rate proportional to the product of the population that has heard the rumour and the population that has not yet heard the rumour. Initially, at 8:00 a.m., 100 people had heard the rumour. By this time, 10 people per hour had heard the rumour.

    (a) Write a differential equation for this scenario.

    (b) i. Determine how many people will have heard the rumour by 8:00 a.m. the next day.

    ii. When will 900 people have heard the rumour?

35. (a) Find $\int \cos^2 3y \sin y$ dy

    (b) Using the substitution $2x = \sin u$, evaluate

$$\int_0^{\frac{1}{4}} \sqrt{\frac{1-2x}{1+2x}} dx$$

    (c)

$$\int_0^1 \frac{\tan^{-1} x}{1-x^2} dx$$

36. (a) Differentiate $\sec x$ with respect to x from first principles.

    (b) Given that $\frac{(2y-x)e^x}{y+2x} = 1$, find $\frac{dy}{dx}$ in terms of x

37. The parametric equations of a curve are $x = t + 4\text{In}t$, $y = t + \frac{9}{t}$,for $t > 0$.

(a) Show that

$$\frac{dy}{dx} = \frac{t^2 - 9}{t^2 + 4t}$$

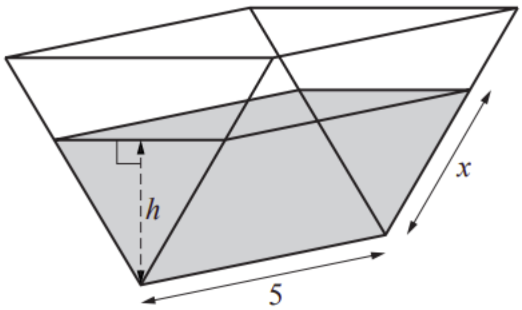
(b) The curve has one stationary point.Find the y - coordinate of this point and determine whether it is a maximum or a minimum point.

38. (a) Differentiate e^{kx} from first principles

(b) Use small changes to show that $(16.02)^{\frac{1}{4}} = 2\frac{1}{1600}$

(c) Differentiate $\frac{e^{5x}\cos 2x}{\ln(1-x)}$ with respect to x

39. The diagram shows a water container in the shape of a triangular prism. The depth of water in the container is h. The container has length 5. The water in the container forms a prism with a uniform cross-section that is an equilateral triangle of side x.In this question all lengths are in metres.



(a) Show that the volume ,V ,of the water is given by

$$V = \frac{5\sqrt{3}h^2}{3}$$

(b) Water is pumped into the container at a rate of $0.5m^3$ per minute. Find the rate at which the depth of the water is increasing when the depth of the water is 0.1m.

40. Show that

(a)

$$\int \frac{1}{1 + 2\sin^2 x} dx = \frac{1}{\sqrt{3}} \tan^{-1}(\sqrt{3}\tan x) + c$$

(b)

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{3\sin x + 4\cos x} dx = \frac{3\pi}{50} + \frac{4}{25}\ln\left(\frac{4}{3}\right)$$

(c)

$$\int_0^{36} \frac{1}{\sqrt{x}(\sqrt{x} + 2)} dx = \ln|16|$$

(d)

$$\int \frac{1}{\operatorname{cosec}2x - \cot 2x} dx = \ln|\sin x| + c$$

41. The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding .Initially the temperature of the body is 80^0C .After 10 minutes the temperature of the body is 60^0C .The temperature of the surrounding is 15^0C

    (a) Form a differential equation for the temperature of the body

    (b) Determine the time it takes for the temperature of the body to reach 40^0C

42. Maize dwarf mosaic virus(MDMV) has infected a number of maize plants Mr Ronalds' garden .The growth in the number of maize plants infected is proportional to the number already infected .Initially 20 maize plants were infected

    (a) Form a differential equation that models the growth in the number infected

    (b) Thirty days after the initial number of infections ,60 maize plants were infected .After how many further days does the model predict that 200 maize plants will be infected?

43. (a) Prove that

$$\int_1^3 \left(\tfrac{3-x}{x-1}\right)^{\frac{1}{2}} dx = \pi$$

    Hint : Use the substitution $x = 3\sin^2\theta + \cos^2\theta$

    (b) Show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin 3x \sin x = \frac{1}{8}\sqrt{3}$$

44. Show that

$$\int_0^{\ln 2} \frac{e^{2x}}{(1+e^x)(2e^x+1)} dx = \frac{1}{2}\ln\frac{27}{20}$$

45. (a) Show that $\int_2^4 4x \ln x dx = 56\ln 2 - 12$.

    (b) Use the substitution $u = \sin 4x$ to find the exact value of

$$\int_0^{\frac{1}{24}\pi} \cos^3 4x dx$$

END

APPLIED MATHEMATICS P425/2

TOPICS	SECTION A	SECTION B
NUMERICAL ANALYSIS	2	2
STATISTICS AND PROBABILITY	3	3
STATIC MECHANICS	1	1
DYNAMIC MECHANICS	1	1
KINEMATIC MECHANICS	1	1

Mechanics

1. A particle moves a long a horizontal line. The particle's displacement ,in metres ,from an origin O at time t seconds is given by $r(t) = 4 - 2\sin 2t$.

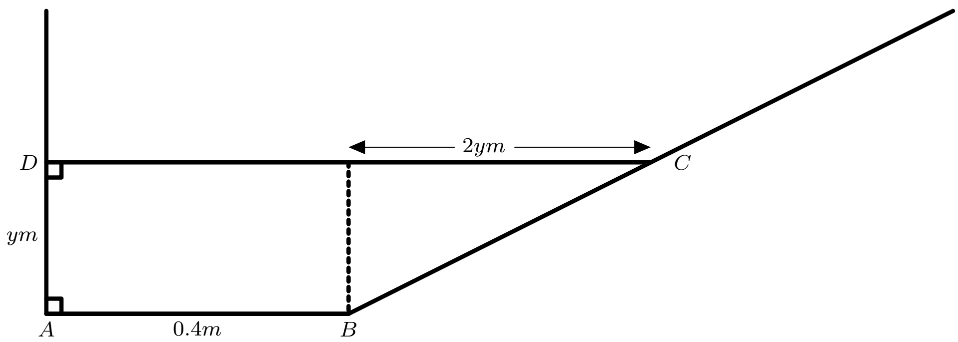
   (a) Find the particle's velocity and acceleration at any time t.

   (b) Find the particle's initial displacement,velocity and acceleration

   (c) Find when the particle is at rest,moving to the right and moving to the left during the time $0 \leq t \leq \pi$

   (d) Find the displacement of the particle after πseconds

2. A light container has a vertical cross-section in the form of a trapezium. The container rests on a horizontal surface.



   Grain is poured into the container to a depth of y m. As shown in the diagram, the cross-section ABCD of the grain is such that $AB = 0.4m$ and $DC = (0.4 + 2y)m$

   (a) When $y = 0.3m$, find the vertical height of the centre of mass of the grain above the base of the container.

   (b) Find the value of y for which the container is about to topple.

3. An object of mass $4kg$ is initially at rest at a point whose position vector is $(-4i + 2j)m$.If its acted upon by a force $F = (14i + 21j + 24k)N$. Find

   (i) acceleration of the object

   (ii) its velocity and speed after 2seconds

   (iii) its distance from the origin after 4seconds

4. A particle is projected from a point on level ground such that its initial velocity is $60ms^{-1}$ at an angle of elevation 30^0 and taking $g = 10ms^{-2}$,find

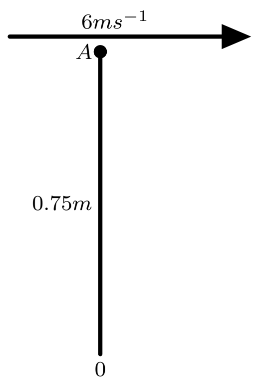
(a) the time taken for the particle to reach its maximum height

(b) the maximum height

(c) the time of flight

(d) the horizontal range of the particle

5. One end of a light inextensible string of length $0.75m$ is attached to a particle A of mass 2.8 kg. The other end of the string is attached to a fixed point O. A is projected horizontally with speed 6 ms^{-1} from a point 0.75 m vertically above O (see Fig below). When OA makes an angle θ with the upward vertical ,the speed of A is vms^{-1} .



(a) Show that $v^2 = 50.7 - 14.7\cos\theta$.

(b) Given that the string breaks when the tension in it reaches 200 N, find the angle that OA turns through between the instant that A is projected and the instant that the string breaks.

6. (a) i. Forces of $6i+5j, -10i-4j, 7i-7j, -8i+2j, 5i+4j$N act at the points $(2, 2), (5, 0), (-4, -4), (0, -$ $(6, 0)$ rspectively.Show that the system reduces to a couple and state the moment of the couple

ii. Forces of magnitude 10 N, 15 N and 20 N act away from a common point in the directions $S30^0E$, E 60^0N and North-West respectively. Find the resultant force.

(b) Forces of magnitude $2N, 2N, 3N, 4N, 2\sqrt{2}N$ and $\sqrt{2}N$ act along sides **AB,BC,DC,AD,AC** and **DB** respectively. Where the square is of side $2m$.Find the ;

i. Resultant force

ii. Equation of the line of atcion of the resultant force and where it cuts the $x-$ axis

7. At time $t = 0$,the position vector **r** and velocity **v** of two trains A and B are as follows.

Trains	Velocity vector	Position vector
A	$V_A = (-6i + k)ms^{-1}$	$r_A = (i + 2j + 3k)m$
B	$V_B = (-5i + j + 7k)ms^{-1}$	$r_B = (4i - 14j + k)m$

If the trains maintain these velocities,find the :

(a) Position of B relative to A at time t

(b) time that elapses before the trains are closest to each other

(c) least distance between the trains in the subsequent motion

8. A horizontal turn table rotates with constant angular speed $\omega rads^{-1}$ about its centre O. A particle P of mass $0.08kg$ is placed on the turntable. The particle moves with the turntable and no sliding takes place.

   (a) It is given that $\omega = 3$ and that the particle is about to slide on the turntable when $OP = 0.5m$. Find the coefficient of friction between the particle and the turntable.

   (b) Given instead that the particle is about to slide when its speed is $1.2ms^{-1}$, find ω.

9. A car of mass 1000kg is moving on a straight horizontal road. The driving force of the car is $\frac{28000}{v}$N and the resistance to motion is $4v$N, where vms^{-1} is the speed of the car t seconds after it passes a fixed point on the road.

   (a) Show that

$$\frac{dv}{dt} = \frac{7000 - v^2}{250v}$$

   (b) The car passes points A and B with speeds $10ms^{-1}$ and $40ms^{-1}$ respectively. Find the time taken for the car to travel from A to B.

10. (a) A particle of mass 5kg resting at point $(1, -4, 4)$ is acted upon by three forces $.F_1 = 3i + 3j, F_2 = 2j + 4k, F_3 = 2i + 6k$. Find the position and momentum of the particle after 4 seconds.

    (b) A particle of mass 4kg moves with a velocity of $e^t i + 2e^{2t} j - \sin tk$.Find the power developed after 4 seconds.

Statistics and Probability

11. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{36}(9 - x^2) & ; -3 \leq x \leq 3 \\ 0 & ; \text{otherwise} \end{cases}$$

    (a) Find the mean and variance of X.

    (b) Calculate

       i. $P(X > 2)$

       ii. $P(|X| > \sigma)$,where σ is the standard deviation of X

12. The probability that Mawejje will play in the CECCAFA challenge champion for his team ,Uganda cranes is 0.8.The probability that Uganda cranes wins the tournament when Mawejje is playing is 0.75 ,otherwise it is 0.5.Find the probability that

    (i) Uganda cranes wins the tournament

    (ii) Mawejje is playing if they lose the tournament

13. The random variable X denotes the number of hours of cloud cover per day at a weather forecasting center.The probability density function of X is given by

$$f(x) = \begin{cases} \frac{(x-18)^2}{k} & ; 0 \leq x \leq 24 \\ 0 & ; \text{otherwise} \end{cases}$$

where k is a constant

(a) Show that $k = 2016$

(b) On how many days in a year of 365 days can the centre expect to have less than 2 hours of cloud cover

(c) Find the mean number of hours of cloud cover per day.

14. The continuous random variable T represents the time in hours that students spend on homework. The cumulative distribution function of T is

$$F(t) = \begin{cases} 0 & ; t < 0 \\ k(2t^3 - t^4) & ; 0 \leq t \leq 1.5 \\ 1 & ; t > 1.5 \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{16}{27}$

(b) Find the proportion of students who spend more than 1 hour on homework.

(c) Find the probability density function, $f(t)$ of T.

(d) Show that $E(T) = 0.9$.

15. A box contains 1 red bead and I white bead. When a bead is drawn from the box, it is returned together with a bead of the other colour. If three such random draws are made,

(a) Find the probability that the second and third beads drawn are of the same colour.

(b) i. Construct a probability distribution for the number of red beads in the box after the experiment.

ii. Find the expected number of red sweets in the box after the draws.

16. Bottles of wine are stacked in racks of 12 . The weights of these bottles are normally distributed with mean $1.3kg$ and standard deviation of 0.06kg.The weights of the empty racks are normally distributed with mean $2kg$ and standard deviation of $0.3kg$.

(a) Find the probability that the total weight of a full rack of 12 bottles of wine is between 17kg and 18kg.

(b) Two bottles of wine are chosen at random .Find the probability that they differ in weight by morethan $0.05kg$

17. Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all the four coins at once. The number of heads he obtains is denoted by a random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$P(X = X)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find b and c.

(b) Find $E(X)$

(c) Jacob throws all four coins together 10 times

i. Find the probability that he obtains exactly one head on fewer than 3 occasions.

ii. Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the four coins.

18. There are three sets of traffic lights on Masandugu's journey to work. The independent probabilities that Masandugu has to stop at the first, second, and third set of lights are $0.4, 0.8$, and 0.3 respectively.

(a) Draw a tree diagram to show this information.

(b) Find the probability that Masandugu has to stop at each of the first two sets of lights but does not have to stop at the third set.

(c) Find the probability that Masandugu has to stop at exactly two of the three sets of lights.

(d) Find the probability that Masandugu has to stop at the first set of lights, given that she has to stop at exactly two sets of lights.

19. The weight of male leopards in a particular region are normally distributed with mean 55kg and standard deviation 6kg.

(a) Find the probability that a randomly chosen leopard from this region weighs between 46kg and 62kg

(b) The weight of female leopards in this region are normally distributed with mean 42kg and standard deviation σ kg. it is known that 25% of female leopards in the region weigh less than 36kg. Find the value of σ .

20. The lengths , in cm, of the leaves of a particular type are modelled by the distribution $N(5.2, 1.5^2)$.

(a) Find the probability that a randomly chosen leaf of this type has length less than 6cm

(b) The length of leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3cm long and 95 are more than 8cm long. Find estimates for the mean and standard deviation of the length of leaves of this type.

21. Given that $P(AUB) = \frac{9}{10}, P(A/B) = \frac{1}{3}$ and $P(B/A) = \frac{2}{5}$,find :

(a) P(A)

(b) $P(A^1/B^1)$

(c) P(A or B but not both A and B)

22. The weights in kg of 25 boys were as follows.

Weights,x(kg)	Frequency
20 – 24	3
25 – 29	5
30	2
31 – 34	6
35 – 49	9

(a) Calculate the

i. Mean weight

ii. number of boys weighing between 26.5kg and 32.5kg

(b) i. Draw a histogram for the data

ii. Use your histogram to estimate the modal weight

23. (a) The table below shows the marks of mathematics (x) and SST (y) obtained by certain students

Math(x)	35	65	55	25	45	75	20	90	51	60
SST(y)	86	70	84	92	79	68	96	58	86	77

i. Plot a scatter diagram for the data, draw a line of best fit and comment on the relationship between the scores in the two tests.

ii. Estimate mathematics mark (x) when SST mark (y) is 84

(b) Calculate the rank correlation coefficient and comment at 5% level of significance

24. The lifetime in hours of 80 solar bulbs were as follows

Lifetime(Hours)	Number of bulbs
Below 10	14
10 and under 20	19
20 and under 30	15
30 and under 40	20
40 and under 50	12

(a) Calculate the mean life time and variance

(b) i. Draw an Ogive for the data .

ii. Use it to estimate the lifetime exceeded by 75% of the bulbs

25. The table below shows the prices and price indices for three items in the years 2023 and 2024 respectively.

Item (kg)	Price in 2023	Price indices in 2024 using 2023 as base
X	3200	125
Y	4000	105
Z	4500	120

(a) Calculate the price of each item in 2024

(b) Taking Y as the base item ,calculate the price indices for 2023

(c) Using weights 3,5 and 2 for items X,Y and Z respectively .Calculate the:

i. Composite price index for the items in 2024

ii. Weighted aggregate price index of the items in 2024

Numerical Analysis

26. (a) Use the trapezium rule with 7 ordinates to estimate $\int_0^6 xe^{-x}dx$ correct to 3 significant figures

(b) Find the percentage error made in your estimation ,giving your answer to 2 deciml places .Suggest how this error may be reduced.

27. (a) Find the range within which the exact value of z lies, given that

$$Z = \frac{1}{x} + \frac{1}{y} + xy, \qquad x = 4.165 \pm 0.001, \qquad y = 6.72 \pm 0.01$$

(b) Use the trapezium rule with 11 ordinates to find the approximate value of $\int_1^2 x\log_{10} xdx$ to 4 d.p

(c) Find the exact value of $\int_1^2 x\log_{10} xdx$ .Hence calculate the error in (b) above. How can this error be reduced when using the trapezium rule.

28. By constructing a table of values for $f(x) = 3xe^x - 1$ in the range $0.1 \leq x \leq 1.1$, using intervals of 0.2, obtain;

(i) the value of f (1.13) using Linear extrapolation.

(ii) the root of f(x) correct to 3d.p using Newton-Raphson's formula.

29. (a) Show that the iterative formula for solving the equation $4x - \sec^2 x = 0$ is $x_{n+1} = \frac{1}{4}\sec^2 x_n$. Starting with $x_0 = 0.2$, find the solution of the equation to 4 significant figures.

(b) The numbers x,y and z were estimated with errors δx,δy and δz respectively .Show that the maximum relative error in $\frac{xy^2}{z}$ is

$$\left|\frac{\delta x}{x}\right| + 2\left|\frac{\delta y}{y}\right| + \left|\frac{\delta z}{z}\right|$$

.State the assumptions made

30. The table values of $\tan\Theta$ have been extracted from four figure tables

Θ	75	76	77	78	79
$\tan\Theta$	3.7321	4.0108	4.3315	4.7046	5.1446

Estimate

(i) $\tan^{-1}(4.6500)$

(ii) $\tan 79^0 36'$

31. (a) Use Newton Raphson's formula to show that the sixth root of a number N is given by

$$x_{n+1} = \frac{5}{6}\left[x_n + \frac{N}{5x_n^5}\right]$$

(b) Draw a flow chart that:

(i) Reads N and the initial approximation x_0

(ii) Computes and prints the roots to 3 d.p

(iii) Print N and the root

(c) Taking N=30.5 and $x_0 = 1.2$ perform a dry run for the flow chart.

32. The numbers $M = 5.83$ and $N = -2.456$ were rounded off to the given number of decimal places.

(a) State the maximum possible errors in M and N

(b) Find the range in which $\frac{M}{N-M}$ lies.

33. The radius r ,heigh h of a cone are measured with errors Δr and Δh respectively.

(a) Show that the maximum relative error in its volume is

$$2\left|\frac{\delta r}{r}\right| + \left|\frac{\Delta h}{h}\right|$$

(b) If $r = 3.55$ and $h = 12.4cm$.Find the percentage error in its volume.

34. (a) Use the graphical approach method to estimate the root of the equation $2\sin x - Inx = 0$ in the interval $2 \leq x \leq 3$.

(b) Use Newton Rahson method ,find the root of the equation $2\sin x - Inx = 0$ taking the approximate root obtained in (a) as the initial value of x_o.Give your answer to 3 decimal places.

35. The charges of sending parcels by JEFF distributing company depends on the weights of the parcels. For the parcels of weight 500g, 1kg, 1.5kg, and 2kg the charges are 1000/=, 2000/=, 3500/=, 4000/= respectively. Estimate

(a) What the distributor would charge for a parcel of weight 450g

(b) What the distributor would charge for a parcel of weight 1.8kg

(c) If the sender pays 6200/= what is the weight of his parcel

36. (a) Show that the equation $\pi \sin x - x = 0$ has a root between $\frac{\pi}{2}$ and π .Hence use linear interpolation only once to find the root,correct to three significant figures.

(b) Use the trapezium rule with 6 ordinates ,estimate $\int_0^{\frac{1}{3}\pi} \frac{1}{2+\tan x}$,correct to 3 decimal places.

(c) Calculate ther percentage error in your estimation in (b) above and state two ways of reducing the error.

PURE MATHEMATICS P425/1

1. Algebra

   (a) For the quadratic equation $ax^2 + bx + c = 0$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   (b) For an arithmetic series (A.P)

   $$u_n = a + (n-1)d$$
   $$S_n = \frac{1}{2}n\{2a + (n-1)d\}$$

   (c) For a geometric series (G.P)

   $$u_n = ar^{n-1}$$
   $$S_n = \frac{a(1-r^n)}{1-r} \qquad r \neq 1$$
   $$S_\infty = \frac{a}{1-r} \qquad |r| < 1$$

   (d) Binomial expansion

   $$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \binom{n}{3} a^{n-3}b^3 + \cdots + b^n$$

   where n is a positive integer

   $$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$
   $$^nC_r = \frac{n!}{r!(n-r)!}$$
   $$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1(n-2)}{3!}x^3 \cdots$$

   Where n is rational and $|x| < 1$

   $$^nP_r = \frac{n!}{(n-r)!}$$

   Where $r \leq n$

   (e) Summations

   $$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$
   $$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$
   $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

2. Trigonometry

No	**Identity**
1	$\tan\theta = \frac{\sin\theta}{\cos\theta}$,$\sec\theta = \frac{1}{\cos\theta}$,$co\sec\theta = \frac{1}{\sin\theta}$
2	$\cos^2\theta + \sin^2\theta = 1$
3	$1 + \tan^2\theta = \sec^2\theta$
4	$\cot^2\theta + 1 = co\sec^2\theta$
5	$\sin(A+B) = \sin A\cos B + \cos A\sin B$
6	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
7	$\cos(A+B) = \cos A\cos B - \sin A\sin B$
8	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
9	$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$
10	$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$
11	$\sin 2A = 2\sin A\cos A$
12	$\cos 2A = \cos^2 A - \sin^2 A$
13	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(a) For the t -formula

$$t = \tan\frac{1}{2}\theta$$
$$\sin\theta = \frac{2t}{1+t^2}$$
$$\cos\theta = \frac{1-t^2}{1+t^2}$$

(b) For any triangle with angles ,A,B and C and with sides a,b,and c .

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{Cosine rule}$$
$$s = \frac{a+b+c}{2}$$

3. Differentiation

No	y	$\frac{dy}{dx}$
1	x^n	nx^{n-1}
2	$\ln x$	$\frac{1}{x}$ for $x \neq 0$
3	e^x	e^x
4	$\sin x$	$\cos x$
5	$\cos x$	$-\sin x$
6	$\tan x$	$\sec^2 x$
7	uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
8	$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
9	$f(x)$	$\frac{f(x+\delta x)-f(x)}{\delta x}$
10	$\sec x$	$\sec x\tan x$
11	y=u,u=x	$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$
12	$e^{f(x}$	$f^1(x)e^{f(x)}$

4. Integration

No	$f(x)$	$\int f(x)dx$
1	x^n	$\frac{x^{n+1}}{n+1}+c$ for $n \neq -1$
2	$\frac{1}{x}$ for $x \neq 0$	$\ln\|x\|+c$ n
3	e^x	e^x+c
4	$\sin x$	$-\cos x+c$
5	$\cos x$	$\sin x+c$
6	$\sec^2 x$	$\tan x+c$
7	$\int u\frac{dv}{dx}dx$	$uv-\int v\frac{du}{dx}dx$
8	$\int \frac{f^1(x)}{f(x)}dx$	$\ln\|f(x)\|+c$
9	$co\sec x\cot x$	$-co\sec x+c$
10	$\sec x\tan x$	$\sec x+c$
11	$co\sec^2 x$	$-\cot x+c$
12	$\tan x$	$\ln\|\sec x\|+c$
13	$co\sec x$	$-\ln\|co\sec x+\cot x\|+c$
14	$\cot x$	$\ln\|\sin x\|+c$

(a)

$$\int \frac{1}{a^2-b^2x^2}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)+c$$

(b)

$$\int \frac{1}{a^2+b^2x^2}dx = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)+c$$

(c)

$$\int \frac{a}{p+qx}dx = \frac{a}{q}\ln|p+qx|+c$$

5. Vectors

(a) If $a = a_1i + a_2j + a_3k$ and $b = b_1i + b_2j + b_3k$ then

$$\begin{aligned} a.b &= a_1b_1 + a_2b_2 + a_3b_3 \\ &= |a||b|\cos\theta \end{aligned}$$

(b) $i.i = j.j = k.k = 1$ and $i.j = i.k = j.k = 0$

(c) $|a.a| = |a|^2$

(d) $a.(b+c) = a.b + a.c$

(e) $a.(kb) = (ka).b = k(a.b)$ where k is a constant

(f) $a.b = |a||b|\cos\theta$

(g) The cartesian equation of the line

$$\frac{x-a}{x_1} = \frac{y-b}{y_1} = \frac{z-c}{z_1}$$

APPLIED MATHEMATICS P425/2

1. Numerical Methods

   (a) Trapezium rule

   $$\int_a^b f(x)dx \approx \frac{1}{2}h\{y_o + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n\}$$

   Where $h = \dfrac{b-a}{n}$

   (b) Newton Raphson Method

   $$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)} \qquad \text{Where } x = 0, 1, 2, \cdots$$

   (c) Ordinates and sub intervals

   The number of sub intervals = Number of ordinates − 1

   (d) The maximum possible error made due to rounding off is given by

   $$\text{Error} = 0.5 \times 10^{-n}$$

   Where **n** is the number of decimal places rounded off to

   (e) Error

   $$\text{Error} = \text{Exact value} - \text{Approximate value}.$$

   (f) Absolute error This is the actual size of the error and is always positive .It is the magnitude of the error

   $$\text{Error} = |\text{Exact value} - \text{Approximate value}|.$$

   (g) Relative error

   $$\text{Relative Error} = \frac{\text{Absolute error}}{\text{Exact value}}$$

   The relative error must always be positive

   $$\text{Relative Error} = \frac{|\text{Error}|}{\text{Exact value}}$$

   (h) Percentage error

   $$\text{Percentage Error} = \frac{|\text{Error}\ |}{\text{Exact value}} \times 100$$

   (i) The interval or range with in which the exact value lies is given by Min value≤ Exact value ≤ Max value or [Min,Max]

(j) Absolute error $=\frac{\text{Maximum value}-\text{Minimum value}}{2}$

2. Probability and Statistics

(a) The mean for ungrouped data is calculated using the formula

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of values in the data}}$$
$$\overline{X} = \frac{\sum x}{n}$$

(b) The mean for grouped data is calculated using the formula

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

Where **x** is the class mark and **f** is the frequency

(c) The mean for grouped data when given an assumed means is calculated using the formula

$$\text{Mean} = A + \frac{\sum fd}{\sum f}$$

Where **A** is the assumed mean or working mean

d is the deviation given as $d = x - A$

(d) For grouped data ,the median is calculated using

$$\text{Median } = L_1 + \left(\frac{\frac{\sum f}{2} - CF_b}{f_m}\right) \times C$$

Where

$$L_1 = \text{Lower class boundary of the median class}$$
$$CF_b = \text{Cummulative frequency before the median class}$$
$$f_m = \text{frequency within the median class}$$
$$C = \text{Class width}$$
$$\sum f = \text{Total frequency}$$

(e) For grouped data with equal class width the mode is calculated using

$$\text{Mode} = L_1 + \left(\frac{d_1}{d_1 + d_2}\right) \times C$$

Where

$$L_1 = \text{Lower class boundary of the modal class}$$
$$d_1 = \text{Modal frequency} - \text{Pre modal frequency}$$
$$d_2 = \text{Modal frequency} - \text{Post modal frequency}$$
$$C = \text{Class width}$$

(f) For grouped data ,the lower quartile is calculated using

$$q_1 = L_1 + \left(\frac{\frac{\sum f}{4} - CF_b}{f_m}\right) \times C$$

Where

$$L_1 = \text{Lower class boundary of the } q_1 \text{ class}$$
$$CF_b = \text{Cummulative frequency before the } q_1 \text{ class}$$
$$f_m = \text{frequency within the } q_1 \text{ class}$$
$$C = \text{Class width}$$
$$\sum f = \text{Total frequency}$$

(g) For grouped data ,the upper quartile is calculated using

$$q_3 = L_1 + \left(\frac{\frac{3\sum f}{4} - CF_b}{f_m}\right) \times C$$

Where

$$L_1 = \text{Lower class boundary of the } q_3 \text{ class}$$
$$CF_b = \text{Cummulative frequency before the } q_3 \text{ class}$$
$$f_m = \text{frequency within the } q_3 \text{ class}$$
$$C = \text{Class width}$$
$$\sum f = \text{Total frequency}$$

(h) Inter quartile range $= q_3 - q_1$

(i) For grouped data ,the variance is calculated using

$$\text{Var(x)} = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$$

(j) Standard deviation $= \sqrt{\text{Var(x)}}$

3. Index numbers

(a)

$$\text{Price relative } = \frac{p_n}{p_0} \times 100$$

Where

p_n = Price of the commodity in the given year(new year)

p_0 = Price of the commodity in the base year(old year)

(b)

$$\text{Simple price index} = \frac{\text{Sum of the price relatives}}{\text{Number of items (N)}} \times 100$$
$$= \frac{\sum \left(\frac{p_n}{p_0}\right) \times 100}{N}$$

(c)

$$\text{Simple aggregate price index} = \frac{\text{Current year price total}}{\text{Base price total}} \times 100$$
$$= \frac{\sum p_n}{\sum p_0} \times 100$$

(d)

$$\text{Weighted price index} = \text{Price relatives} \times \text{weights}$$
$$= \frac{p_n}{p_o} \times w \times 100$$

(e)

$$\text{Weighted aggregate price index} = \frac{\sum p_n w}{\sum p_0 w} \times 100$$

(f)

$$\text{Weighted average price index} = \frac{\sum \frac{p_n}{p_0} \times 100 \times w}{\sum w}$$

(g)

$$\text{Value index} = \frac{\sum p_n q_n}{\sum p_o q_o} \times 100$$

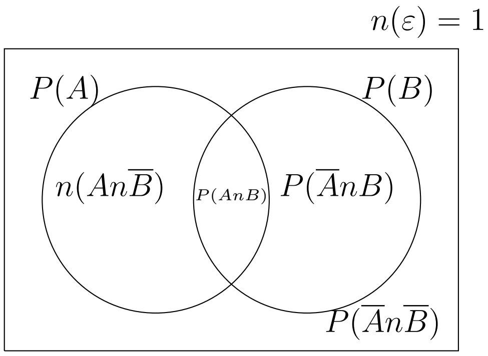
4. Spearman's rank correlation coefficient

$$\rho = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

Where n is the difference between the rankings of a given scores and n is the number of pairs

5. Probability theory

(a) For any events A and B



$$P(A) = P(A \cap \overline{B}) + P(A \cap B)$$
$$P(\overline{A}) = P(\overline{A} \cap B) + P(A \cup B)^1$$
$$P(B) = P(\overline{A} \cap B) + P(A \cap B)$$
$$P(\overline{B}) = P(A \cap \overline{B}) + P(A \cup B)^1$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(b) $P(A) + P(\overline{A}) = 1$

(c) $P(A \cup B)^1 = P(\overline{A} \cap \overline{B})$

(d) $P(\overline{A} \cup \overline{B}) = P(A \cap B)^1$

(e) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ for $P(B) \neq 0$

6. Mechanics

(a) For projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2}sec^2\theta$$

(b) For calculus

Physical quantity	Formula	units	Formula	units	Formula
Force	F=ma	N	$a = \frac{dv}{dt}$	ms^{-2}	k.e$=\frac{1}{2}mv^2$
Power	P=F.v	W	$v = \frac{dr}{dt} or \frac{ds}{dt}$	ms^{-1}	Avg accel$=\frac{v(t_2)-v(t_1)}{t_2-t_1}$
Work done	W=F.s or F.r	j	W$=\int_{t_1}^{t_2} f.vdt$	j	speed$=\|v\|$
Impulse	I=F.t	Ns	$v = \int adt$	ms^{-1}	Avg vel$=\frac{r(t_2)-r(t_1)}{t_2-t_1}$
Momentum	momentum =m.v	Kgms^{-1}	$r = \int vdt$	m	distance$=\|r\|or\|s\|$

STATISTICAL TABLES

SIGNIFICANCE LEVELS FOR CORRELATION COEFFICEINTS

	Product-moment coefficient of correlation (r_{xy})		Spearman's rank Correlation coefficient (ρ)		Kendall's rank coefficient of correlation (τ)	
No. of pairs	Significance if $\lvert r_{xy} \rvert$ exceeds at 5%	at 1%	Significance if $\lvert \rho \rvert$ exceeds at 5%	at 1%	Significance if $\lvert \tau \rvert$ exceeds at 5%	at 1%
3	1.00	1.00				
4	0.95	0.99				
5	0.88	0.96	1.00			
6	0.81	0.92	0.89	1.00	0.87	1.00
7	0.75	0.88	0.75	0.89	0.71	0.81
8	0.71	0.83	0.71	0.86	0.64	0.79
9	0.67	0.80	0.68	0.83	0.56	0.72
10	0.63	0.77	0.65	0.79	0.51	0.64
11	0.60	0.74	0.60	0.74	0.49	0.60
12	0.58	0.71	0.58	0.71	0.45	0.58
13	0.55	0.68	0.55	0.68		
14	0.53	0.66	0.53	0.66		
15	0.51	0.64	0.51	0.64		
16	0.50	0.62	0.50	0.62		
17	0.48	0.61	0.48	0.61		
18	0.47	0.59	0.47	0.59		
19	0.46	0.58	0.46	0.58		
20	0.44	0.56	0.44	0.56	0.33	
30	0.35	0.45	0.35	0.45		
40	0.31	0.39	0.31	0.39		
50	0.27	0.35	0.27	0.35		
60	0.25	0.33	0.25	0.33		
70	0.23	0.31	0.23	0.31		
80	0.22	0.29	0.22	0.29		
90	0.21	0.27	0.21	0.27		
100	0.20	0.25	0.20	0.25		

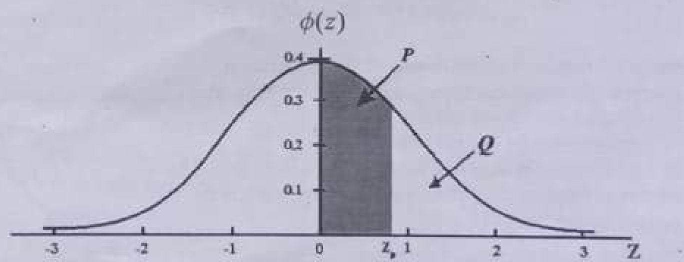
CUMULATIVE NORMAL DISTRIBUTION $P(z)$

											ADD								
z	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673							3	6	9	12	15	19	22	25	28
					2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023					3	6	8	11	14	17	20	22	25
							3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
0.9	0.3159	3186	3212	3238	3264	3289					3	5	8	10	13	16	18	21	23
							3315	3340	3365	3389	2	5	7	10	12	15	17	20	22
1.0	0.3413	3438	3461	3485	3508						2	5	7	10	12	14	17	19	22
						3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
1.1	0.3643	3665	3686	3708							2	4	6	8	11	13	15	17	19
					3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
1.2	0.3849	3869	3888	3907	3925						2	4	6	8	10	11	13	15	17
						3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives $P(z) = \int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P$(Shaded Area)
2. $P(Z > Z_p) = Q = ½ - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$



CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION) $\sum_{i \geq r} p_i$

n	r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	1	0.0199	0975	1900	2775	3600	4375	5100	5775	6400	6975	7500
	2	0.0001	0025	0100	0225	0400	0625	0900	1225	1600	2025	2500
3	1	0.0297	1426	2710	3859	4880	5781	6570	7254	7840	8336	8750
	2	0.0003	0072	0280	0608	1040	1562	2160	2818	3520	4252	5000
	3		0001	0010	0034	0080	0156	0270	0429	0640	0911	1250
4	1	0.0394	1855	3439	4780	5904	6836	7599	8215	8704	9085	9375
	2	0.0006	0140	0523	1095	1808	2617	3483	4370	5248	6090	6875
	3		0005	0037	0120	0272	0508	0837	1265	1792	2415	3125
	4			0001	0005	0016	0039	0081	0150	0256	0410	0625
5	1	0.0490	2262	4095	5563	6723	7627	8319	8840	9222	9497	9688
	2	0.0010	0226	0815	1648	2627	3672	4718	5716	6630	7438	8125
	3		0012	0086	0266	0579	1035	1631	2352	3174	4069	5000
	4			0005	0022	0067	0156	0308	0540	0870	1312	1875
	5				0001	0003	0010	0024	0053	0102	0185	0312
6	1	0.0585	2649	4686	6229	7379	8220	8824	9246	9533	9723	9844
	2	0.0015	0328	1143	2235	3446	4661	5798	6809	7667	8364	8906
	3		0022	0158	0473	0989	1694	2557	3529	4557	5585	6562
	4		0001	0013	0059	0170	0376	0705	1174	1792	2553	3438
	5			0001	0004	0016	0046	0109	0223	0410	0692	1094
	6					0001	0002	0007	0018	0041	0083	0156
7	1	0.0679	3017	5217	6794	7903	8665	9176	9510	9720	9848	9922
	2	0.0020	0444	1497	2834	4233	5551	6706	7662	8414	8976	9375
	3		0038	0257	0738	1480	2436	3529	4677	5801	6836	7734
	4		0002	0027	0121	0333	0706	1260	1998	2898	3917	5000
	5			0002	0012	0047	0129	0288	0556	0963	1529	2266
	6				0001	0004	0013	0038	0090	0188	0357	0625
	7						0001	0002	0006	0016	0037	0078
8	1	0.0773	3366	5695	7275	8322	8999	9424	9681	9832	9916	9961
	2	0.0027	0572	1869	3428	4967	6329	7447	8309	8936	9368	9648
	3	0.0001	0058	0381	1052	2031	3215	4482	5722	6846	7799	8555
	4		0004	0050	0214	0563	1138	1941	2936	4059	5230	6367
	5			0004	0029	0104	0273	0580	1061	1737	2604	3633
	6				0002	0012	0042	0113	0253	0498	0885	1445
	7					0001	0004	0013	0036	0085	0181	0352
	8							0001	0002	0007	0017	0039
9	1	0.0865	3698	6126	7684	8658	9249	9596	9793	9899	9954	9980
	2	0.0034	0712	2252	4005	5638	6997	8040	8789	9295	9615	9805
	3	0.0001	0084	0530	1409	2618	3993	5372	6627	7682	8505	9102
	4		0006	0083	0339	0856	1657	2703	3911	5174	6386	7461
	5			0009	0056	0196	0489	0988	1717	2666	3786	5000
	6			0001	0006	0031	0100	0253	0536	0994	1658	2539
	7					0003	0013	0043	0112	0250	0498	0898
	8						0001	0004	0014	0038	0091	0195
	9								0001	0003	0008	0020
10	1	0.0956	4013	6513	8031	8926	9437	9718	9865	9940	9975	9990
	2	0.0043	0861	2639	4557	6242	7560	8507	9140	9536	9767	9893
	3	0.0001	0115	0702	1798	3222	4744	6172	7384	8327	9004	9453
	4		0010	0128	0500	1209	2241	3504	4862	6177	7340	8281
	5		0001	0016	0099	0328	0781	1503	2485	3669	4956	6230
	6			0001	0014	0064	0197	0473	0949	1662	2616	3770
	7				0001	0009	0035	0106	0260	0548	1020	1719
	8					0001	0004	0016	0048	0123	0274	0547
	9							0001	0005	0017	0045	0107
	10									0001	0003	0010
11	1	0.1047	4312	6862	8327	9141	9578	9802	9912	9964	9986	9995
	2	0.0052	1019	3026	5078	6779	8029	8870	9394	9698	9861	9941
	3	0.0002	0152	0896	2212	3826	5448	6873	7999	8811	9348	9673
	4		0016	0185	0694	1611	2867	4304	5744	7037	8089	8867
	5		0001	0028	0159	0504	1146	2103	3317	4672	6029	7256

CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION)

n	*r*	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
11	6			0003	0027	0117	0343	0782	1487	2465	3669	5000
	7				0003	0020	0076	0216	0501	0994	1738	2744
	8					0002	0012	0043	0122	0293	0610	1133
	9						0001	0006	0020	0059	0148	0327
	10								0002	0007	0022	0059
	11										0002	0005
12	1	0.1136	4596	7176	8578	9313	9683	9862	9943	9978	9992	9998
	2	0.0062	1184	3410	5565	7251	8416	9150	9576	9804	9917	9968
	3	0.0002	0196	1109	2642	4417	6093	7472	8487	9166	9579	9807
	4		0022	0256	0922	2054	3512	5075	6533	7747	8655	9270
	5		0002	0043	0239	0726	1576	2763	4167	5618	6956	8062
	6			0005	0046	0194	0544	1178	2127	3348	4731	6128
	7			0001	0007	0039	0143	0386	0846	1582	2607	3872
	8				0001	0006	0028	0095	0255	0573	1117	1938
	9					0001	0004	0017	0056	0153	0356	0730
	10							0002	0008	0028	0079	0193
	11								0001	0003	0011	0032
	12										0001	0002
15	1	0.1399	5367	7941	9126	9648	9866	9953	9984	9995	9999	1.0000
	2	0.0096	1710	4510	6814	8329	9198	9647	9858	9948	9983	9995
	3	0.0004	0362	1841	3958	6020	7639	8732	9383	9729	9893	9963
	4		0055	0556	1773	3518	5387	7031	8273	9095	9576	9824
	5		0006	0127	0617	1642	3135	4845	6481	7827	8796	9408
	6		0001	0022	0168	0611	1484	2784	4357	5968	7392	8491
	7			0003	0036	0181	0566	1311	2452	3902	5478	6964
	8				0006	0042	0173	0500	1132	2131	3465	5000
	9				0001	0008	0042	0152	0422	0950	1818	3036
	10					0001	0008	0037	0124	0338	0769	1509
	11						0001	0007	0028	0093	0255	0592
	12							0001	0005	0019	0063	0176
	13								0001	0003	0011	0037
	14										0001	0005
20	1	0.1821	6415	8784	9612	9885	9968	9992	9998	1.0000	1.0000	1.0000
	2	0.0169	2642	6083	8244	9308	9757	9924	9979	9995	9999	1.0000
	3	0.0010	0755	3231	5951	7939	9087	9645	9879	9964	9991	9998
	4		0159	1330	3523	5886	7748	8929	9556	9840	9951	9987
	5		0026	0432	1702	3704	5852	7625	8818	9490	9811	9941
	6		0003	0113	0673	1958	3828	5836	7546	8744	9447	9793
	7			0024	0219	0867	2142	3920	5834	7500	8701	9423
	8			0004	0059	0321	1018	2277	3990	5841	7480	8684
	9			0001	0013	0100	0409	1133	2376	4044	5857	7483
	10				0002	0026	0139	0480	1218	2447	4086	5881
	11					0006	0039	0171	0532	1275	2493	4119
	12					0001	0009	0051	0196	0565	1308	2517
	13						0002	0013	0060	0210	0580	1316
	14							0003	0015	0065	0214	0577
	15								0003	0016	0064	0207
	16									0003	0015	0059
	17										0003	0013
	18											0002

To obtain $p(i \le r)$ use: $p(i \le r) = 1 - p(i \ge r + 1)$

Where a space in the table is empty the probability is less than 0.00005.