SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

(SMATA)

A'LEVEL MATHEMATICS

4TH ANNUAL POST - MOCK SEMINAR 2023

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ST JOSEPH OF NAZARETH HIGH SCHOOL

Sunday 24th September 2023

PURE MATHS P425/1

1. ALGEBRA
2. GEOMETRY
3. TRIGONOMETRY
4. ANALYSIS
5. VECTORS

APPLIED MATHS P425/2

1. STATISTICS
2. PROBABILITY
3. NUMERICAL METHOD
4. MECHANICS
   - DYNAMICS
   - STATICS

SEMINAR QUESTIONS 2023

PURE MATHEMATICS

ALGEBRA

1. Find the values of x for which $3^x + 6(3^{-x}) = 5$

2. Solve the pair of simultaneous equations:
$$\log_5(2x + y) = 0$$
$$2\log_5 x = \log_5(y - 1)$$

3. Solve the equation $\sqrt{2x-3} + \sqrt{x+2} = 3$

4. If $\boldsymbol{\alpha}$ *and* $\boldsymbol{\beta}$ are roots of the equation $4x^2 + 5x - 1 = 0$, find the equation whose roots are $\left(2 - \frac{\beta}{\alpha}\right)$ and $\left(2 - \frac{\alpha}{\beta}\right)$

5. Find the term independent of x in the expansion of $\left(2x + \frac{1}{2x^3}\right)^8$.

6. Expand $(3 - 2x)^{12}$ in ascending powers of x up to and including the term in x^3. Hence, evaluate $(2.998)^{12}$ correct to the nearest whole number.

7. Find the range of values of x can take for the inequality

   $\left|\frac{2x-4}{x+1}\right| < 4$ to be true.

8. The polynomial $8x^3 + ax^2 + bx - 1$, where a *and* b are constants is denoted by *P(x)*. It is given that $(x + 1)$ is a factor of $P(x)$ and that when the polynomial is divided by $(2x + 1)$, the remainder is 1.
   (i) Find the values of a and b.
   (ii) When a and b have these values, factorize *P(x)* completely.

9. (a) Find the possible number of ways of arranging the letters of the word **DIFFERENTIATION** in a line.

   (b) A committee of 5 people is to be chosen from 4 men and 6 women. Wilson is one of the 4 men and Martha is one of the 6 women. Find the number of different committees that can be chosen if Wilson and Martha refuse to be on the committee together.

10. (a) The first, second and last terms in an arithmetic progression (A.P) are 56, 53 and -22 respectively. Find the sum of all the terms in the progression.

(b) The first, second and third terms of a geometric progression (G.P) are $2k + 6$, $2k$ and

$k + 2$ respectively, where k is appositive constant.

(i) Determine the value of k.

(ii) the common ratio

(iii) Find the sum to infinity of the progression.

(c). The 1st, 3rd and 13th terms of an A.P are also the 1st, 2nd and 3rd terms respectively of a G.P. The first term of each progression is 3. Find the common difference of the A.P and the sum of the first 10 terms of a G.P.

11. (a) Given that, $z = \frac{1+2i}{1-3i}$. Find;

(i) modulus of Z.

(ii) argument of Z.

(iii). Express Z in polar form

(iv) Represent Z on a complex plane.

(b) If a complex number Z lies on the curve $|Z - (-1 + i)| = 1$, find the locus of the complex number, $w = \frac{Z+i}{1-i}$.

(c) Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

(d) Find x and y if $(x + 2i)(1 - yi) = (3 - i)^2$

DIFFERENTIATION

12. Given the curve $y = \frac{12}{x^2-2x-3}$

Determine the;

a) range of values for y in which the curve does not lie and hence find the coordinates of the turning point.

b) asymptotes and sketch the curve $y = \frac{12}{x^2-2x-3}$

13. Differentiate the following with respect to x

i) $y = x^2\, sin\left(\frac{1}{x}\right)$ ii). $y = x(1n^3x)$ iii). $\sqrt{\frac{(2x+3)^3}{(1+x)^2}}$

14. Given that $y = cosec^{-1}(x)$ prove that $\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}}$

15. An inverted cone with vertical angle 60^0 has water in it dripping out through a hole at the vertex at the rate of $9cm^3$ per minute. Find the rate at which it's level will be decreasing at an instant when the volume of water left in the cone is $9\pi cm^3$.

16. If $y = e^{2x}\sin 3x$, prove that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

17. Find the equation of the tangent to the curve $\frac{y}{1-y} + \frac{x}{1-x} + 5x - 3y = 0$ At the point (2,2).

18. If $T = 2\pi\sqrt{\frac{L}{10}}$. Find the approximate increase in T if l increases from 10:0m to 10.1m.

19. A circular cylinder open at the top is made so as to have a volume of $1cm^3$. If r is the radius of the base, prove that the total outside surface is $\pi r^2 + \frac{2}{r}$. Hence prove that this surface area is minimum when $h = r = \frac{1}{\sqrt[3]{\pi}}$.

INTEGRATION

20. Evaluate

i). $\int_2^6 \frac{\sqrt{x-2}}{x}dx$ ii). $\int_0^{\pi/4} \frac{sec\, x^2}{1+tan\, x}dx$

21. Solve the differential equation $x\frac{dy}{dx} = 2x - y$

22. By using a suitable substitution $x = sin\theta$

Evaluate $\int_0^{\frac{\sqrt{3}}{2}} \frac{x^3}{\sqrt{1-x^2}}dx$

23. a)i). Form a differential equation by eliminating the constant A from $y = Ae^{x2}$

ii) State the order of the differential equation formed.

b). A chapatti had reached at a temperature of 160 degrees in an oven. It was pulled out and allowed to cool in a room of temperature 70 degrees. After 20 minutes the chapatti had a temperature of 140 degrees. Given that the rate of cooling of the chapatti was directly proportional to the difference between its temperature T and that of its surrounding. How much longer would it take for chapatti to cool to 120 degrees?

24. Express $f(x) = \frac{x^3 - x^2 - 3x + 5}{(x-1)\,(x^2-1)}$ in partial fractions. Hence, find $\int f(x)\mathrm{dx}$.

TRIGONOMETRY

25. (a) Given that $\sin \mathrm{P} = \frac{3}{5}$ and $\cos \mathrm{Q} = \frac{15}{17}$ where P is acute and Q is obtuse, find the exact value of

(i) $sin(P+Q)$ (ii) $cos(P-Q)$ (iii) $cot(P+Q)$.

(b) Solve the equations (i) $7\,sin\,2\,A - 6\,cos\,2\,A = 7$

(ii) $cot\,A + tan\,A = 2\,cos\,e\,c^2 A$ for $0^0 \leq A \leq 360^0$.

26. (a) Given that $cot\,\beta = \frac{4+3\,tan\,\alpha}{3-4\,tan\,\alpha}$, deduce that $sin(\alpha+\beta) = \frac{3}{5}$.

(b) Show that if A, B and C are angles of a triangle, then $tan(A+B+C) = \frac{tan\,A + tan\,B + tan\,C - tan\,A\,tan\,B\,tan\,C}{1 - tan\,A\,tan\,B - tan\,A\,tan\,C - tan\,B\,tan\,C}$ and $cot\,A\,cot\,B + cot\,A\,cot\,C + cot\,B\,cot\,C = 1$.

27. (a) Solve the equation $7\,tan^2 A + 5\,sec\,A\,tan\,A + 1 = 0$

for $0^0 \leq A \leq 360^0$.

(b) Prove the following (i) $cot^{-1}\frac{1}{3} - cot^{-1} 3 = cos^{-1}\frac{3}{5}$

(ii) $4\,tan^{-1}\frac{1}{5} - tan^{-1}\frac{1}{239} = \frac{\pi}{4}$.

(c) In a trianglePQR, prove that $tan(Q-R) = \frac{2(q^2-r^2)\,cot\frac{P}{2}}{(q+r)^2-(q-r)^2\,cot^2\ \frac{P}{2}}$.

28. Express $10\,sin\,x\,cos\,x\ +\ 12\,cos\,2\,x$ in the form $R\,sin(2x\ +\ \alpha)$, hence or otherwise

(a) solve $10\,sin\,x\,cos\,x\ +\ 12\,cos\,2\,x\ +\ 7\ =\ 0$ in the range $0^o\ \leq\ x\ \leq\ 360^o$.

(b) determine the maximum and minimum values of $\frac{3}{10\,sin\,x\,cos\,x\ +\ 12\,cos\,2x - 17}$. State also the values of x for which they occur.

29. (a) Solve the equation $3\,tan^3 x - 3\,tan^2 x = tan\,x - 1$

for $0^0 \leq x \leq 360^0$

(b) In the triangle ABC, AB = 9 cm, AC = 12 cm, angle ABC = 2θ and angle ACB = θ. Find the (i) length of BC, (ii) area of the triangle ABC.

(c) The area of a triangle is 336 m^2. The sum of the three sides is 84 m and one side is 28 m. Calculate the lengths of the other two sides.

(d) A right- angled triangle has perpendicular sides of lengths t and r. If t and r are adjacent and opposite to one of the non – right angle β, respectively,

prove that $\frac{r+t}{r-t} = sec\, 2\,\beta + cot\, 2\,\beta$

GEOMETRY

30. The point C lies on the perpendicular bisector of the line joining the points A(4, 6) and B(10, 2). C also lies on the line parallel to AB through (3, 11).
    (i) Find the equation of the perpendicular bisector of AB.
    (ii) Calculate the coordinates of C.

31. A point P moves in such a way that the sum of its distance from (0, 2) and (0, - 2) is 6. Find the equation of the locus of P.

32. (a) Determine the equation of a circle which passes through the points $A(1,2), B(-1,6)$ and $C(-5,4)$. Hence calculate the length of the tangent from the point $T(5,4)$.

(b) Determine the equation of the circle with centre at $(1,5)$ and has a tangent passing through the points $A(-1,2)$ and $B(0,-2)$.

(c) Find the co-ordinates of the point of intersection of the common chord to the circles $x^2 + y^2 - 4y - 4 = 0$ and $x^2 + y^2 - x + y - 12 = 0$ and the line $y = 7 - 3x$.

(d) Determine the equation of a circle which passes through the point $(0,-1)$ and the intersection of the circles

$x^2 + y^2 + 2x - y - 5 = 0$ and $x^2 + y^2 + 3x + 4y + 1 = 0$.

33. (a) Show that the curve $y^2 - 8y = -4x - 4$ represents a parabola. Sketch the parabola and state its focus and equation of the directrix.

(b) The chord PQ of the parabola in (a) above subtends a right angle at the vertex, P and Q being $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$. Prove that $pq + 4 = 0$ and that the locus of the point of intersection of the normal at P and Q is $y^2 = 16a(x - 6a)$.

34. P is the point $(aT^2, 2aT)$ and Q is the point $(at^2, 2at)$ on the parabola $y^2 = 4ax$ .The tangents at P and Q intersect at R.

(a) Find (i) the equation of the chord joining the points P and Q and hence, deduce the equation of the tangents at P and Q,

(ii) the co-ordinates of R.

(b) Obtain the perpendicular distance of the point R from the straight line PQ and deduce that the area of the triangle PQR is $\frac{1}{2}a^2(T - t)^3$.

35 (a). Show that the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$. Hence determine the tangents at the points where the line $2x + y = 3$ cuts the ellipse $4x^2 + y^2 = 5$.

(b) Find the equations of the tangents from the point (4,4) to the hyperbola $9x^2 - 9y^2 = 16$.

(c) Determine the foci and equations of the directrices of the hyperbola $4x^2 - 25y^2 = 15$. Find also the asymptotes to the hyperbola.

VECTORS

36. (a) The points P, Q and R have position vector $-5\boldsymbol{i} + 3\boldsymbol{j} - 7\boldsymbol{k}$, $\boldsymbol{i} + 6\boldsymbol{j} + 2\boldsymbol{k}$ and $-\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$respectively. The point S divides PQ externally in the ratio$1:4$. The point T divides QR internally in the ratio $1:2$. Determine the distance ST.

(b) In a triangle OAB, $\overrightarrow{\boldsymbol{OA}} = \boldsymbol{a}$and $\overrightarrow{\boldsymbol{OB}} = \boldsymbol{b}$. C lies on OA where $OC = \frac{2}{3}\boldsymbol{OA}$, D is the mid-point of AB and BC and OD intersect at M. Find the ratios $OM:MD$and $BM:MC$.

37. (a) Find a vector $\mathbf{r}$ which makes an angle of 45° with $\mathbf{p}$ and is of magnitude$3\sqrt{10}$ units.

(b) Find the perpendicular distance of the point from the point $T(-2,-2,-1)$to the line joining the points $A(3,1,2)$and $B(-1,5,1)$.

(c) Calculate the angle between the line $\frac{2-x}{3} = y = \frac{6+3z}{-6}$ and the plane $\boldsymbol{r} = \begin{pmatrix}1\\1\\7\end{pmatrix} + \lambda\begin{pmatrix}5\\-4\\8\end{pmatrix} + \mu\begin{pmatrix}-2\\0\\3\end{pmatrix}$.

38. The lines L_1 and L_2 have vector equations $\boldsymbol{r} = -2\boldsymbol{j} + 5\boldsymbol{k} + \lambda(\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$ and $\boldsymbol{r} = \boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k} + \boldsymbol{\mu}(-\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$ respectively. Determine the:

(a) co-ordinates of the point of intersection of the lines L_1 and L_2

(b) Cartesian equation of the plane containing the lines L_1 and L_2.

(c) angle between the lines L_1 and L_2.

39. (a) Determine the co-ordinates of the foot of the perpendicular from the point $M(11,-13,8)$ to the plane $2x - 3y + z + 1 = 0$.

(b) Find the vector equation of the line of intersection of the planes $8x + 12y - 13 \rightleftarrows z = 32$and $4x + 4y - 5z - 12 = 0$.

40. The line L has equation $x - 7 = \frac{y-1}{2} = \frac{z+5}{-2}$ and the plane P has equation $\boldsymbol{r}.\begin{pmatrix}1\\2\\-2\end{pmatrix} = 37$.

(a) Find the point of intersection of L and P.

(b) Show that the coordinates of the two points on the line whose distances from the plane are of magnitude 3 units, are $(8,3,-7)$ and $(10,7,-11)$.

Part 1 of 2