

Holiday package. S5 2025**Principal mathematics. (PROBABILITY)****Os e-learning platform**

In rural areas of Uganda, like Kasese or Bundibugyo, students in S5 (Senior Five) are studying statistics to understand patterns of water collection. A continuous random variable X represents the time (in hours) villagers spend walking to collect water from a nearby stream. The probability density function of X is given by:

$$f(x) = \begin{cases} \frac{1}{4}x(4 - x^2) & ; 0 \leq x \leq 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

Real-Life Issue:

In many Ugandan rural communities, access to clean water is a challenge. Villagers often walk long distances to fetch water, affecting their time for other activities like farming or studying. Understanding the distribution of time spent on water collection can help in planning community water projects.

**Task.**

Task using the knowledge of principal mathematics determine ;

- (a) The most common time spent walking to collect water.
- (b) The time by which half of the villagers have collected water.
- (c) The time by which 60% of villagers have collected water.

Expected response.

- a) The most common time spent walking to collect water. Is obtained by calculating the mode.

The mode is the value of x that maximizes $f(x)$.

To find the mode, we differentiate $f(x)$ with respect to x and set it to zero.

$$f'(x) = \frac{1}{4}(4 - 3x^2)$$

Setting $f'(x) = 0$:

$$4 - 3x^2 = 0$$

$$x^2 = \frac{4}{3}$$

$$x = \frac{2}{\sqrt{3}} \approx 1.155 \quad (\text{since } 0 \leq x \leq 2)$$

Therefore The most common time spent walking to collect water is approximately 1.155 hours

(b) The time by which half of the villagers have collected water is obtained by calculating the median

The median m is found by solving:

$$\int_0^m \frac{1}{4}x(4 - x^2)dx = 0.5$$

$$\frac{1}{4} \int_0^m (4x - x^3)dx = 0.5$$

$$\frac{1}{4} \left[2x^2 - \frac{x^4}{4} \right]_0^m = 0.5$$

$$\frac{1}{4} \left(2m^2 - \frac{m^4}{4} \right) = 0.5$$

$$2m^2 - \frac{m^4}{4} = 2$$

Let $u = m^2$:

$$2u - \frac{u^2}{4} = 2$$

$$u^2 - 8u + 8 = 0$$

$$u = \frac{8 \pm \sqrt{64 - 32}}{2}$$

$$u = \frac{8 \pm 4\sqrt{2}}{2} = 4 \pm 2\sqrt{2}$$

$$m^2 = 4 - 2\sqrt{2} \quad (\text{since } m^2 \leq 4)$$

$$m = \sqrt{4 - 2\sqrt{2}} \approx 1.082$$

(c) The time by which 60% of villagers have collected water is obtained by calculating the 6th decile.

The 6th decile d is found by solving:

$$\int_0^d \frac{1}{4}x(4 - x^2)dx = 0.6$$

$$\frac{1}{4} \left[2x^2 - \frac{x^4}{4} \right]_0^d = 0.6$$

$$\frac{1}{4} \left(2d^2 - \frac{d^4}{4} \right) = 0.6$$

$$2d^2 - \frac{d^4}{4} = 2.4$$

Let $v = d^2$:

$$2v - \frac{v^2}{4} = 2.4$$

$$v^2 - 8v + 9.6 = 0$$

$$v = \frac{8 \pm \sqrt{64 - 38.4}}{2}$$

$$v = \frac{8 \pm \sqrt{25.6}}{2}$$

$$v = \frac{8 \pm 5.06}{2}$$

$$d^2 = 1.47 \quad (\text{valid solution})$$

$$d = \sqrt{1.47} \approx 1.212$$