MASAKA SECONDARY SCHOOL

PRE-REGISTRATION

TERM 1, 2026

SENIOR 6 PURE MATHEMATICS

DURATION: 3 HOURS

Instructions:

- Answer all ITEMS in section A and ANY 4 in section B.
- Show your working clearly.
- Calculators and mathematical tables can be used.
- Each ITEM in section A has a score of 5 and each in section B has a score of 12.

SECTION A (40 SCORES)

Item 1.

Algebra

A software developer at a financial technology company models a profit function using the polynomial $2x^3 + 6x^2 + qx - 5$. During system testing, it was discovered that whenever the input value is $x = -2$, the output becomes zero. This confirms that $x + 2$ is a factor of the polynomial. For further system calibration, the developer now wants to know what value remains when the same expression is divided by $2x - 1$.

Task: Help this developer determine what remains when the expression is divided by $2x - 1$.

Item 2.

Algebra

Teddy operates a mobile money business in her trading centre. For security reasons, she trains her assistant never to write her 4-digit Mobile Money PIN anywhere. One day, she tells the assistant that the first three digits are already known to him, but the last digit is hidden in a mathematical condition. She explains that the last digit is the value of x which satisfies the equation $3^x - 3^{2--x} = 8$.

Task: Determine the last digit of Teddy's Mobile Money PIN.

Item 3.

Trig

David is installing a solar panel support on a roof. One metal bar makes an angle of 45° with the horizontal, and he knows that $\cos 45° = \frac{\sqrt{2}}{2}$. He fixes another brace exactly halfway between the horizontal and the 45° bar, so it makes an angle of 22.5°. For accurate measurement, he needs the exact value of $\cos 22.5°$. He recalls that, using trigonometric identities, it can be expressed as $\cos 22.5° = \frac{\sqrt{\sqrt{2}+2}}{2}$ but he has no calculator.

Task:

Help David determine if what he recalls is the exact value of $\cos 22..5°$ using the information given.

diff 1

Item 4.

An engineer designing a parabolic metallic arch for a modern bridge models the shape of the arch using the equation $y = 2x^2 + 5x + 3$. For structural reinforcement, a supporting beam is to be fixed at the point (2,21). To determine the direction of the support and the line perpendicular to it for stability analysis, the engineer needs the equations of the tangent and the normal at that point.

Task: Help this engineer determine the equation of the tangent and the normal to the curve at the given point.

Series A.P

Item 5.

During preparation for the annual Mathematics Challenge at a secondary school, the organizing committee decided to reward participants using a structured token system. The number of tokens awarded to a participant follows an arithmetic progression. It was recorded that the 10th participant received 29 tokens, while the 15th participant received 44 tokens. The committee plans to reward the first 60 participants using the same pattern.

Task: Find the common difference of the progression and the number of tokens awarded to the first participant. Hence determine the total number of tokens required to reward the first 60 participants.

Analysis diff 1

Item 6.

Mark, a road construction engineer, models the vertical profile of a new road section using the quadratic expression $y = ax^2 - bx + c$. He explains to his team that such a curve can represent either a valley or a hill depending on the values of the constants. From his survey results, he has established that the stationary point of the curve is (2, 4), and that the road crosses the vertical axis at 8 units, meaning the y-intercept is 8. He now needs to determine the exact values of a, b, and c in order to decide whether the road section forms a valley or a hill.

Task: Help Mark determine the values of a, b, and c, and hence establish whether the road section represents a valley or a hill.

Item 7.

not masaka SS equalities

A group of computer programmers use the function $f(x) = 2x^2 - 7x - 15$ to control when a certain software feature is activated or deactivated. They explain that the feature switches on when the function gives positive values and switches off when the function gives negative values. They want to know the exact range of values of x for which the function produces positive results and the range for which it produces negative results, but they challenge themselves to determine this mathematically without using a computer.

Task: Help the programmers determine the range of values of x for which the function is;

a) Positive.
b) Negative.

Item 8.

dif 2

A student defined a function by $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$. He was required to show that its derivative simplifies to $\frac{1}{1-\sin x}$. However, he encountered difficulty in applying the appropriate differentiation techniques and simplifications.

Task: Show that the stated derivative is correct.

Section B. (48 SCORES)

Attempt ANY 4 ITEMS from this section.

Item 9.

Algebra

Two students, Aine and Okello, were analysing a quadratic equation whose roots are expressed in terms of squares of unknown positive numbers. They discovered that α^2 and β^2 are the roots of the equation $x^2 - 21x + 4 = 0$ where α and β are both positive real numbers. For further study, they needed to determine the vale of certain constants, but they were unsure how to proceed systematically from the given quadratic equation.

Tasks:

Help Aine and Okello determine;

a) The value of;

i) $\alpha - \beta$

ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

b) The equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

Item 10.

diff 2

Ms. Kemigisa introduced her class to differentiation of composite and quotient trigonometric functions. She defined two functions: $f(x) = sin^2x \cos 2x$ and $g(x) = \frac{\sin x}{x}$. The students were required to apply appropriate differentiation techniques to analyse the behaviour of these functions. However, several learners were unsure which rules should be applied at each stage.

Tasks:

Help them;

a) Differentiate $f(x)$ with respect to x.

b) Differentiate $g(x)$ with respect to x.

Item 11.

ITEM 2

A mountain rescue team is conducting an emergency operation to deliver medical supplies to an injured climber. The rescue package of mass 40 kg must be lowered down a rocky slope inclined at 25° to the horizontal. The package is connected by a strong, lightweight rope to an anchor point at the top of the slope.

Due to the steepness and rough terrain, the rescue team cannot simply let the package slide down freely. They need to control the descent carefully. The coefficient of kinetic friction between the package and the rocky surface is 0.15.

The rescue team applies a tension force through the rope to control the package's motion. They want the package to move down the slope with a constant acceleration of $0.5\ m/s^2$. During the descent, the rope suddenly snaps when the package is halfway down the 80-meter slope. The package continues sliding down the remainder of the slope, and then moves horizontally across a rough plateau before coming to rest.

Task:

As the rescue team's consultant, analyze this emergency situation and provide comprehensive guidance for the operation. Your analysis should determine all necessary forces, predict the package's motion throughout the entire journey, and assess the safety implications for the rescue operation. **Note:** Assume $g = 9.8\ m/s^2$ throughout your calculations.

ITEM 3

A non-governmental organization (NGO) has been running a farmer support program in Masaka district aimed at improving coffee yields. The NGO assumes that increased fertilizer application leads to higher coffee production.

To evaluate this assumption, a random sample of 20 household farmers who participated in the program were interviewed. Each farmer used a different amount of fertilizer. For each farmer, the NGO recorded:

- Total fertilizer applied (kg per acre)
- Average coffee yield (bags per acre)

The NGO has set the following criteria for future action:

- Yield Prediction: The NGO also wants to estimate the amount of fertilizer required to achieve a target yield of 36 bags per acre.
- Support Plan: If the assumption that fertilizer increases yield is supported by the data using more than one approach, the NGO will provide more fertilizer to farmers.
- Expansion Condition: The NGO plans to expand the program to a new group only if the probability of finding at least 4 selected farmers achieving a yield of 30 bags or more per acre is greater than 0.75.

At a carpentry workshop in Mbarara, two different decorative wooden panels are being designed for the reception area of a hotel. The edges of the panels are shaped using computer-controlled cutting machines, and each edge follows a mathematical model. The first panel has its edge modelled by $y = x^3 - x^2 - 5x + 6$ while the second panel has its edge modelled by $y = x^4 + 2x^3$. Before production begins, the designer needs to identify the exact points where each curve changes direction, in order to smoothen sharp bends and ensure structural stability.

diff 1

Task: For each curve, find the coordinates of the turning points and distinguish between their nature.

Item 12.

Three students; Achan, Kato and Ssemanda were given a set of advanced trigonometry challenges. They were first required to analyse the expression $6\sin x - 3\cos x$ in order to determine its greatest possible value. They were then presented with the expression $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$ which was claimed to be equal to $\tan 56°$, and they were asked to justify the claim. Finally, in a triangle ABC, they were required to establish that $\sin B + \sin C - \sin A = \frac{1}{4}\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. Although they attempted the problems, they could not complete the arguments satisfactorily.

Trig

Tasks:

Help them;

a) Determine the required maximum value.

b) Establish the stated trigonometric relationship.

c) Prove the given result in triangle ABC.

Item 13.

Algebra

Namusoke, Okello and Tumuhimbise were given rational algebraic models arising from different applied contexts. The first model represented the response of a mechanical system and was given by $\frac{6}{(x+3)(x-3)}$. The second model described a simplified electrical transfer function: $\frac{x}{(2+x)(2-x)}$. The third model arose from an economic growth adjustment equation: $\frac{x-1}{3x^2-11x+10}$. To analyse the behaviour of each model separately, they were required to decompose the expressions into simpler rational components.

Task:

Help them express all the three model equations into simpler rational components.

END