END OF YEAR EXAMINATIONS 2025

Senior Five

APPLIED MATHEMATICS

P425/2

Time: 3 hours

SECTION A

Answer all items in this section

Item 1

Two ordinary die are thrown. Find the probability that the sum of scores obtained is,

a) Multiple of 5
b) Greater than 9

Item 2

The government of Uganda is conducting a demographic study in a specific tribal clan in Fort Portal city where the probability of a baby being a boy is 0.4. The clan has a tradition of large families, and the government is analyzing a random sample of 10 children from different households to understand gender distribution trends.

Task

As a mathematics student, you have been approached, help the government to determine the expected number of girls in this random sample of 10 children and the chance of getting at most two girls in this sample of 10 children.

Item 3

The dimensions of a rectangle are 6.5 cm and 5.26 cm

(i) State the maximum possible error in each dimension
(ii) Find the percentage error made in the area of the triangle

Item 4

You are a healthcare administrator responsible for resource allocation at a regional hospital. The daily number of patients visiting the hospital is uniformly distributed between 150 and 210, and you need to ensure adequate staffing and supplies based on this data.

(i) Write down the probability distribution function (p.d.f.) of the number of patients visiting the hospital.

(ii) Find the probability that between 170 and 194 patients visit the hospital on a particular day, to assist in scheduling staff effectively.

Item 5

Two hunters **A** and **B** shoot an animal. The probability of that **A** hits the animal is ½ and the probability that **B** doesn't hit the animal is $^1/_3$. **A** shoots at animal first, then **B**. Find the probability that

i) Both A and B hit the animal
ii) Only one hits the animal

Item 6

You are an environmental scientist tasked with analyzing the temperature-dependent growth rate of a specific algae species in a controlled experiment. The growth rate, represented by the function $f(x)$, is measured at various temperatures x (in °C), and the data is recorded in the table below. To estimate the growth rate at intermediate temperatures and identify the temperature corresponding to a target growth rate, you decide to use linear interpolation.

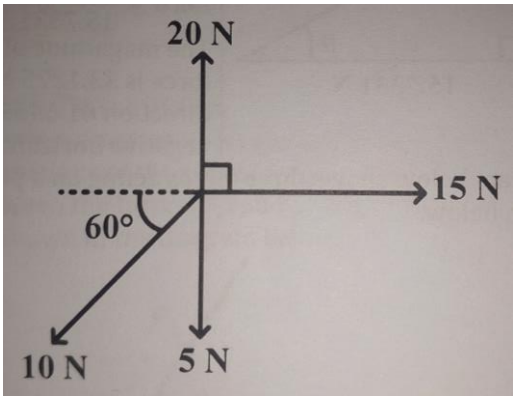
x(°C)	1.8	2.0	2.2	2.4
$f(x)$	0.532	0.484	0436	0.384

(i) Use linear interpolation to find the value of f (2.08)

(ii) Use linear extrapolation to find the temperature x corresponding to f(x) = 0.551.

Item 7

The figure below shows a system of four forces in acting in the directions shown below



Find the magnitude and direction of the resultant force in the above system

Item 8

A discrete random variable X has a probability distribution function given by

$$P(X = x) = \begin{cases} kx^3 & ; x = 1,2,3,4 \\ 0 & ; otherwise \end{cases}$$

Find (i) the value of the constant

(ii) $P(X \geq 2)$

SECTION B

Answer only 5 items from this section

Item 9

RONALD is a quality control engineer at a manufacturing plant in JINJA CITY producing alloy components for aerospace applications. To ensure the material meets specifications, he measures the density of a sample alloy block, which is calculated as the $\boldsymbol{\rho = \frac{m}{V}}$, where m is the mass and V is the volume.

The mass m is measured as 456.7 grams with a possible error of $\Delta m = 0.5$ grams, and the volume V is measured as 123.4 cm^3 with a possible error of $\Delta V = 0.2\ cm^3$.

(a) Show that the maximum absolute error in the density **ρ** is given by

$$\boldsymbol{\Delta\rho = \frac{(|V|\,\Delta m + |m|\,\Delta V)}{V^2}}$$

(b) Find the interval within which the exact value of the density is expected to lie, based on these measurements.

Item 10

A hydraulic engineer was tasked to estimate the average water flow rate through a narrow channel, where the flow rate can be modeled by the function $f(x) = \frac{1}{2x+1}dx$ with x representing the channel width in meters. The flow rate needs to be approximated over a range from 0.1 m to 0.5 m to optimize irrigation system design. He has decide to use the trapezium rule with 6 ordinates for the approximation.

(a) Use the trapezium rule with 6 ordinates to estimate $\int_{0.1}^{0.5} \frac{1}{2x+1}dx$, correct to three significant figures.

(b) Evaluate the exact value of $\int_{0.1}^{0.5} \frac{1}{2x+1}dx$, correct to three significant figures.

(c) (i) Determine the percentage error in the estimation in (a) above, correct to two decimal places.

(ii) Suggest how the percentage error may be reduced.

Item 11

The mathematics department of **St Leo's college** recently administered a standardized mathematics test to 100 students. The school aims to analyze the performance data to identify trends in student achievement, set academic improvement goals and determine how evenly the marks are distributed. The table below shows marks obtained by students (out of 100)

Marks	Number of students
20-<40	5
40-<50	15
50-<55	10
55-<60	15
60-<70	25
70-<90	25
90-<100	5

The head of department has asked you, as a data analyst, to help prepare a detailed performance summary report using the following tasks;

Tasks

a) Calculate the mean mark of students. Based on this mean, explain whether the overall performance is closer to the passing threshold. (assume 50 marks is the pass mark)
b) Construct a cumulative frequency table and draw cumulative frequency curve (ogive). Use the graph to determine
    i) The median mark, which divides the students into two equal halves
    ii) Find the range of scores between the 30^{th} and 70^{th} percentiles

Item 12

Fort portal city council is conducting a study to understand the life style and political behavior of its residents. The research team observes the following

Meal habits:

10% of the residents regularly have hot Breakfast, 20% have a hot Lunch, and 25% have either a hot breakfast or a hot lunch (or both)

Political alignment and voting

30% of the residents are NUP supporters, 50% are NRM supporters and 20% are independents. In the last election, 65% of the NUP supporters voted, 82% of the NRM supporters voted, and 50% of the independents voted.

If a resident is chosen at random from city;

a) What is the probability that the person has both hot breakfast and hot lunch
b) What is the probability that the person has a hot lunch given that they already had a hot breakfast?
c) What is the probability that this person voted in the last election?
d) If it is known that this person did not vote, what is the probability that they are NUP supporters?

Item 13

SLECK SACCO is a financial institution based in Fort Portal City. The institution is conducting a study to evaluate the performance of 12 secretaries. The management is interested in understanding the relationship between the **typing speeds** of the secretaries and the **number of errors** they make in their typed scripts

The data collected from the 12 secretaries is summarized below, where

- x represents the number of errors
- y represents the typing speed in seconds

	A	B	C	D	E	F	G	H	I	J	K	L
x	12	24	20	10	32	30	28	15	18	40	27	35
y	130	136	120	120	153	160	155	142	145	172	140	157

TASK

As a data analyst for the institution, you are tasked with the following;

a) i) construct a scatter diagram to visually represent the relationship between typing speed(y) and number of errors (x) for the 12 secretaries

ii),. Draw the line of best fit on your scatter diagram.

iii) Based on your diagram and line of best fit, comment on the nature of the relationship between typing speed and errors

iv) Use the line of best fit to estimate the number of errors x when the typing speed y = 142 seconds

b) i) Calculate the Spearman's rank correlation coefficient to quantify the relationship between typing speed and errors

iii) Test the significance of your correlation coefficient at the 5% significance level and comment on the strength and direction of the relationship

Item 14

A toy company is conducting a quality inspection using two storage boxes labelled A and B, each containing a mix of white and pink marbles representing different product types.

- Box A is three times as likely to be selected as Box B due to larger size
- Box A contains 5 white marbles and 6 pink marbles
- Box B contains 2 white marbles and 5 pink marbles

During the inspection, one box is chosen at random, and from it, two marbles are drawn one at a time without replacement to test for quality defects

As a quality analyst using the principles of probability, determine the likelihood of the following events

a) Both marbles are of the same colour
b) At least one white marble is picked
c) A pink marble is picked
d) Given that a pink marble is picked, what is the probability that it comes from box A

END