

NAME: COMB: STREAM:

DEPARTMENT OF MATHEMATICS

S.5 END OF YEAR PROMOTIONAL ASSESSMENT

PURE MATHEMATICS

TIME: 3 Hours

INSTRUCTIONS TO LEARNERS

- This paper consists of **7** items.
- All items carry **equal** scores.
- Attempt any **FIVE (5)** items of your choice.
- Attempt each item starting on a fresh page.
- All necessary working must be shown clearly.

FOR EXAMINER'S USE

ITEM	SCORES	INITIALS
TOTAL		

ITEM 1

Maize farmers in Bulambuli district have been greatly affected by a swarm of locusts and agricultural research experts have been sent to assess the extent of the damage. They plan to use drones to take aerial photographs but, the photographs can be taken clearly only if the argument of the flight path of the drone remains within 0° and 180° . The drones are set to take different flight paths represented by the expressions; $Z = \frac{1-i}{1+i}$ for **drone 1**, $Z = \frac{2+3i}{i}$ for **drone 2** and $Z = 1 + i\sqrt{3}$ for **drone 3**. The experts need to know the polar forms of each flight path in order to ensure that the drones will take clear photographs.

Task:

Provide mathematical assistance to the experts so as to enable them conclude on the suitability of each drone for the work. (20 scores)

ITEM 2

Mr. John sells honey in cubical containers. He has a rectangular sheet of metal measuring **80cm** by **60cm**. He has been advised to improve this metal sheet by turning it into a packing material by removing equal squares of dimensions of x cm from each corner and then fold the remaining sheet to form an open box.

After harvesting, honey is initially put in a right circular cone with its vertex pointing downwards. The cone has a radius of **10cm** and vertical height of **30cm**. honey leaks into the box from the cone at a rate of $2\text{cm}^3/\text{s}$ and john is interested in knowing the rate at which the honey level in the cone falls.

Task:

As a student of principal mathematics, help Mr. john to determine the;

- a) The dimensions x cm of the squares that should be removed in order to maximize the volume of the box and hence find the volume and surface area of the formed box.
- b) i) the rate at which the depth of the honey in the cone reduces at an instant when the honey is **halfway** down the height of the cone.

ii) the rate at which the radius of the honey in the cone is reducing at an instant when the radius is **5cm** (20 scores)

ITEM 3

RVA Project, Africa is selecting a group of disadvantaged children from the Rhino Refugee Camp for education sponsorship. The number of children available for selection and their ages are summarized in the table below;

Age (X)	4	5	6	7	8	9
Number of Children	13	17	20	25	10	15

Due to limited funding the organization has come up with selection criteria based on the age(x) of the child to be selected, that is $\frac{7X+12}{x+6} < \frac{12}{x-1}$ and $|2X - 3| < |X + 3|$. The Project Officer has been tasked to carry out the selection process based on the selection criteria set and update the organization with the number of children that will have passed the selection criteria.

Task:

- Guide the project officer to conduct the selection process. (17 scores)
- If the organization is to spend **USD 10,000** on each selected child from 4years to 6years and **USD 15,000** on each selected child from 7 years to 9 years, guide the Project Officer to provide the Organization with total amount of money that will be spent on the selected children. (03 scores)

ITEM 4

The ministry of Education and Sports has initiated a special task to develop innovative Mathematics curriculum for Ugandan schools. The task force, comprising of top Mathematicians and educators, has been assigned to derive the derivative of tangent function, $\tan(x)$, using first principles. The team leader Dr. kyotalye, has challenged her team to demonstrate the step-by- step process of differentiating $\tan(x)$ from scratch, without relying on pre-existing formulas or shortcuts. The team leader also gave the task force to provide the derivative of the function $y = 15 - 8x - 6x^2$ from first principles and thereafter provide the tangent equation to this curve at a point where $x = -1$.

Task:

Help the task force to;

- Provide the step-by-step derivative of $\tan(x)$.

- (b) Derive the derivative of the function, y and also find the equation of the tangent. (20 scores)

ITEM 5

The Engineering Brigade of the UPDF has taken up a contract to construct a bridge whose total construction cost is expressed as $T = \frac{\sin(A+2B)+\sin(A)}{\cos(A+2B)+\cos(A)}$ and the architectural setup of the bridge is modeled by $\sin x + \sin 2x + \sin 3x = 0$, where x is for the angles of elevation of the bridge. The construction plan is to be changed if the total cost, (T) is equal to the Net Income earned from the contract, $N = \tan(A + B)$ and if less than five angles of elevation, (x) lie in the range of 0° to 180° inclusive.

Task:

Guide the head of the Engineering Brigade to come up with a conclusion on the construction plan. (20 scores)

ITEM 6

A sports equipment company manufactures high quality spherical basket balls of radius r and given volume v . Due to machine calibration error, the basketballs are produced with an 8% error in volume of each ball manufactured for a full week. Given that the company manufactures 500 balls daily and it costs the company UGX. 15, 000 of the error made in the surface area of each ball to modify it.

One of the features of these basketballs is their unique surface design which is created using parametric equations. The x and y coordinates of the ball's surface are defined by the parametric equations $x = \frac{t}{1+t}$ and $y = \frac{t^3}{1+t}$ where t represents the arc length of the curved surface. The company intends to use an expression for the second derivative of the curve $\frac{d^2y}{dx^2}$ as the magnification of the curve's curvature revealing how the ball's surface is shaped and curved.

Task:

Help the company manager to obtain;

- a) The percentage error in surface area of each basketball manufactured and how much the company will incur to modify the errored balls.

- b) An expression for the magnification of the curve's curvature of the balls.
(20 scores)

ITEM 7

In an effort to reduce deforestation in lake Mbuoro forest reserve, a NEMA forest surveyor has been tasked to come up with a sketch map of the forest that is bounded by the straight horizontal road (x -axis) with its origin at a point O on the certain plane map and a curved chain link fence given by the equation $y = x(x - 8)(x - 15)$. Due to limited mathematical knowledge, the forest surveyor needs guidance from an expert.

Task:

- (a) Guide the forest surveyor to come up with a sketch map of the forest and the total area bounded by the curved fence and the straight horizontal road in square kilometers.
- (b) The area bounded by the straight road and the curved fence from $x = 0$ to $x = 8$ is rotated about the horizontal road through one complete revolution, find the volume of the solid generated. (20 scores)