

**P425/1**

**PURE MATHEMATICS**

PAPER ONE

**SENIOR FIVE**

3 HOURS

BOMBO ARMY SECONDARY SCHOOL

**END OF YEAR ASSESSMENT 2025**

MATHEMATICS

PAPER 1

**TIME ALLOWED: 3 HOURS**

**INSTRUCTIONS:**

Attempt *ALL* questions in section **A** and any *TWO* from section B

Answers to all questions must begin on a fresh page.

**SECTION A**

**(Attempt all items in this section)**

**Item 1**

Alice, a farmer in Mbale district is planning to plant sorghum in parallel lines. The first proposed line is to pass through the earth coordinates **(10, 50)** and **(30, 10)**, while the second line is to pass through the earth coordinates **(15, 60)** and **(25, 40)**. However, she is not sure whether these lines are actually parallel to each other. To ensure access to sunlight a reasonable distance between these lines is needed. The minimum distance needed between them is **100cm**, but Alice does not know if the given coordinates do not meet this requirement.

**Task**

Help Alice find

a) The equations of the first and second lines. Convince Alice that these lines are surely parallel to each other.

b) Whether Alice should increase the distance between these lines.

## Item 2

A local community in a rural area has a large water storage tank that supplies water to households through two distribution pipes. The rate at which water flows out of the tank through these pipes depends on how full the tank is. After conducting measurements, a group of physics students models the rate of water flow,  $R(x)$ , with respect to time using a rational function:

$$R(x) = \frac{5x+3}{(x+3)(x-3)^2}$$

Where  $R(x)$  is the rate of water flow (in litres per minute) and  $x$  is the number of hours after the start of the day. The engineers must first decompose it into partial fractions before they determine the volume of the tank.

### Task:

By expressing the function as partial fractions, help the students to:

Simplify the integration process by expressing it into partial fraction.

## Item 3

The chairperson of ward B who is suffering from blood pressure is given a sleeping pill. This has caused an initial level of 4mg of a drug per litre of his blood. After hours, the number of mg per litre in the blood is  $N$ , where

$$N = 4(0.76)^t.$$

### Task:

a) Help the doctor to estimate the number of mg per litre after four (4) hours.  
(Correct your answer to one (1) d.p)

b) The doctor also wants to know how long it'll take the amount of the drug per litre of blood to reduce to half its initial value?

c) If the mg per litre of blood increases to thrice the initial amount, the patient has to be put on drip, advise the doctor, after how long can this happen?

**Item 4**

James has a challenge of differentiating from first principles, as a mathematician help james to differentiate  $\tan x$  from first principles.

**Item 5**

John has applied for a job in a construction company and he has been selected among those to sit for interviews. Among the areas to be considered, is how good one's knowledge of trigonometry is. John has tried the task and has failed. Given that  $\tan \theta + \sin \theta = x$  and  $\tan \theta - \sin \theta = y$ , then;  $(x^2 - y^2)^2 = 16xy$

**Item 6**

In a certain manufacturing company, a right circular cone is to be made such that its slant height is  $\pi$  meters. Show that the maximum volume of the cone is  $\frac{2\pi^4}{9\sqrt{3}}$  cubic meters.

**Item 7**

An engineer is designing a hydraulic piston whose pressure,  $y$  depends on the piston displacement,  $x$  according to the formula;  $y = (3x - 1)^7$ . Find the rate of change of pressure with respect to displacement.

**Item 8**

A road engineer is designing a ramp that must rise at an angle  $\theta$  satisfying the condition;  $2 \sin \theta = 3 \cos \theta + 1$

Find the value of  $\theta$  for  $0^\circ \leq \theta \leq 360^\circ$  that satisfies the above condition.

**SECTION B**

**(Attempt only *two* items in this section)**

**Item 9**

A farmer wants to buy a fertilizer for his farm that lies between a curved boundary and a straight route. The boundary is modeled by the curve

$$y = x(2 - x)(x + 1)$$

You are contracted to survey the land and prepare a report to present to the farm manager to guide the decision making

### **Hint**

A tin of fertilizer covers 2 square unit of land.

### **Task**

As a learner of mathematics help

- a) make a sketch of the farm to the present to the farm manager
- b) estimate the area of the land
- c) establish the number of tins of fertilizer needed.

### **Item 10**

Calculus is one of the widest branches of principle mathematics which is widely applied in the fields of engineering. Among the parts detailed in calculus there is partial fractions which are used to break down complex expressions into simplified fractions for simplifying further operations like integration, differentiation and so on. Mr Nayebare Brian is a chemical engineer working on a design of a continuous stirred tank reactor (CSTR) whose area can only be obtained by integrating the function  $f(x) = \frac{3x-1}{(2-x)(1-x)^2}$ . And to locate regions where to put reactants in the (CSTR) he is required to base on the inequality  $\frac{x}{x-4} < 5$

### **TASK**

- a) Help Mr. Nayebare Brian
- b) Simplify the process of integration by expressing  $f(x)$  into partial fractions.
- c) Find regions where to put reactants in the (CSTR).

### **Item 11**

You have participated in an annual math contest in which all schools within your district have to take part by sending a single representative per school. In the group stages, each participant randomly chooses three questions from the box, selecting a question after the other.

Qualified candidates move to semi-finals, and at this stage, all competitors answer two similar questions. The best two qualify for the final round which declares the winner of the contest.

### Task

(a) Suppose you qualified for the semi-finals through picking and answering the questions below;

(i) Given  $\cos 2A - \cos 2B = -p$  and  $\sin 2A - \sin 2B = q$ , prove that

$$\sec(A + B) = \frac{1}{q} \sqrt{p^2 + q^2}$$

(ii) Prove that in any triangle  $\frac{b^2 - c^2}{a^2} = \frac{\sin(B - C)}{\sin(B + C)}$

(iii) Show that  $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2 \tan 2\theta$

Write your responses.

(b) You emerged the winner of the competition after correctly responding to the question below;

Prove the identity:  $\cos 3x = 4 \cos^3 x - 3 \cos x$

Write your response.

**END**