NATIONAL MATHEMATICS CONTEST

O-LEVEL-LOWER (S1&S2) PAPER X .

SATURDAY 28t MARCH, 2026 9:00 AM - 12:15 PM

INSTRUCTIONS TO CANDIDATES AND SUPERVISORS:

(a) This competition is conducted on the assumption that proper security is maintained. UMS reserves the right, should there be evidence or suspicion of any malpractice, to reject scripts from a particular school/institution/centre.

(b) This paper has **two** *sections* ***A*** *and* ***B*** *each carrying 50 marks. Attempt all questions in exactly three hours fifteen minutes (9 : 00am to 12 : 15pm).*

(c) Indicate your names, date of birth, gender, class, school and district where your school is located ***on all*** *your answer sheets.*

(d) Marks will be awarded for only answers for which a clear and logical layout of the working is provided. A correct solution, poorly presented, will not earn full marks.

(e) Answer scripts of only registered students should be returned to the contest coordinator, latest **Monday 30th March, 2026**.

(f) National Mathematics Contest 2026 Paper 2 will be done on 27th June, 2026 at various centres and **ALL** *participants who qualify for Paper 2 are cordially invited to the certificate and prize giving ceremony on Saturday 1st August, 2026 at 9:00am at Makerere University, Kampala.*

SECTION A: (Each question carries 5 marks).

1. Three interior angles of a certain triangle are in the ratio of 1 : 3 : 5. Determine the ratio of its corresponding exterior angles.

Solution:

Let the measures of the three angles be $1x$, $3x$, and $5x$. We know that the sum of the interior angles of a triangle is always 180°.

$$\begin{aligned} 1x + 3x + 5x &= 180 \\ 9x &= 180 \\ x &= 20 \end{aligned}$$

The interior angles are 20°, 60° and 100°. An exterior angle is computed as 180° - interior angle. The corresponding exterior angles are 160°, 120° and 80°, respectively. The ratio of exterior angles is 160° : 120° : 80° which simplifies to 4 : 3 : 2

2. A positive integer n is called "twoprime" if it has exactly three different positive factors: $1, 2$, and the number n itself. Determine how many such twoprime numbers exist.

Solution:

For a number n to have exactly three factors, it must be the square of a prime number (p^2). For example, 4 has factors $\{1, 2, 4\}$, and 9 has factors $\{1, 3, 9\}$.
The question specifies that the three factors must be 1, 2, and n. Since 2 is one of the factors, and we know that for a number to have exactly three factors it must be a perfect square of a prime, n must be 2^2.
The factors of 4 are $1, 2$, and 4. If we try any other prime square, such as $3^2 = 9$, the factors are $1, 3$, and 9, which does not include 2. If we try a non-prime square like 16, it has more than three factors. Therefore, there is only 1 such twoprime number (which is 4).

3. If

$$\frac{x+5}{4} = \frac{y - \frac{3}{2}}{2},$$

what is the numerical value of the expression $\sqrt[3]{2y - x}$?

Solution:

We first obtain $2y - x$.To eliminate the fractions, we multiply both sides of the equation by the least common multiple of the denominators, which is 4.

$$4\left(\frac{x+5}{4}\right) = 4\left(\frac{y - \frac{3}{2}}{2}\right)$$

$$x + 5 = 2(y - \frac{3}{2})$$

Next, we expand the right side and rearrange the terms to isolate the expression $2y - x$:

$$\begin{aligned} x + 5 &= 2y - 3 \\ 5 + 3 &= 2y - x \\ 8 &= 2y - x \end{aligned}$$

Therefore, the value of $\sqrt[3]{2y - x}$ is $\sqrt[3]{8} = 2$.

4. An integer is a perfect cube if it can be written in the form n^3 for some integer n. Determine the smallest positive integer k such that the product $2^2 \times 3^5 \times 7^1 \times k$ results in a perfect cube.

Solution:

For a number to be a perfect cube, the exponent of every prime factor in its prime factorization must be a multiple of 3. We examine the current exponents in the expression $2^2 \times 3^5 \times 7^1$.
To reach the next multiple of 3 for each prime factor, we determine the missing factors needed in k:

- For the factor 2^2: The next multiple of 3 is 3. We need one more factor of 2 (2^1).

- For the factor 3^5: The next multiple of 3 is 6. We need one more factor of 3 (3^1).
- For the factor 7^1: The next multiple of 3 is 3. We need two more factors of 7 (7^2).

The smallest integer k is the product of these missing pieces:

$$k = 2^1 \times 3^1 \times 7^2 = 294$$

Therefore, the smallest positive integer k is 294.

5. A factory produces $2^{11} \times 3^{17}$ chocolates and packed them evenly into 72 large jars. How many chocolates are in each jar?

Solution:

To divide the chocolates evenly, we need to divide the total number ($2^{11} \times 3^{17}$) by the number of jars (72). First, we express 72 as powers of 2 and 3 to make the division easier:

$$72 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$$

We set up the division using the laws of exponents (subtracting the powers):

$$\text{Chocolates per jar} = \frac{2^{11} \times 3^{17}}{72} = \frac{2^{11} \times 3^{17}}{2^3 \times 3^3} = 2^{11-3} \times 3^{17-3} = 2^8 \times 3^{14}$$

6. During a family reunion, three grandchildren try to guess their grandfather's age. The first grandchild guesses 70 years, the second guesses 75 years, and the third guesses 76 years.
The grandfather tells them that one guess is off by 1 year, the other guess is off by 4 years, and the remaining guess is off by 5 years. Based on this information, determine the grandfather's exact age.

Solution:

Let grandfather's actual age be x. The difference between his age and each guess must match one of the values in the set of errors: $\{1, 4, 5\}$. We can test the possible ages by looking at the margins of each guess.
First, we list the possible ages if the first guess (70) is off by each error:

- If the error is 1: x could be 69 or 71.
- If the error is 4: x could be 66 or 74.
- If the error is 5: x could be 65 or 75.

Next, we check these potential ages against the other two guesses (75 and 76) to see if their errors match the remaining values from the set $\{1, 4, 5\}$.
Let us test $x = 71$:

$$|71 - 70| = 1$$
$$|71 - 75| = 4$$
$$|71 - 76| = 5$$

In this case, the errors are $1, 4$, and 5, which exactly matches the set provided by the grandfather. If we test other possibilities, such as $x = 74$:

$$|74 - 70| = 4$$
$$|74 - 75| = 1$$
$$|74 - 76| = 2$$

Here, the error 2 is not in our set. Similarly, testing $x = 75$ gives errors of $\{5, 0, 1\}$, where the error 0 is not allowed. Through this logical testing, we find that 71 is the only age that satisfies all conditions.
Therefore, the grandfather is 71 years old.

7. A student is exploring properties of square roots using the current year, 2026. He writes down a long expression where the value 2026^2 is added to itself several times under a single square root sign:

$$\sqrt{2026^2 + 2026^2 + \cdots + 2026^2} = 2026^8$$

Determine exactly how many times the term 2026^2 must appear under the square root for this statement to be true.

Solution:

Let k be the number of times the term 2026^2 appears under the square root. We can rewrite the sum inside the square root as a product. So

$$\sqrt{k \cdot 2026^2} = 2026^8.$$

To find the value of k, we simplify the left side of the equation using the property $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$, and have

$$\begin{aligned}
\sqrt{k} \cdot \sqrt{2026^2} &= 2026^8 \\
\sqrt{k} \cdot 2026 &= 2026^8 \\
\sqrt{k} &= \frac{2026^8}{2026^1} \\
\sqrt{k} &= 2026^7 \\
(\sqrt{k})^2 &= (2026^7)^2 \\
k &= 2026^{14}
\end{aligned}$$

Therefore, the term 2026^2 must appear 2026^{14} times.

8. How many positive three-digit numbers are divisible by 17?

Solution:

A three-digit number ranges from 100 to 999. We need to find the first and last numbers in this range that are divisible by 17. First, we find the smallest three-digit multiple of 17:

$$100 \div 17 \approx 5.88$$
$$17 \times 6 = 102$$

Next, we find the largest three-digit multiple of 17:

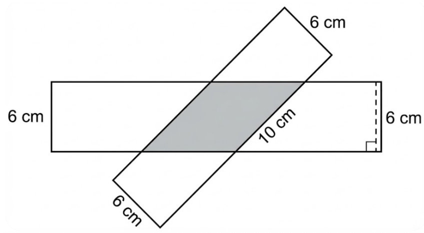
$$999 \div 17 \approx 58.76$$
$$17 \times 58 = 986$$

The multiples of 17 form an arithmetic sequence: $102, 119, 136, \ldots, 986$. The number of terms n in such a sequence can be calculated by comparing the multipliers. The multipliers are $6, 7, 8, \ldots, 58$. The number of terms is:

$$n = 58 - 6 + 1 = 53$$

Therefore, there are 53 three-digit numbers divisible by 17.

9. Two identical rectangular strips of paper, each with a width of 6 cm, overlap each other as shown in the diagram. The overlapping region (shaded) forms a parallelogram with a side length of 10 cm along the edge of the strip. Calculate the area of this overlapping region.



Solution:

The shaded region is a parallelogram (specifically a rhombus, since the strips have equal width). To find the area of a parallelogram, we use the formula:

$$\text{Area} = \text{Base} \times \text{Perpendicular Height}$$

From the diagram and the information given:

- The **Base** of the parallelogram corresponds to the side length lying along the edge of the strip, which is 10 cm.
- The **Perpendicular Height** corresponds to the width of the horizontal strip, which is 6 cm.

Calculating the area:

$$\text{Area} = 10 \text{ cm} \times 6 \text{ cm} = 60 \text{ cm}^2$$

10. Given that $x, y,$ and z are real numbers such that $x + y + z = 0$ and $xyz = 92$. Find the numerical value of the expression $(x + y)(y + z)(z + x)$.

Solution:

From the given equation $x + y + z = 0$, we can express each sum of two variables in terms of the third variable,

$$x + y = -z$$
$$y + z = -x$$
$$z + x = -y$$

We substitute these three expressions into the required product and have

$$(x + y)(y + z)(z + x) = (-z)(-x)(-y) = -(xyz) = -92$$

Therefore, the value of the expression is -92.

SECTION B: (Each question carries 10 marks).

11. In a village youth club, 20 chairs are arranged in a large circle and numbered 1 to 20 in a specific order. The chairs are placed such that the numbers on any two adjacent chairs always have a difference of either 1 or 2.
A "Special Link" is formed between two adjacent chairs if their numbers differ by exactly 1. Determine the minimum number of Special Links that must exist in this circle and justify your answer.

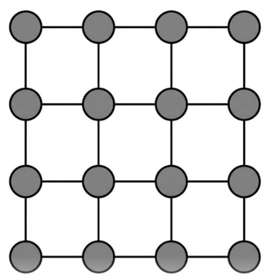
Solution:

To solve this, we look at the parity (whether a number is odd or even) of the chair labels. In the set of numbers from 1 to 20, there are 10 odd numbers and 10 even numbers.
When we move from one chair to the next, the difference between the numbers is either 1 or 2. If the difference is 2, the parity stays the same (e.g., 3 to 5 or 4 to 6). If the difference is 1 (a Special Link), the parity must change (e.g., 3 to 4 or 8 to 9).
In a circular arrangement of 20 chairs, we must eventually return to the starting chair. This means we must switch from the "odd group" to the "even group" and then switch back at least once to complete the loop. Each switch between groups requires one Special Link.
Since we must change parity to get from an odd number (like 1) to an even number (like 20) and then change back to return to the start, there must be an even number of Special Links. We cannot have zero Special Links because then all chairs would have the same parity, which is impossible since we have both odds and evens.
The smallest even number greater than zero is 2. We can verify this by arranging the chairs as follows:

$$1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2$$

In this arrangement, the only Special Links are between 19 and 20, and between 2 and 1.
Therefore, the minimum number of Special Links is 2.

12. The diagram on the right shows a map of 16 villages connected by a road network. The government intends to build mobile network masts in some of these villages. Each mast provides a signal to the village where it is located and to all of its immediate neighbouring villages (those directly connected by a single road).
Determine the minimum number of network masts required to ensure that every village receives a signal.



Solution:

To cover all 16 villages with the minimum number of masts, we place them so that their coverage overlaps as little as possible. Let us denote the villages by coordinates $(row, column)$.

We can cover all villages by placing masts at the following 4 locations.

- **Mast 1 at (1, 2):** Covers (1,1), (1,2), (1,3), and (2,2).
- **Mast 2 at (2, 4):** Covers (1,4), (2,4), (3,4), and (2,3).

- **Mast 3 at (3, 1):** Covers (2,1), (3,1), (4,1), and (3,2).
- **Mast 4 at (4, 3):** Covers (4,2), (4,3), (4,4), and (3,3).

Any attempt to use only 3 masts will always leave at least one corner or central village without a signal.
Therefore, the minimum number of masts is 4.

13. Ocen is filling a 3×3 grid with nine numbers such that the sum of the numbers in any two adjacent cells (cells that share a common side) is always the same constant value. He has already placed the number 5 in the top-left corner and the number 8 in the middle cell of the bottom row. Calculate the total sum of all nine numbers in the grid.

Solution:

Let the constant sum of two adjacent cells be S. If the top-left cell has a value x, any cell adjacent to it must have a value y such that $x + y = S$. For any cell adjacent to y to also satisfy the sum S, it must have the value x. This results in a checkerboard pattern where the grid alternates between two values.

$$\text{Grid Layout:} \quad \begin{bmatrix} x & y & x \\ y & x & y \\ x & y & x \end{bmatrix}$$

From the information provided in the problem:

- The top-left corner is an x cell, so $x = 5$.
- The middle cell of the bottom row is a y cell, so $y = 8$.

The constant sum S for any two adjacent cells is $5 + 8 = 13$. The completed grid contains five cells with the value 5 and four cells with the value 8.
To find the total sum of all nine numbers, we calculate the sum of these two groups of cells:

$$\begin{aligned} \text{Total Sum} &= 5(x) + 4(y) \\ \text{Total Sum} &= 5(5) + 4(8) \\ \text{Total Sum} &= 25 + 32 \\ \text{Total Sum} &= 57 \end{aligned}$$

The total sum of all nine numbers in the grid is 57.

14. At Ssezibwa Falls, there are two types of lizards: speckled and plain. Over a mating season, the number of speckled lizards increases by 60%, while the number of plain lizards decreases by 60%. Interestingly, the new ratio of speckled lizards to plain lizards is the same as the original ratio of plain lizards to speckled lizards. Determine the percentage change in the total population of lizards.

Solution:

Let s be the original number of speckled lizards and p be the original number of plain lizards. After the changes,

- The new number of speckled lizards is $1.6s$.
- The new number of plain lizards is $0.4p$.

According to the problem, the new ratio $\dfrac{1.6s}{0.4p}$ is equal to the original inverse ratio $\frac{p}{s}$.
We set up the equation and solve for the relationship between s and p:

$$\begin{aligned}\frac{1.6s}{0.4p} &= \frac{p}{s}\\ \frac{4s}{p} &= \frac{p}{s}\\ 4s^2 &= p^2\\ 2s &= p\end{aligned}$$

This means there were originally twice as many plain lizards as speckled lizards.
To find the total change, we compare the original total population $(s + p)$ to the new total population $(1.6s + 0.4p)$.

$$\begin{aligned}\text{Original Total} &= s + 2s = 3s\\ \text{New Total} &= 1.6s + 0.4(2s)\\ &= 1.6s + 0.8s = 2.4s\end{aligned}$$

The change in population is $2.4s - 3s = -0.6s$. To find the percentage change,

$$\begin{aligned}\text{Percentage Change} &= \frac{-0.6s}{3s} \times 100\\ &= -0.2 \times 100\\ &= -20\%\end{aligned}$$

Therefore, the total number of lizards has decreased by 20%.

15. The sum of the digits of a two-digit number is 9. If the digits are interchanged, the new number formed is 9 less than twice the original number. Find the original number.

Solution:

Let the original number be represented by $10x + y$, where x is the tens digit and y is the units digit. From the first statement, the sum of the digits is 9.

$$x + y = 9 \quad \ldots \text{(i)}$$

When the digits are interchanged, the new number becomes $10y + x$. The problem states that this new number is 9 less than twice the original number. So,

$$10y + x = 2(10x + y) - 9$$

We simplify this equation as

$$\begin{aligned} 10y + x &= 20x + 2y - 9 \\ 10y - 2y &= 20x - x - 9 \\ 8y &= 19x - 9 \quad \ldots\text{(ii)} \end{aligned}$$

From equation (i), we know that $y = 9 - x$. We substitute this into equation (ii):

$$\begin{aligned} 8(9 - x) &= 19x - 9 \\ 72 - 8x &= 19x - 9 \\ x &= 3 \end{aligned}$$

Since $x = 3$, we find y using equation (i):

$$y = 9 - 3 = 6$$

The original number is $10(3) + 6 = 36$. (Check: Reverse is 63. Twice original is 72. $72 - 9 = 63$. Correct). Therefore, the original number is 36.

END