

**JK BEGUMISA RESOURCES**

**IDENTITY:** “A” LEVEL VECTORS – REVISION QUESTIONS:

**DATE:** 07/11/2025.

**TEL CONTACTS:** [+256786108453](tel:+256786108453)/[+256700798389](tel:+256700798389)

**EMAIL:** [jkbegumisa2025@gmail.com](mailto:jkbegumisa2025@gmail.com)

*“A Physics/Mathematics Teacher”*



Author of **“Advanced Level Pure Mathematics Item Bank”** (New curriculum) 2025

**NOTES:**

- This Pure Mathematics Problems leaflet is completely free and available online.
- Error(s) (if any) could have occurred while typing (or and) proof – reading, and **JK Begumisa** is responsible for any error in this leaflet.
- **JK’s** vision is simple; *“To make high - quality, context appropriate learning tools accessible to Every Ugandan Learner and Teacher”*.

**DISPLACEMENT VECTORS, LENGTH OF A VECTOR AND UNIT VECTORS:**

1. Given that points A, B, C and D have position vectors;  $2\mathbf{k} - 3\mathbf{j}$ ,  $-\mathbf{j} + 4\mathbf{i} - 5\mathbf{k}$ ,  $3\mathbf{i} - (2\mathbf{j} - \mathbf{k})$  and  $4\mathbf{i} + 5\mathbf{j} - \mathbf{k}$  respectively.  
Find the;
  - i).  $3\overrightarrow{AB} - 4\overrightarrow{CD}$
  - ii). unit vector in the direction of  $4\overrightarrow{BD} - \overrightarrow{OC}$
  - iii).  $|5\overrightarrow{BA} - \overrightarrow{CD}|$
2. Find the co-ordinates of  $P_2$  if  $2\mathbf{P}_0\mathbf{P}_1 = 3\mathbf{P}_1\mathbf{P}_2$  where  $P_0(-2,7,4)$  and  $P_1(7, -2,1)$ .
3. Points; P, R, S and T are such that;  $\mathbf{OP} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{OS} = 6\mathbf{i} - 9\mathbf{j}$  and  $3\overrightarrow{PR} = 4\overrightarrow{SP} = 2\overrightarrow{RT}$ . Find the position vector of;
  - a) Point R **Ans:**  $-\mathbf{i} + \frac{29}{3}\mathbf{j} + \frac{14}{3}\mathbf{k}$

- YouTube Channel: **JK Maths**
- WhatsApp Channel: **TR. JK (A’ Level Maths)**

b) Point T Ans:  $-7\mathbf{i} + \frac{77}{3}\mathbf{j} + \frac{26}{3}\mathbf{k}$

**RATIO THEOREM:**

4. A point D internally divides the points A and B with position vectors;  $\mathbf{a}$  and  $\mathbf{b}$  respectively, in the ratio;  $\lambda: \mu$ .

Show that;  $\overrightarrow{OD} = \left(\frac{\mu}{\lambda+\mu}\right)\mathbf{a} + \left(\frac{\lambda}{\lambda+\mu}\right)\mathbf{b}$

5. Two points; A and B have respective position vectors;  $\mathbf{a}$  and  $\mathbf{b}$ . A point N divides A and B externally in the ratio;  $\beta: \mu$ .

Show that;  $\overrightarrow{ON} = \left(\frac{\mu}{\mu-\beta}\right)\mathbf{a} + \left(\frac{-\beta}{\mu-\beta}\right)\mathbf{b}$ .

6. A point T divides the line AB internally in the ratio 5:2  
Given that; A(-4,10) and B(10,3), find the position vector of T.

7. The position vectors of points A and B are;  $\begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$  and  $\begin{pmatrix} 8 \\ -6 \\ 6 \end{pmatrix}$  respectively.

A point P divides AB in the ratio 2:3. Find the position vector of point P

8. Given that;  $\mathbf{OA} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$  and  $\mathbf{OB} = 4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$  and a point P divides line AB in the ratio; 3: -2, find the exact magnitude of  $\mathbf{OP}$ .

9. Find the co – ordinates of the point which divides the line segment joining the points; (-2, 3, 5) and (1, -4, 6) in the ratio;

- a) 2:3 externally.  
b) 3:4 internally.

10. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are;  $(2\mathbf{a} + \mathbf{b})$  and  $(\mathbf{a} - 3\mathbf{b})$  externally in the ratio 1:2.

Also, show that P is the midpoint of the line segment RQ.

11. Consider two points P and Q with position vectors;  $\overrightarrow{OP} = 3\mathbf{a} - 2\mathbf{b}$  and  $\overrightarrow{OQ} = \mathbf{a} + \mathbf{b}$ .

Find the position vector of a point R which divides the line joining P and Q in the ratio; 2:1

- a) internally  
b) externally

12. Using vector methods, find the position of point C that divides the line segment AB where

A and B have respective position vectors;  $\begin{pmatrix} 2 \\ -3 \\ 10 \end{pmatrix}$  and  $\begin{pmatrix} -3 \\ 12 \\ 0 \end{pmatrix}$  in the ratio 2:3

13. A point  $P \begin{bmatrix} -1 \\ -7 \\ 4 \end{bmatrix}$  externally divides the line segment TR in the ratio 1: -2

Relative to the origin, T has position vector;  $\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ . Find the co – ordinates of the point R.

14. A point  $G \begin{pmatrix} -8.5 \\ -20 \\ -11 \end{pmatrix}$  externally divides the line segment ST in the ratio; 7:3

Relative to the origin O, a point T has a co – ordinates (-4, -8, -5).

Find the position vector of point S.

15. P and M are two points such that;  $OP = i + 2j + 3k$  and  $OM = -5k + 10i + 11j$ .  
Find the co – ordinates of point D such that point M divides the line segment PD externally in the ratio 3:2.
16. Find the position vectors of the point which divides two points with respective position vector (relative to the origin, O);  $5a - 3b$  and  $3a - 5b$ ;
- Internally,
  - Externally, in the ratio 2:3

### COLLINEARITY OF POINTS AND PARALLELITY OF VECTORS:

17. Points R, S and T have position vectors;  $3i - 4k + 2j$ ,  $5i + 4j - 6k$  and  $-10k + 9i + 8j$  respectively. Show that points R, S and T lie on a straight line.
18. Show that the points;  $P(-2, 3, 5)$ ,  $Q(1, 2, 3)$  and  $R(7, 0, -1)$  are collinear. Hence or otherwise; find the ratio in which;
- Point Q divides line PR.
  - Point R divides line PQ
19. Points A, B and C have position vectors;  $10k - 4i + 6j$ ,  $2i + 4j + 6k$  and  $14i - 2k$  respectively, relative to the origin, O.  
Prove that the three points; A, B and C are collinear. Hence find the ratio in which point B divides  $\overline{AC}$ .
20. Three points with respective position vectors;  $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$ ,  $\begin{pmatrix} m \\ -4 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$  lie on a straight line.  
Find the exact value of  $m$ .

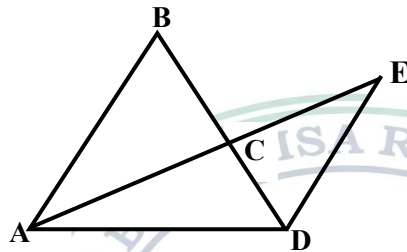
21. Relative to the origin, O, points; M, N and T have position vectors;  $\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ ,  $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \\ 3 \\ 2 \end{pmatrix}$  respectively.

Clearly, show that the points M, N and T are collinear. Hence, find the ratio in which point T divides MN.

22. Show that the points A(1,2,3), B(3,8,1) and C(7,20,-3) lie on a straight line.
23. Vectors;  $\mathbf{a}$  and  $\mathbf{b}$  are such that;  $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$  and  $\mathbf{b} = -4\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$ . Show that  $\mathbf{a}$  and  $\mathbf{b}$  are parallel.
24. Relative to the origin, points; R, S and T have position vectors;  $\begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$ ,  $\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} 3 \\ 10 \\ -1 \end{pmatrix}$  respectively. Show that points R, S and T lie on a straight line.
25. Find the value of  $\lambda$  and  $m$  given that the points;  $\mathbf{P} \begin{pmatrix} 2 \\ 3 \\ \lambda \end{pmatrix}$ ,  $\mathbf{N} \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$  and  $\mathbf{W} \begin{pmatrix} m \\ 8 \\ 7 \end{pmatrix}$  lie on a straight line.
26. Relative to the origin O, three points; W, C and D are such that;  $\mathbf{OW} = 4\mathbf{i} - 3\mathbf{j}$ ,  $\mathbf{OC} = t\mathbf{i} + \mathbf{j}$  and  $\mathbf{OD} = 10\mathbf{i} + 5\mathbf{j}$ . Find the value of  $t$  given that points; W, C and D are collinear.
27. Given that points; A( $h$ , -2,3), B(7, -5,2) and C(1,  $d$ , 5) are collinear, find the values of  $h$  and  $d$ .
28. Relative to the origin, points P, R, S and T are such that;  $\mathbf{OP} = 5\mathbf{j} + a\mathbf{k} + \mathbf{i}$ ,  $\mathbf{OR} = 3\mathbf{i} + 9\mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{OS} = 7\mathbf{j} + 10\mathbf{k} + b\mathbf{i}$  and  $\mathbf{OT} = 4\mathbf{i} + 9\mathbf{j} + 7\mathbf{k}$ . Given that  $\mathbf{AB}$  and  $\mathbf{CD}$  are parallel, find the values of  $a$  and  $b$ .
29. The points; A, B and C have position vectors;  $\mathbf{OA} = 4\mathbf{i} + 14\mathbf{j} - 5\mathbf{k}$ ,  $\mathbf{OB} = \mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$  and  $\mathbf{OC} = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ .
- Show that  $\mathbf{BC}$  and  $\mathbf{CA}$  are parallel.
  - Hence, deduce that the points A, B and C are collinear.
30. Show that the points whose position vectors are  $-2\mathbf{a} + 3\mathbf{b} + 5\mathbf{c}$ ,  $\mathbf{a} + 2\mathbf{b} + 3\mathbf{c}$  and  $7\mathbf{a} - \mathbf{c}$  are collinear, when  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are coplanar vectors.
31. If the vectors;  $-3\mathbf{i} + 4\mathbf{j} + \lambda\mathbf{k}$  and  $\mu\mathbf{i} + 8\mathbf{j} + 6\mathbf{k}$  are collinear vectors, find  $\lambda$  and  $\mu$ .
32. The position vectors of three points A, B and C are  $\mathbf{p}$ ,  $3\mathbf{q} - \mathbf{p}$  and  $9\mathbf{q} - 5\mathbf{p}$  respectively. Show that the points are collinear.

### APPLICATIONS OF COLLINEARITY:

33. In a triangle, OPR,  $\mathbf{OP} = \mathbf{p}$  and  $\mathbf{OR} = \mathbf{r}$ . A is the midpoint of  $\mathbf{OP}$  and B lies on  $PR$  such that  $\mathbf{RP} : \mathbf{BP} = 3 : 4$ . Line AR and line OB intersect at point D. Find the position vector of point D in terms of  $\mathbf{p}$  and  $\mathbf{r}$ .
34. In a triangle OAB,  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OB} = \mathbf{b}$ . Point M lies on OP such that  $\mathbf{OM} : \mathbf{OP} = 2 : 5$  and point N lies on OQ such that  $\mathbf{ON} : \mathbf{NQ} = 5 : 1$ . Line AN intersects line MB at X.
- Express  $\mathbf{OX}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Determine the ratio in which X divides the line;
    - MB
    - NA
35. In the figure below,  $\mathbf{AB} = \mathbf{r}$ ,  $\mathbf{AD} = \mathbf{p}$ ,  $\mathbf{DE} = \frac{1}{2}\mathbf{AB}$  and  $\mathbf{BC} = \frac{2}{3}\mathbf{BD}$



- Find, in terms of  $\mathbf{p}$  and  $\mathbf{r}$ , the vectors;
    - BC
    - AC
  - Express  $\mathbf{AE}$  in terms of  $\mathbf{p}$  and  $\mathbf{r}$ .
36. In a triangle OPR,  $\overrightarrow{OP} = \mathbf{p}$  and  $\overrightarrow{OR} = \mathbf{r}$ . Points A and B lie on  $\overrightarrow{OP}$  and  $\overrightarrow{RP}$  respectively such that  $\mathbf{RB} : \mathbf{RP} = 3 : 7$  and  $\mathbf{OP} : \mathbf{AP} = 3 : 1$ . Lines AB and OR are both produced to meet at point M. Find the position vector of M in terms of  $\mathbf{r}$ .
37. In a triangle OAB,  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ . A point L is on the side AB and M on the side OB. OL and AM meet at S.  $\overrightarrow{AS} = \overrightarrow{SM}$  and  $\overrightarrow{OS} = \frac{3}{4}\overrightarrow{OL}$ . Given that  $\overrightarrow{OM} = n\overrightarrow{OB}$  and  $\overrightarrow{AL} = m\overrightarrow{AB}$ . express the vectors;
- $\mathbf{AM}$  and  $\mathbf{OS}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $n$ .
  - $\mathbf{OL}$  and  $\mathbf{OS}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $m$ .
- Hence, find the values of  $n$  and  $m$ .

38. In a triangle OAB, X is a point on OB such that  $\overrightarrow{OX} = 2\overrightarrow{XB}$  and Y is a point on AB such that  $2\overrightarrow{BY} = 3\overrightarrow{YA}$ .
- Express  $\overrightarrow{OX}$  and  $\overrightarrow{OY}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$  where;  $\mathbf{a} = \overrightarrow{OA}$  and  $\mathbf{b} = \overrightarrow{OB}$ .
  - Find the position vector of any point on XY, and hence, find the position vector of point Z, where  $\overrightarrow{XY}$  produced meets  $\overrightarrow{OA}$  produced.
  - Calculate the ratio of  $\overrightarrow{AZ} : \overrightarrow{OZ}$
39. Given that;  $\mathbf{OP} = \mathbf{p}$  and  $\mathbf{OQ} = \mathbf{q}$ . Point T lies on  $\overrightarrow{OQ}$  such that  $\mathbf{OT} : \mathbf{TQ} = 4 : 1$ , point S lies on  $\overrightarrow{QP}$  such that  $\mathbf{QS} : \mathbf{SP} = 2 : 3$ . Both line TS and line OP are produced to meet at point D.  
Show that;  $\mathbf{OD} = \frac{8}{5}\mathbf{p}$
40. Triangle OAB has  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OB} = \mathbf{b}$ . C is a point on OA such that  $\mathbf{OC} = \frac{2}{3}\mathbf{a}$ . D is the midpoint of line AB. When line CD is produced, it meets line OB produced at T such that  $\mathbf{DE} = n\mathbf{CD}$  and  $\mathbf{BE} = k\mathbf{b}$ .
- Find the values of  $n$  and  $k$ .
  - Express  $\mathbf{OE}$  in terms of  $\mathbf{b}$ .
41. In a triangle OPS,  $\mathbf{OS} = 2\mathbf{r}$  and  $\mathbf{OP} = \mathbf{p}$ . Point R is the midpoint of line OS, and point K lies on line PS. Both line OP and line RK are produced to intersect at point Q such that  $\mathbf{OQ} = \frac{3}{2}\mathbf{p}$ .
- Express  $\mathbf{OK}$  in terms of  $\mathbf{r}$  and  $\mathbf{p}$ .
  - Show that the ratio;
    - $\mathbf{QK} : \mathbf{KR} = 1 : 1$
    - $\mathbf{PK} : \mathbf{KS} = 1 : 3$
42. OABC is a parallelogram with  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OC} = \mathbf{c}$ . S is a point on AB such that  $\mathbf{AS} : \mathbf{SB} = 3 : 1$   
T is the point on BC such that  $\mathbf{BT} : \mathbf{TC} = 1 : 3$   
Line CS and line OT intersect at point G.
- Express each of the following in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .
    - $\mathbf{AC}$
    - $\mathbf{SB}$
    - $\mathbf{BT}$
    - $\mathbf{ST}$
  - Express  $\mathbf{OG}$  in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .

43. In a triangle OAB,  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OB} = \mathbf{b}$ . M is the midpoint of AB and N is the point on OB such that  $\mathbf{ON}:\mathbf{NB} = 1:4$ . OM meets AN at P.
- Express  $\mathbf{OP}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Deduce that  $\mathbf{AP}:\mathbf{PN} = 5:1$
44. In a trapezium, OABC,  $\mathbf{OA} = \mathbf{a}$ ,  $\mathbf{OC} = \mathbf{c}$  and  $\mathbf{CB} = 3\mathbf{a}$ . T is the point on BC such that  $\mathbf{BC}:\mathbf{TC} = 2:1$ . OT meets AC at P
- Express  $\mathbf{OP}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$
  - Deduce that P is the point of trisection of both AC and OT
45. In a triangle OAB, M is the midpoint of AB. N is the point on OB such that  $\mathbf{ON}:\mathbf{NB} = \lambda:\mu$ . OM meets AN at H. Find the ratio of;
- $\mathbf{AH}:\mathbf{HN}$  Ans:  $(\lambda + \mu):\lambda$
  - $\mathbf{OH}:\mathbf{HM}$  Ans:  $2\lambda:\mu$
46. In a triangle OAB, point C is such that;  $\overrightarrow{\mathbf{OC}} = 2\overrightarrow{\mathbf{OA}}$  and point M is the midpoint of AB. The straight line through C and M cuts OB at point N. Given that  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OB} = \mathbf{b}$ ,
- Find  $\mathbf{CM}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Show that;  $\mathbf{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$ , where  $\lambda$  is a scalar constant.
  - Hence, prove that;  $\mathbf{ON}:\mathbf{NB} = 2:1$

### COPLANARITY OF POINTS AND VECTORS:

47. Show whether the following points are coplanar or not.
- A(2, 5, 10), B(6, -7, 0), C(0, 3, -6) and D(16, -29, -4)
  - P(4, 5, 1), R(0, -1, -1), S(3, 9, 4) and T(-4, 4, 4)
48. Show that the vectors;  $\begin{pmatrix} -4 \\ -6 \\ -2 \end{pmatrix}$ ,  $\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} -8 \\ -1 \\ 3 \end{pmatrix}$  lie in the same plane.
49. Four points; G, F, R and T are such that;  $\mathbf{OG} = -\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ ,  $\mathbf{OF} = -5\mathbf{k} + 3\mathbf{i} + 2\mathbf{j}$ ,  $\mathbf{OR} = -3\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$  and  $\mathbf{OT} = \mathbf{k} - 3\mathbf{i} + 2\mathbf{j}$ . Show that the points; G, F, R and T are coplanar.
50. Given that vectors;  $2\mathbf{i} - \mathbf{k} + 4\mathbf{j}$ ,  $8\mathbf{i} - 10\mathbf{j} + 5\mathbf{k}$  and  $5\mathbf{i} - m\mathbf{j} + 2\mathbf{k}$  lie in the same plane, find the value of  $m$ .

51. Four points with position vectors;  $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ ,  $\begin{pmatrix} 8 \\ t \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ 10 \\ -6 \end{pmatrix}$  and  $\begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix}$  lie in the same plane.

Find the value of  $t$ .

52. Determine whether the three vectors:  $\mathbf{p} = 3\mathbf{i} + 4\mathbf{k} - \mathbf{j}$ ,  $\mathbf{q} = 6\mathbf{i} - 8\mathbf{k} - 4\mathbf{j}$  and  $\mathbf{r} = 7\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$  are coplanar or not.
53. Given that the vectors:  $\mathbf{s} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ ,  $\mathbf{t} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$  and  $\mathbf{r} = 3\mathbf{i} - 4\mathbf{j} + D\mathbf{k}$  are coplanar, find the value of  $D$ .

### THE DOT / SCALAR PRODUCT AND CROSS / VECTOR PRODUCT:

54. Given that vectors;  $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{b} = 6\mathbf{i} - 3(\mathbf{k} - 2\mathbf{j})$  and  $\mathbf{c} = \mathbf{i} + \mathbf{j} + 3\mathbf{k}$ . Find;
- $\mathbf{a} \cdot \mathbf{b}$
  - $(4\mathbf{a} - 2\mathbf{c}) \cdot (3\mathbf{b} + 2\mathbf{a})$
55. Points A and B have position vectors;  $3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$  and  $\mathbf{k} + 3(\mathbf{i} + \mathbf{j})$  respectively, relative to the origin, O. Obtain the;
- Lengths of  $\mathbf{OA}$  and  $\mathbf{OB}$ .
  - Scalar product of  $\mathbf{OA}$  and  $\mathbf{OB}$
  - Acute angle between  $\mathbf{OA}$  and  $\mathbf{OB}$ , correct to two decimal places.
56. Two points; A and B are such that;  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OB} = \mathbf{b}$ . Given that the acute angle between  $\mathbf{OA}$  and  $\mathbf{OB}$  is  $\theta$ , show that;  $\theta = \cos^{-1} \left\{ \frac{|\mathbf{a} \cdot \mathbf{b}|}{|\mathbf{a}| |\mathbf{b}|} \right\}$ .
57. Using dot product, find the normal (**orthogonal**) vector to each of the following pair of vectors.
- $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$  and  $-2\mathbf{i} + 4\mathbf{j}$
  - $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\mathbf{i} + \mathbf{j} + \mathbf{k}$
58. Given that;  $\mathbf{m} = 2\mathbf{i} + \beta\mathbf{j}$ ,  $\mathbf{n} = \mu\mathbf{i} - 3\mathbf{j}$  and  $\mathbf{c} = \mathbf{i} - \mathbf{j}$ . Find the value of;
- $\beta$  if  $\mathbf{m}$  is perpendicular to  $\mathbf{c}$ .
  - $\mu$  if  $\mathbf{n}$  is parallel to  $\mathbf{c}$ .
59. Given the position vectors  $\mathbf{n} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $\mathbf{r} = -2\mathbf{i} + \mathbf{k}$ , find the;
- a unit vector normal to both  $\mathbf{n}$  and  $\mathbf{r}$ .
  - angle between vectors  $\mathbf{n}$  and  $\mathbf{r}$ .

60. Given that  $\mathbf{a} = \alpha\mathbf{i} + (2 + \alpha)\mathbf{j} + 3\mathbf{k}$  and  $\mathbf{b} = 3\mathbf{j} - \mathbf{i} + (4 - \alpha)\mathbf{k}$  are orthogonal to one another, find the value of  $\alpha$ .
61. If  $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ ,  $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and  $\mathbf{c} = 3\mathbf{i} + \mathbf{j}$  are such that;  $\mathbf{a} + \lambda\mathbf{b}$  is normal to  $\mathbf{c}$ , find the value of  $\lambda$ .
62. Using cross product, find the vector that's normal to each of the following pair of vectors.
- $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  and  $3\mathbf{j} + 4\mathbf{k} + \mathbf{i}$
  - $5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$  and  $\mathbf{i} + \mathbf{j} + \mathbf{k}$
  - $3\mathbf{i} - \mathbf{k}$  and  $2\mathbf{j} - \mathbf{i}$
63. Find the unit vector that is orthogonal to each of the following pairs of vectors:
- $2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$  and  $-\mathbf{k} + 2(3\mathbf{j} + \mathbf{i})$
  - $2\mathbf{k} + \mathbf{i} + \mathbf{j}$  and  $2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$
64. Three points; R, S, T and U are such that;  $\mathbf{OR} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ ,  $\mathbf{OS} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ ,  $\mathbf{OT} = 6\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$  and  $\mathbf{OU} = 13\mathbf{i} - 3\mathbf{k} + 8\mathbf{j}$ .  
Show that;
- $\mathbf{RS}$  is normal to  $\mathbf{RT}$
  - $\mathbf{RT}$  is parallel to  $\mathbf{SU}$
65. Three points P, Q and R have position vectors  $7\mathbf{i} + 10\mathbf{j}$ ,  $3\mathbf{i} + 12\mathbf{j}$  and  $-\mathbf{i} + 4\mathbf{j}$ .
- Write down the vectors  $\overrightarrow{PQ}$  and  $\overrightarrow{RQ}$ , and show that they are normal to one another.
  - Using a scalar product or otherwise, find angle PRQ
  - Find the position vector of S, the midpoint of PR
  - Show that  $|\overrightarrow{QS}| = |\overrightarrow{RS}|$ . Using your previous results or otherwise, find angle PSQ
66. If  $\mathbf{a} = 2\mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = \beta\mathbf{i} + 4\mathbf{j} - \frac{\gamma}{2}\mathbf{k}$  are equal, find the values of  $\lambda$ ,  $\beta$  and  $\gamma$ .
67. The position vectors of points A and B are  $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$  and  $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$  respectively, relative to the origin O.
- Calculate angle AOB.
  - Point C is such that  $\mathbf{AC} = 3\mathbf{AB}$ , find the unit position vector in the direction of  $\overrightarrow{OC}$ .

68. Relative to the origin O, the position vectors of P and Q are given by  $\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$  and  $\overrightarrow{OQ} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ .
- Use vector product to find angle POQ, correct to the nearest degree.
  - Find the unit vector in the direction of  $\overrightarrow{PQ}$ .
  - The point R is such that  $\overrightarrow{OR} = 6\mathbf{j} + m\mathbf{k}$ , where  $m$  is a constant. Given that the lengths of  $\overrightarrow{PQ}$  and  $\overrightarrow{PR}$  are equal. Find the possible values of  $m$ .

### THE TRIANGLE:

69. Relative to origin, points A, B and C have position vectors;  $\mathbf{i} + 3\mathbf{k} + 2\mathbf{j}$ ,  $4\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$  and  $2\mathbf{i} + 7\mathbf{j} + 8\mathbf{k}$ . Prove that A, B and C are the vertices of a triangle.
70. Show that the vectors;  $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ ,  $\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$  and  $3\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$  form the vertices of a right-angled triangle.
71. In each of the following, show whether the given set of points form the vertices of a triangle.
- (2, 2, 2), (-1, 3, -1) and (1, 1, 4)
  - (1, 5, 9), (2, 6, 10) and (3, 7, 12)
  - (7, 7, 7), (8, 6, 5) and (9, 8, 8)
72. Show, using vector methods, that the points; R, S and T with respective position vectors;  $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$  form the vertices of a right – angled triangle.
73. In a triangle OAB, points A and B are such that;  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OB} = \mathbf{b}$ .  
Show that the area, A of the triangle OAB is given by;  $A = \frac{1}{2}\sqrt{|\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2}$
74. In a triangle OAB, points A and B are such that;  $\mathbf{OA} = \mathbf{a}$  and  $\mathbf{OB} = \mathbf{b}$ , and  $\angle AOB = \theta$ ,
- Show that the area of  $\Delta OAB$  given by;  $\frac{1}{2}|\mathbf{a}||\mathbf{b}|\sin\theta$  sq. units
  - Hence, deduce that the area of  $\Delta OAB$  can be obtained as;  $\frac{1}{2}|\mathbf{a} \times \mathbf{b}|$  sq. units.

75. Points A, B and C have position vectors  $4\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}$ ,  $6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$  and  $\mathbf{i} + 10\mathbf{j} + 3\mathbf{k}$  respectively.
- Show A, B and C are the vertices of a triangle
  - Prove that a triangle ABC is right angled.
  - Using vector product, find the size of  $\angle ABC$
  - Solve the triangle ABC.
76. Three points; N, R and T are such that;  $\mathbf{ON} = \mathbf{1} + 3\mathbf{k} + 2\mathbf{j}$ ,  $\mathbf{OR} = 2\mathbf{j} + \mathbf{i} + 7\mathbf{k}$  and  $\mathbf{OT} = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}$ .  
Show that the triangle NRT is isosceles. Hence, calculate its exact area.
77. A triangle PQR is such that;  $\mathbf{OP} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{OQ} = 3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$  and  $\mathbf{OR} = 12\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ .
- Show that PQR is isosceles.
  - Hence, calculate its exact area.
78. Given that points; A(1, 2, 3), B(5, 6, 7) and C(4, -1, 9) form the vertices of a triangle ABC. Clearly, show that ABC is an isosceles triangle.  
Hence, calculate its exact area.
79. Points; X, Y and T have position vectors;  $\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ ,  $3\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$  and  $-2\mathbf{i} + 7\mathbf{k} + 5\mathbf{j}$  respectively.
- Show that the triangle XYT is isosceles.
  - Calculate the exact area of isosceles triangle XYT.
80. Given the points; R, S and T with position vectors (relative to origin, O);  $10\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ ,  $12\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$  and  $5\mathbf{i} - 2\mathbf{j} + 13\mathbf{k}$  respectively.
- Show that RST is an isosceles triangle.
  - Calculate the exact area of triangle RST.
81. Three points; K, L and J are such that;  $\mathbf{OL} = 2\mathbf{j} + 3\mathbf{k} + 8\mathbf{i}$ ,  $\mathbf{OJ} = \mathbf{i} + 2(5\mathbf{k} + \mathbf{j})$  and  $\mathbf{OK} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ .
- Show that KLM is an isosceles triangle.
  - Prove that the exact area of  $\Delta KLM$  is; 24.5 sq. units
82. Given that the points; M(1, 2, 3), X(4, 5, 6) and Y(9, 6, 7) are the vertices of a triangle.

- a) Show that  $\Delta MXY$  is isosceles.
- b) Prove that the exact area of a  $\Delta MXY$  is;  $6\sqrt{2}$  sq. units.
83. Points; D, F and G have position vectors (relative to the origin, O);  $\begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix}$ ,  $\begin{pmatrix} 11 \\ 22 \\ 33 \end{pmatrix}$  and  $\begin{pmatrix} 13 \\ 21 \\ 32 \end{pmatrix}$  respectively.
- a) Show that  $\Delta DFG$  is isosceles.
- b) Prove that the exact area of  $\Delta DFG$  is;  $\frac{5\sqrt{3}}{2}$  sq. units.
84. Points R, M and T are such that;  $\mathbf{OR} = 11\mathbf{i} + 32\mathbf{k} + 21\mathbf{j}$ ,  $\mathbf{OM} = 10\mathbf{i} + 20\mathbf{j} + 30\mathbf{k}$  and  $\mathbf{OT} = 8\mathbf{i} + 24\mathbf{j} + 29\mathbf{k}$ .
- a) Show that RMT is a right – angled triangle.
- b) Calculate the exact area of a triangle RMT.
85. A triangle; GHT has vertices; G, H and T with position vectors (relative to the origin, O);  $\begin{pmatrix} 11 \\ 22 \\ 33 \end{pmatrix}$ ,  $\begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix}$  and  $\begin{pmatrix} 14 \\ 15 \\ 32 \end{pmatrix}$  respectively. Show that the;
- a) triangle GHT is right – angled at H.
- b) exact area of triangle GHT is;  $\frac{3\sqrt{70}}{2}$  square units.
86. Points; N, M and T are such that;  $\mathbf{ON} = 23\mathbf{k} + 11\mathbf{i} + 16\mathbf{j}$ ,  $\mathbf{OT} = 19\mathbf{j} + 12\mathbf{i} + 18\mathbf{k}$  and  $\mathbf{OM} = 10\mathbf{i} + 15\mathbf{j} + 20\mathbf{k}$ .
- a) Prove that NMT is a right – angled triangle.
- b) Hence, show that the exact area of a right – angled triangle NMT is;  $\sqrt{66}$  sq. units.
87. Show that the vectors;  $21\mathbf{i} + 32\mathbf{j} + 44\mathbf{k}$ ,  $20\mathbf{i} + 30\mathbf{j} + 40\mathbf{k}$  and  $22\mathbf{i} + 27\mathbf{j} + 41\mathbf{k}$  form the vertices of a right – angled triangle. Hence prove that its exact area is;  $\frac{7\sqrt{6}}{2}$  sq. units
88. Calculate the exact area of a triangle whose two adjacent sides are represented by vectors;
- a)  $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$  and  $\mathbf{i} - 4\mathbf{k} + 3\mathbf{k}$
- b)  $-5\mathbf{j} + \mathbf{i} - 2\mathbf{j}$  and  $6\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

89. A right – angled triangle has vertices; A, B and C such that;  $\mathbf{OA} = m\mathbf{i} + 7(\mathbf{j} + \mathbf{k})$ ,  $\mathbf{OB} = \mathbf{i} + 6\mathbf{j} + 5\mathbf{k}$  and  $\mathbf{OC} = -2\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}$ .

Find the possible values of  $m$ .

90. The vectors  $\mathbf{a}$  and  $\mathbf{b}$  are such that  $|\mathbf{a}| = 3$ ,  $|\mathbf{b}| = 12$  and  $\mathbf{a} \cdot \mathbf{b} = 18$ .

Show clearly that;  $|\mathbf{a} - \mathbf{b}| = 3\sqrt{13}$ .

### QUADRILATERALS:

These include the following:

- i). Rectangle.
  - ii). Parallelogram.
  - iii). Square.
  - iv). Rhombus.
91. A quadrilateral ABC has vertices A, B and C with position vectors;  $-4\mathbf{i} + 6\mathbf{j}$ ,  $3\mathbf{i} + 5\mathbf{j}$ ,  $4\mathbf{i} - 2\mathbf{j}$  and  $-3\mathbf{i} - \mathbf{j}$  respectively.

Verify whether the quadrilateral is a rhombus or a parallelogram.

92. The vertices of a quadrilateral ABCD are; A(-3, 2), B(1, -2), C(5, 2) and D(1, 6).

Prove whether ABCD is a rectangle, rhombus, parallelogram or a square.

93. In a parallelogram OABC,  $\mathbf{OA} = \mathbf{a}$ ,  $\mathbf{OC} = \mathbf{c}$  and  $\angle AOC = \theta$ .

Clearly show that the area of a parallelogram OABC can be obtained as;

- a)  $Area = |\mathbf{a}||\mathbf{c}|\sin\theta$  sq. units
  - b)  $Area = \sqrt{|\mathbf{a}|^2|\mathbf{c}|^2 - (\mathbf{a} \cdot \mathbf{c})^2}$  sq. units
  - c)  $Area = |\mathbf{a} \times \mathbf{c}|$  sq. units
94. Calculate the exact area of a parallelogram whose two adjacent sides are given by;
- a)  $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and  $-3\mathbf{k} + 4\mathbf{j} + 3\mathbf{i}$
  - b)  $2\mathbf{i} + \mathbf{k} - 4\mathbf{j}$  and  $\mathbf{i} + \mathbf{j} - \mathbf{k}$
95. Find the exact area of a rectangle whose adjacent sides are represented by position vectors;  $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  and  $5\mathbf{i} - \mathbf{j} - 8\mathbf{k}$ .
96. Points; P, R, S and T are such that;  $\mathbf{OP} = \mathbf{1} + 2\mathbf{j} + 3\mathbf{k}$ ,  $\mathbf{OR} = 5\mathbf{i} + 7\mathbf{j} + 9\mathbf{k}$ ,  $\mathbf{OS} = 7\mathbf{i} + 11\mathbf{j} + 17\mathbf{k}$  and  $\mathbf{OT} = 3\mathbf{i} + 6\mathbf{j} + 11\mathbf{k}$ .

Show that the;

- a) Quadrilateral PRST (in that order) is a parallelogram
  - b) area of a quadrilateral PRST is;  $2\sqrt{173}$  sq. units.
97. A parallelogram ABCD has vertices with position vectors;  $\mathbf{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{OB} = 4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$  and  $\mathbf{OC} = 5\mathbf{i} + 8\mathbf{j} + 6\mathbf{k}$ .
- a) Find the position vector of vertex D
  - b) Show that the exact area of the parallelogram ABCD is;  $11\sqrt{2}$  sq. units.
98. The points; X, R and S have position vectors;  $\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\begin{pmatrix} -2 \\ 0 \\ 5 \end{pmatrix}$  and  $\begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix}$  respectively.
- a) Find the position vector of point T such that XRST is a parallelogram.
  - b) Calculate the exact area of the parallelogram XRST.
99. Points; P, R and T have position vectors;  $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ ,  $5\mathbf{j} + 4\mathbf{i}$  and  $2\mathbf{k} + 5\mathbf{i} + 2\mathbf{j}$  respectively.
- a) Find the co – ordinates of point S such that PRST is a parallelogram.
  - b) Calculate the exact area of the parallelogram PRST.
100. A quadrilateral ABCD has vertices with position vectors;  $\mathbf{OA} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ,  $\mathbf{OB} = 6\mathbf{i} - \mathbf{j} + \mathbf{k}$ ,  $\mathbf{OC} = \mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$  and  $\mathbf{OD} = 6\mathbf{i} - 3\mathbf{k}$ .  
Show whether ABCD is a rectangle or parallelogram.
101. A quadrilateral RXYZ has vertices; R(-3, 0), X(-1, 3), Y(5, 2) and Z(3, -1).
- a) Show whether a quadrilateral ABCD is a rhombus, a square or a parallelogram.
  - b) Hence, calculate the exact area of the Quadrilateral RXYZ.
102. The diagonals of a parallelogram are determined by the vectors;  $3\mathbf{j} + 3\mathbf{k}$  and  $-(\mathbf{i} + 2\mathbf{k}) + \mathbf{j}$ .
- a) Show that this parallelogram is a rhombus.
  - b) Determine the vectors representing its sides. Hence, determine the length of its sides.
103. The diagonals of a quadrilateral ABCD are represented by vectors;  $3\mathbf{i} + 3\mathbf{j} + 10\mathbf{k}$  and  $-\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$ .  
Show that the quadrilateral ABCD is a rectangle.

104. Four points; A, B, C and D are such that;  $\mathbf{OA} = \mathbf{i} + 3\mathbf{k} + 3\mathbf{j}$ ,  $\mathbf{OB} = 3(\mathbf{i} + \mathbf{j}) + 5\mathbf{k}$ ,  $\mathbf{OC} = 4\mathbf{i} + 7\mathbf{k} + \mathbf{j}$  and  $\mathbf{OD} = 2\mathbf{i} + 5\mathbf{k}$ .
- Using vector methods, prove that vertices ABCD (taken in that order) form a rhombus.
  - Hence, show that the exact area of the rhombus ABCD is;  $\sqrt{65}$  sq. units.
105. A parallelogram PRST has vertices; P at the origin, R at (2, 2, 1), S at (3, 0, 3) and T at (1, -2, 2).
- Using vector methods, prove that the parallelogram PRST is a square.
  - Show that the exact area of square PRST is; 9.0 sq. units
106. Given points; A(0, 1, 0), B(1, 3, 2), C(5, 2, 1) and D(4, 0, -1).
- Prove that points; A, B, C and D are vertices of a rectangle but not a square.
  - Show that the exact area of the rectangle is;  $9\sqrt{2}$  sq. units
107. A parallelogram KLMN has vertex K at the origin, and points; M and N have position vectors;  $7\mathbf{k} - \mathbf{i}$  and  $3\mathbf{k} - 4\mathbf{i}$ .
- Find the position vector of vertex L
  - Prove that the parallelogram KLMN is a square.
  - Hence, show that the exact area of the square KLMN is; 25 sq. units.
108. Three points; R, S and U such that;  $\mathbf{OR} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ ,  $\mathbf{OS} = 3(\mathbf{i} + \mathbf{k}) + 2\mathbf{j}$  and  $\mathbf{OU} = 2 + 3(\mathbf{j} + \mathbf{k})$  form the vertices of a parallelogram RSTU.
- Find the co – ordinates of vertex T
  - Prove that the parallelogram RSTU is a rhombus not a square.
  - Hence, show that the exact area of RSTU is;  $\sqrt{17}$  sq. units.
109. A quadrilateral XYRT has vertices; X, Y, R and T with position vectors;  $\begin{pmatrix} 5 \\ 10 \\ 20 \end{pmatrix}$ ,  $\begin{pmatrix} 7 \\ 11 \\ 23 \end{pmatrix}$ ,  $\begin{pmatrix} 8 \\ 15 \\ 21 \end{pmatrix}$  and  $\begin{pmatrix} 6 \\ 14 \\ 18 \end{pmatrix}$  respectively.
- Prove that the quadrilateral XYRT is a rectangle but not a square.
  - Show that the exact area of the rectangle is;  $7\sqrt{6}$  sq. units.

110. The points; R, S, T and V are vertices of a parallelogram RSTV (in that order) such that;  
 $\mathbf{OR} = 10\mathbf{i} + 20\mathbf{j} + 30\mathbf{k}$ ,  $\mathbf{OS} = 11\mathbf{i} + 22\mathbf{j} + 34\mathbf{k}$  and  $\mathbf{OV} = 16\mathbf{i} + 19\mathbf{j} + 29\mathbf{k}$ .
- Find the coordinates of point T
  - Prove that the parallelogram RSTV is a rectangle.
  - Show that the exact area of the rectangle is;  $\sqrt{798}$  sq. units.

### LINES AND 3D PLANES:

111. The points A, B and C have position vectors;  $-2\mathbf{i} + 3\mathbf{j}$ ,  $\mathbf{i} - 2\mathbf{j}$ ,  $8\mathbf{i} - 5\mathbf{j}$  respectively.
- Find the equation of line AC
  - Determine the co-ordinates of point D, if ABCD is a parallelogram
  - Write down the vector equation of the line through point B perpendicular to AC.

Hence, find where it meets AC.

112. The cartesian equation of the line is given by;  $\frac{2-x}{-3} = \frac{y-3}{1} = \frac{4-2z}{2}$ . Find the;
- direction vector of the line
  - two points that lie on a line
  - vector equation of the line that passes through a point with position vector;  $-\mathbf{k} + 4\mathbf{j}$  and is parallel to the given line.

113. Find the cartesian equation of a line that passes through the following three collinear points;

- A(-2, 0, 4), B(5, 2, 6) and C(3, -1, -4).
- R(1, 2, 3), S(6, 7, 8) and T(16, 17, 18)
- X(1, 2, 4), Y(3, 5, 10) and V(9, 14, 28)

(Hint: For three collinear points,  $\mathbf{r} = \overline{OA} + \beta\overline{BC}$ ).

114. A line  $l$  passes through a point with a position vector;  $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$  and is parallel to the two vectors;  $-4\mathbf{j} + \mathbf{i}$  and  $3\mathbf{i} + 5\mathbf{k}$ . Find the;
- Vector,

- b) Parametric,  
 c) Symmetrical, equation(s) of the line  $l$ .
115. The points with co-ordinates; A(7, 6, 10), B(6, 5, 6) and C(1, 0, 4) are the vertices of a parallelogram ABCD.
- a) Find the;  
 i). co-ordinates of point D  
 ii). vector equation of the line  $l$  that passes through points; A and C.
- b) Show that the;  
 i). shortest distance of  $l$  from B is;  $\sqrt{6}$  sq. units  
 ii). exact area of a parallelogram ABCD is;  $18\sqrt{2}$  sq. units

116. The point A has position vector;  $-\mathbf{i} + 7\mathbf{j} - \mathbf{k}$ .

- a) Find the vector equation of the straight line  $l_1$  which passes through A and parallel to the vector  $3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ .

The straight line  $l_2$  has equation:  $\mathbf{r}_2 = 9\mathbf{i} - 9\mathbf{j} + 8\mathbf{k} + \beta(3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$ .

- b) Show that;  
 i).  $l_1$  and  $l_2$  do not intersect. Hence, find the least distance between them.  
 ii). Vector  $2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$  is perpendicular to both  $l_1$  and  $l_2$ .

Point P lies on  $l_1$  and M lies on  $l_2$  so that  $\overline{PM}$  is least.

- c) find the co-ordinates of P and M.

117. Determine whether the following pairs of lines are parallel, concurrent (coincident) or skew:

$$\mathbf{r}_1 \equiv \begin{cases} x = 4 + 5t \\ y = 3 + 2t \\ z = 3t \end{cases} \quad \text{and} \quad \mathbf{r}_2 \equiv \begin{cases} x = -5 + 2\beta \\ y = 4 - \beta \\ z = 1 \end{cases}$$

118. Lines  $l_1, l_2$  and  $l_3$  are given by the equations:

$$\frac{x+5}{2} = \frac{y-14}{-10} = \frac{z+13}{11} = \alpha, \quad \frac{x-3}{2} = \frac{y+5}{-3} = \frac{z+17}{-5} = \beta \quad \text{and} \quad x = y + 5 = \frac{z-7}{-2} = \gamma \quad \text{respectively.}$$

$l_1$  intersects  $l_2$  at A while  $l_1$  intersects  $l_3$  at B.

Find the distance AB.

119. The straight lines;  $l_1$  and  $l_2$  have respective vector equations;

$$\mathbf{r}_1 = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r}_2 = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \beta(\mathbf{i} + 2\mathbf{k})$$

Where;  $\mu$  and  $\beta$  are scalar parameters.

Show that;

- a)  $l_1$  and  $l_2$  are skew
- b) shortest distance between  $l_1$  and  $l_2$  is  $\frac{5}{\sqrt{14}}$  units using;
  - i). dot product.
  - ii). cross product.

120. The straight lines  $l_1$  and  $l_2$  have the following vector equations:

$$\mathbf{r}_1 = \mathbf{i} - 5\mathbf{j} + \alpha(4\mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r}_2 = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \beta(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

Where;  $\alpha$  and  $\beta$  are scalar parameters.

- a) Given that  $l_1$  and  $l_2$  intersect at point Q. Find the position vector of Q.
- b) Given that P lies on  $l_1$  and has position vector;  $\mathbf{i} + p\mathbf{j} - 3\mathbf{k}$ , find the value of  $p$ .

The point T lies on  $l_2$  so that  $|\overrightarrow{PQ}| = |\overrightarrow{QT}|$ ,

- c) Determine the possible position vectors of T.

121. Relative to a fixed origin O, the straight lines  $l$  and  $m$  have vector equations;

$$\mathbf{r}_1 = \begin{pmatrix} p \\ 4 \\ 5 \end{pmatrix} + \gamma \begin{pmatrix} q \\ -1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 9 \\ 0 \\ 16 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}$$

Where;  $\gamma$  and  $\beta$  are scalar parameters,  $p$  and  $q$  are scalar constants.

Point A is the intersection of  $l$  and  $m$ , and cosine of acute angle  $\theta$  between  $l$  and  $m$  is  $\frac{1}{3}\sqrt{6}$

- a) Find the values of;  $p$  and  $q$ , given that  $q$  is a positive integer.
- b) Determine the co-ordinates of A.

122. The straight line  $l_1$  passes through the points; A(2, 5, 9) and B(6, 0, 10).

Another line  $l_2$  has equation:  $\mathbf{r} = \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ .

- a) Show that Point A is the intersection of;  $l_1$  and  $l_2$ .
- b) Show further that;  $l_1$  and  $l_2$  are perpendicular to each other.

123. Relative to a fixed origin O, point A has position vector;  $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$ , point B has position vector;  $5(\mathbf{i} + 2\mathbf{k}) + 2\mathbf{j}$  and point D has a position vector;  $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ .

The line  $l$  passes through points; A and B.

- Find the vector  $\overrightarrow{AB}$ .
- Find the cartesian equation of the line  $l$ .
- Show that the size of the angle BAD is  $71^\circ$ , to the nearest degree.

Points; A, B, and C are the vertices of the parallelogram ABCD, where  $\overrightarrow{AB} = \overrightarrow{DC}$ .

- Find the position vector of C.
- Calculate the area of the parallelogram ABCD. Give your answer in simplified surd,
- Find the shortest distance from point D to line  $l$ , giving your answer to 3 significant figures.

124. Relative to the fixed origin O, points; A and B have position vectors;  $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$  and  $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

respectively. The line  $l$  passes through points; A and B.

- Find the parametric equations of  $l$ .

Point P has position vector;  $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$  and angle PBA is  $\theta$ .

- Show that;  $\cos\theta = \frac{1}{3}$

Line  $l_0$  passes through point P and is parallel to line  $l$ .

- Find the vector equation of the line,  $l_0$ .

Points; C and D both lie on line  $l_0$ .

Given that;  $\overrightarrow{AB} = \overrightarrow{PC} = \overrightarrow{DP}$  and the  $x$  –coordinate of C is positive.

- Find the coordinates of C and the coordinates of D
- Find the exact area of the trapezium ABCD, giving your answer as a simplified surd.

125. With respect to a fixed origin O, the lines;  $l_1$  and  $l_2$  are given by the equations;

$$l_1: \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \beta(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

Where  $\lambda$  and  $\beta$  are scalar parameters.

- YouTube Channel: **JK Maths**
- WhatsApp Channel: **TR. JK (A' Level Maths)**

- a) Show that;  $l_1$  and  $l_2$  meet. Hence, find the position vector of their point of intersection.
- b) Show that  $l_1$  and  $l_2$  are normal to one another.

The point A has position vector;  $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$ .

- c) Show that A lies on  $l_2$
126. Given the lines:  $l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ -3 \end{pmatrix}$  and  $l_2: \mathbf{r} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ .
- a) Show that  $l_1$  and  $l_2$  are perpendicular.
- b) Find the position vector of their point of intersection.

127. Show that the least distance of A(0, 3, 5) from the line;  
 $\mathbf{r} = 2\mathbf{i} + \beta\mathbf{k} - \mathbf{j} - \beta\mathbf{i} + 3\mathbf{k} + 2\beta\mathbf{j}$  is zero.

128. Given the lines;

$$l_1: \mathbf{r}_1 = -\mathbf{k} + \mathbf{i} + 2\mathbf{j} + \gamma(4\mathbf{j} + 5\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad l_2: \mathbf{r}_2 = 3\beta\mathbf{k} + 2\mathbf{i} + 4\beta\mathbf{j} + \mathbf{k} + 5\beta\mathbf{i}.$$

Show that;

- a)  $l_1$  and  $l_2$  are parallel.
- b) Perpendicular distance between  $l_1$  and  $l_2$  is;  $\frac{21\sqrt{2}}{10}$  units.
129. Find the angle between vector  $3\mathbf{k} - 4\mathbf{j}$  and the line  $\mathbf{r} = -\beta\mathbf{j}$ .
130. The points A and B have position vectors;  $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$  and  $4\mathbf{i} + \mathbf{j} + \mathbf{k}$  respectively.  
 The line  $l$  has equation:  $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + t(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ .  
 Point J lies on the line  $l$  such that  $\hat{PAB} = 120^\circ$ .

Show that;

- a)  $3t^2 + 8t + 4 = 0$
- b) Position vector of J is;  $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ .
131. With respect to origin O, the points A and B have position vectors given by;

$$\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k} \quad \text{respectively}$$

The line  $l$  has vector equation;  $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + \beta(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ .

- a) Prove that  $l$  does not intersect the line through A and B.
- b) Hence, find the shortest distance between  $l$  and line AB.
132. Points A and B have position vectors;  $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$  and  $-4\mathbf{k} + \mathbf{j}$  respectively.

- a) Find the parametric equations of the line that passes through points; A and B.  
Point C has position vector;  $6\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ .
- b) Find the perpendicular distance of C from the line in a) above
133. The line  $l$  has equation:  $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$  and point P has position vector;  $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ .
- a) Show that P does not lie on the line  $l$ .
- Given that a circle with center P intersects  $l$  at points; A and B.
- Point A has position vector;  $\begin{pmatrix} 0 \\ -3 \\ 4 \end{pmatrix}$ .
- b) Find the position vector of B.
134. The lines  $l_1$  and  $l_2$  have vector equations;  
 $\mathbf{r}_1 = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \alpha(\mathbf{i} + \mathbf{j} - \mathbf{k})$  and  $\mathbf{r}_2 = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \beta(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$   
respectively.
- a) Show that  $l_1$  and  $l_2$  are skew.
- b) Hence, find the distance between  $l_1$  and  $l_2$
- Point P lies on  $l_1$  and point M has a position vector  $2\mathbf{i} - \mathbf{k}$ .
- c) Given that the line PM is perpendicular to  $l_1$ , find the position vector of P.  
Hence, write the cartesian Equation of line PM
- d) Verify that M lies on  $l_2$ , and that PM is perpendicular to  $l_2$ .
135. Two points; A and B have position vectors  $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$  and  $4\mathbf{i} + 3\mathbf{j}$  respectively, relative to the origin O.
- a) Find the lengths OA and OB
- b) Find the scalar product of OA and OB. Hence, find angle AOB.
- c) Find the exact area of triangle AOB.
- The point C divides AB in the ratio  $\beta: 1 - \beta$
- c) Find the expression of  $\overrightarrow{OC}$
- d) Show that  $|\overrightarrow{OC}|^2 = 14\beta^2 + 2\beta + 9$ .

- e) Find the possible position vectors of the two points on AB whose distance from O is  $\sqrt{21}$  units.
- f) Show that the shortest (**perpendicular**) distance of O from AB is approximately 2.99 units.
136. Using vectors, find the acute angle between the following pairs of lines;
- $2y = x + 3$  and  $3y + 2x = 1$
  - $4y + 3x = 10$  and  $5x - 2y = 7$
  - $5x + 2y = 1$  and  $2x + 7y - 12$
  - $4y - 3x = 15$  and  $2x + 11y + 1 = 0$
137. The line  $l$  has Equation:  $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \gamma(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ .  
A point P has position vector  $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ .  
Find the length of the perpendicular from P to  $l$ . (**Use both cross and dot product**)
138. The line equation  $l_0$  has equation:  $\frac{x-1}{2} = \frac{1-y}{-2} = \frac{z+3}{-1}$ .  
Point A has co-ordinates; (1, -2, 1).  
Find the cartesian equation of the line that is perpendicular to  $l_0$  and passes through A.
139. Find the equation of the plane that contains points; (1, 3, 2), (2, 1, -1) and (2, 4, 1) in;
- scalar,
  - cartesian, form.
140. Find the cartesian equation of the plane that contains point; (1, 1, -2) and normal to the vector;  $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ .
141. A plane  $\pi$  contains a point with position vector (relative to the origin, O);  $2\mathbf{i} - 3(\mathbf{k} - 2\mathbf{k})$ .  
Vectors;  $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$  and  $-3\mathbf{i} + 2\mathbf{i} + 5\mathbf{k}$  are parallel to the plane  $\pi$ .
- Write the equation of  $\pi$  in parametric form.
  - Compute the normal vector to the plane.
  - Hence, find the equation of  $\pi$  in scalar product form.
142. Find the cartesian equation of the plane that contains a point whose position vector is;  $\mathbf{i} - 4\mathbf{k} + 2\mathbf{j}$  and normal to the vector;  $3\mathbf{j} + \mathbf{k} - 4\mathbf{i}$ .
143. Two lines;  $l_1$  and  $l_2$  have equations;

$$\mathbf{r}_1 = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \text{ and } \mathbf{r}_2 = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} \text{ respectively, and } p \text{ is a constant.}$$

$l_1$  and  $l_2$  intersect at Q. Find the;

- value of;  $p$
- position vector of Q
- equation of the plane containing  $l_1$  and  $l_2$ .

144. Find the equation of the plane that contains two points with position vectors;  $2\mathbf{i} - 3\mathbf{k}$  and  $-\mathbf{j} + \mathbf{i} - 4\mathbf{k}$ .

145. Three points; (-3, -6, 11), (2, -1, 6) and (6, 0, -10) lie on the plane  $\pi$ .

- Write the parametric equation of  $\pi$ .
- Find the vector normal to the plane  $\pi$ .
- Hence, find the cartesian equation of the  $\pi$ .
- Verify that point; (7, 2, -7) lies on the plane  $\pi$ .

146. Using the general cartesian equation of the plane;  $ax + by + cz + d = 0$ , find the equations of planes that contain the following points;

- (2, 3, 1), (1, -1, 2) and (2, 1, -2)
- (0, 0, 2), (1, 2, 3) and (-2, 0, -1)

147. Show that the lines:  $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z+2}{4}$  and  $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$  are parallel to each other.

- Find the cartesian equation of the plane containing the two lines.
- Calculate the shortest distance between those two lines.

148. A plane  $\pi$  contains points; (1, 2, 5), (1, 0, 4) and (5, 2, 1).

Show whether the point with position vector;  $-4\mathbf{k} + 2\mathbf{i} + \mathbf{j}$  lies on  $\pi$  or not.

149. Show that; (2,4,1) lies on the plane that contains points; (4, 2, 3), (5, 1, 4) and (-2, 1, 1).

150. The plane  $\pi$  has parametric equation:  $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \rho \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ .

Show that the point with position vector;  $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$  lies on the plane  $\pi$ .

151. Given lines:

$$l_1: \mathbf{r} = \mathbf{i} + 2\mathbf{k} + t(2\mathbf{i} - \mathbf{j} + \mathbf{k}) \text{ and } \mathbf{r} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \beta(-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}).$$

Both lines intersect at A.

Find the;

- co-ordinates of A
- cartesian equation of the plane  $\pi_0$  containing  $l_1$  and  $l_2$ .
- cartesian equation of the plane  $\pi$  that contains a point; J(3,-2,1) and is parallel to  $\pi_0$ .
- symmetric equation of line  $l$  that is normal to  $\pi_0$  and passes through; A(0, -1, 1).

152. Given that  $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = -\mathbf{j} + 2\mathbf{k} + 3\mathbf{i}$ . Find the;

- angle between  $\mathbf{a}$  and  $\mathbf{b}$ .
- unit vector normal to both  $\mathbf{a}$  and  $\mathbf{b}$ .
- cartesian equation of plane  $\pi$  that contains  $\mathbf{a}$  and  $\mathbf{b}$ .
- line  $l$  that passes through points; A and B with above position vectors;  $\mathbf{a}$  and  $\mathbf{b}$  respectively.
- plane  $\pi_0$  that is parallel to  $\pi$ , and contains point; A(3, -1, 0).

153. Find the angle between the line:  $\frac{x+2}{2} = \frac{1-z}{-1} = \frac{1+y}{3}$  and the plane;  $3x + y + z = 1$

154. A plane contains points; A(4, -6, 5) and B(2, 0, 1).

A perpendicular to the plane from point; P(0, 4, -7) intersects the plane at point C.

Find the co-ordinates of C.

155. Find the cartesian equation of a plane that is parallel to the vectors;  $\mathbf{r}_1 = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$  and  $\mathbf{r}_2 = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  and passes through a point with position vector;  $2\mathbf{i} + 3\mathbf{k}$ .

156. If M is the foot of a perpendicular from point; A(5, -3, 2) to the line;  $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z-2}{-1}$ .

- Determine the co-ordinates of M.
- Find the equation of the plane through points; M, A and B(-1, 4, 2).

Hence, determine the angle ABM.

157. Find the position vector of the point of intersection of the line;  $x = \frac{y+1}{2} = \frac{z+1}{3}$  and the plane that passes through; A(4, 0, 1), B(0, -3, 0) and C(6, 3, 3).

- YouTube Channel: **JK Maths**
- WhatsApp Channel: **TR. JK (A' Level Maths)**

158. Find the cartesian equation of the plane that passes through points; (1, 1, 0), (4, -2, 1) and is normal to the plane  $3x + y - z = 2$ .

(Ans:  $x + 3y + 6z = 4$ )

159. Find the cartesian equation of the plane containing a line:  $\frac{1-x}{-2} = \frac{1-y}{-1} = \frac{z+2}{-1}$  and is orthogonal to the plane;  $x - 2y + 3z = 1$ .

(Ans:  $x - 7y - 5z = 4$ ).

160. A line  $l$  and a plane  $\pi$  have the following equations:

$$l: \frac{x+1}{2} = \frac{y-3}{5} = \frac{z+2}{-1} \text{ and } \pi: x + y + z = 12.$$

Given that  $l$  and  $\pi$  meet at point A.

- a) Find the position vector of A
- b) Calculate the angle between  $l$  and  $\pi$ .
- c) Find the cartesian equation of a plane  $\pi_1$  that;
  - i). contains  $l$  and a point whose position vector is  $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ .
  - ii). is parallel to plane  $\pi$  and contains a point; A(1, -2, 3).
  - iii). is perpendicular to plane  $\pi$  and contains  $l$ .

161. A plane passes through a point; (3, -6, -7) and is orthogonal to the line parametrically given by;

$$\mathbf{r} = \begin{cases} x = 2 + 3t \\ y = 1 + 4t \\ z = -8t + 7 \end{cases}.$$

Find the equation of the plane in scalar product form.

162. Line  $l$  passes through; R(3, 1, -2) and S(4, -1, 2).

A plane OMN where O is origin, meets  $l$  at A.

Points; M and N have position vectors;  $3\mathbf{j}$  and  $\mathbf{i} + 2\mathbf{j}$ , respectively

Find the co – ordinates of point A

163. A vector;  $-\mathbf{j} + \mathbf{k} - 3\mathbf{i}$  is parallel to the plane that contains two points with position vectors (relative to the origin, O);  $2\mathbf{k} + \mathbf{j} + \mathbf{i}$  and  $3\mathbf{i} + 4\mathbf{k}$ .

- a) Write down the parametric equation of the plane.

Hence, or otherwise, find the cartesian equation of the plane.

b) Calculate the distance of the point; C(5, 4, -5) from the plane in a) above.

164. The co-ordinates of the points A and B are; (0, 2, 5) and (-1, 3, 1) respectively.

Line  $l$  has equation:  $\frac{x-2}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$ .

a) Find the equation of the plane  $\pi$  which contains point A and is perpendicular to  $l$ .

Verify that point B lies on  $\pi$

b) Show that point C at which line  $l$  meets  $\pi$  is (1,4,3).

Hence, find the angle between **CA** and **CB**.

165. Find the cartesian equation of the plane  $\pi_0$  that contains a point whose position vector is;

$i - j$  and is orthogonal to line  $l: \mathbf{r} = \begin{cases} x = 3 + 2t \\ z = -t + 3 \\ y = 1 + 3t \end{cases}$ .

166. Points A, B and C are such that  $\mathbf{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ ,  $\mathbf{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$  and  $\mathbf{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ .

Find the cartesian equation of the plane  $\pi$  that is normal to **AB** and contains point C.

167. Find the position vector of the point of intersection of the line:  $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$  and the plane:  $x - y + z = 16$ .

168. The point M has co-ordinates; (2, 0, -1) and plane  $\pi$  has the equation;  $x + 2y - 2z = 8$ .

The line through M parallel to line  $l: \frac{x}{2} = y = \frac{z+1}{1}$  meets  $\pi$  at point C.

a) find the co – ordinates of C.

b) find the length of MC

169. Find the co-ordinates of the point of intersection of lines:

$l_1: \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$  and  $l_2: \frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$ .

Hence, find the;

a) equation of the plane  $\pi$  containing  $l_1$  and  $l_2$  in the form  $\mathbf{r} \cdot \mathbf{n} = D$

b) vector equation of line  $l_0$  normal to both  $l_1$  and  $l_2$  and passes through; B(1, 2, -4)

c) cartesian equation of the plane  $\pi_1$  that contains A(5, -1, 2) and  $l_0$

d) perpendicular distance of  $\pi_1$  from the origin.

170. A plane  $\pi$  contains a point whose position vector is;  $\mathbf{i} - 2\mathbf{j} - \mathbf{k}$  and is parallel to two lines;  $l_1$  and  $l_2$  parametrically given as:

$$\mathbf{r}_1 = \begin{cases} x = 2 + \beta \\ y = 3\beta - 4 \\ z = 1 - 2\beta \end{cases} \quad \text{and} \quad \mathbf{r}_2 = \begin{cases} z = 6\gamma + 1 \\ x = -3 - 9\gamma \\ y = -3\gamma \end{cases}$$

- Write down the parametric equation of  $\pi$ .
  - Find the;
    - Normal vector of  $\pi$ .
    - Cartesian equation of  $\pi$ .
    - Shortest distance of  $l_1$  from  $\pi$ .
    - Plane  $\pi_1$  that is normal to  $\pi$  and contains  $l$ .
171. Find the co-ordinates of the foot of the perpendicular drawn from point; A(1, 0, -1) to the;
- plane:  $\mathbf{r} \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k}) = 6$ .
  - line:  $\frac{x+1}{2} = \frac{y-4}{-1} = \frac{z+2}{3}$ .

172. A plane  $\pi: 2x - 3y + z = 5$  and the line  $l: \mathbf{r} = \begin{cases} y = 3 - t \\ z = 2 + 3t \\ x = 1 + 2t \end{cases}$  intersect at R.

Find the;

- position vector of R.
  - angle between  $\pi$  and  $l$ .
  - distance of  $l$  from the origin.
  - equation of the plane  $\pi_0$  that contains  $l$  and parallel to vector;  $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ .
173. Find the equation of the plane  $\pi$  containing the following lines;
- $\frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$  and  $\frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ .
  - $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$  and  $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-6}{6}$ .
  - $\mathbf{r}_1 = t\mathbf{i} + (3t - 6)\mathbf{j} + (t + 1)\mathbf{k}$  and  $\mathbf{r}_2 = (2t - 4)\mathbf{i} + (t - 3)\mathbf{j} + \mathbf{k}$ .
  - $\frac{x-1}{-4} = y = z + 1$  and  $\frac{x+1}{-4} = y - 1 = z - 3$ .
  - $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$  and  $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$ . Also, find whether the line;  $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$  lies on the plane.

174. The distance of the point; A(4, -1, 2) from the plane is  $\sqrt{3}$  units.  
 The vector;  $\mathbf{i} + \mathbf{j} + \mathbf{k}$  is normal to the plane  
 Find the cartesian equation of the plane.
175. Find the co-ordinates of point on line whose equation:  $x = \frac{y+1}{2} = \frac{z+1}{3}$  which is 2.0 units from a plane which contains points; A(4, 0, 1), B(0, -3, 2) and C(6, 3, 3).
176.  $l$  is a line:  $\frac{x}{2} = \frac{y-1}{1} = \frac{z+1}{-1}$  and  $\pi$  is the plane;  $x - 4y - 2z = 5$ .
- Show that  $l$  is parallel to  $\pi$ , and calculate the distance between  $l$  and  $\pi$
  - Find the cartesian equation of a second plane  $\pi_0$  which contains  $l$  and is perpendicular to  $\pi$ .
177. Find the vector equation of the line passing through the point; (3, 1, 2) and is perpendicular to the plane;  $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) = 4$ .  
 Hence, find the point of intersection of the line and the plane.
178. The planes  $\pi_1$  and  $\pi_2$  have equations;  $3x + y - 2z = 10$  and  $x - 2y + 2z = 5$  respectively.  
 The line  $l$  has equation;  $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \beta(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ .
- Show that  $l$  is parallel to  $\pi_1$
  - Calculate the acute angle between  $\pi_1$  and  $\pi_2$
- Point P lies on  $l$ , the perpendicular distance of P from the plane  $\pi_2$  is equal to 2.
- Find the two possible position vectors of P.
179. A line  $l$  is given by the vector equation:  $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + \beta \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ .
- Point N has a position vector;  $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ .
- Find the cartesian equation of a plane  $\pi_1$  that contains  $l$  and N.
- A line  $l_0$  passes through a point with position vector;  $-6\mathbf{k} + 2\mathbf{i} - \mathbf{j}$  and is parallel to  $\pi$ .
- Find the cartesian equation of  $l_0$ .
  - Hence, find the least distance between  $l$  and  $l_0$ .
180. Find the equation of the line perpendicular to the plane;  $3x - 7y + 2z = 8$  and passes through a point with position vector;  $-5\mathbf{j} + \mathbf{k} - \mathbf{i}$ .

181. A point M has position vector  $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ .

The line  $l$  has equation;  $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \gamma(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ .

Find the cartesian equation of the plane containing  $l$  and M.

(Ans:  $4x + y - 2z = 8$ )

182. The parametric equations of the two planes are;

$$r_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} \text{ and } r_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

a) Find the cosine of the acute angle between planes.

The line of intersection is  $l$ .

b) Find in the form;  $\mathbf{r} = \mathbf{a} + \beta\mathbf{d}$ , the equation of  $l$ .

c) Show that the length of the perpendicular from the point; (1, 5, 1) to line  $l$  is;  $\sqrt{2}$ .

183. Given the parametric equation of the plane,  $\pi: \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$ .

Find the;

a) normal vector to  $\pi$

b) cartesian equation of  $\pi$

c) angle with which  $\pi$  makes with the line  $l: \frac{1-x}{2} = \frac{y+2}{1} = \frac{z+3}{4}$ .

Hence, find the co-ordinates of the point of contact.

184. Find the cartesian equation of the plane through the points; A(1, 0, -2) and B(3, -1, 1) which is parallel to the line with vector equation;  $\mathbf{r} = 3\mathbf{i} + (2t - 1)\mathbf{j} + (5 - t)\mathbf{k}$ .

Hence, find the co-ordinates of the point of contact of this plane and the line;

$$\mathbf{r} = \mu\mathbf{i} + (5 - \mu)\mathbf{j} + (2\mu - 7)\mathbf{k}.$$

185. Find the distance of the point; (-1, -5, -10) from the point of intersection of the plane;

$$x + 3y - 2z = 3 \text{ and the line } \frac{x-1}{3} = \frac{1-z}{-4} = y - 2.$$

186. Given the equations of three planes:

$$2x - y + 3z = 4, \quad 3x - 2y + 6z = 3 \text{ and } 7x - 4y + 5z = 11.$$

Find their point of intersection.

187. Find the position vector of the point of intersection of the following planes;

- a)  $x + 2y + z = 12$ ,  $2x - y + z = 5$  and  $3x + y - 2z = 1$   
 b)  $4x - y + 2z = 1$ ,  $x + 3y + 5z - 16 = 0$  and  $2x + 3y - 4z - 9 = 0$   
 c)  $x + y + 3z = 5$ ,  $-x + 2y + 2z = 3$  and  $4x + y - 3z = 2$   
 d)  $x + y - 2z + 5 = 0$ ,  $x - 2y - z - 1 = 0$  and  $2x - 3y + z - 10 = 0$

188. Find the vector equation of the line of intersection between the planes;

$$x + 3y + 2z = 6 \text{ and } 2x - y + z + 2 = 0$$

189. Find the position vector of the point of contact of the line and a plane in each of the following;

- a)  $-3x = 4y - 2 = z + 1$  and  $-3x + y + z = 13$   
 b)  $2x - 3y + z + 5 = 0$  and  $\mathbf{r} = (7\mathbf{i} - 9\mathbf{k} + 14\mathbf{k}) + t(3(\mathbf{i} + 2\mathbf{j}) - 4\mathbf{k})$   
 c)  $\frac{x-1}{3} = \frac{y+2}{4} = \frac{3-z}{2}$  and  $2x - y + 3z - 1 = 0$   
 d)  $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$  and  $4x - 3y + 5z + 34 = 0$

190. Determine the cartesian equation of the plane that is parallel to the line with equation;

$$x = -2y = 3z \text{ and contains the line of intersection of two planes: } x - y + z = 1 \text{ and } 2y - z = 0$$

$$(\text{Ans: } 8x + 14y - 3z = 8)$$

191. Points; A, B and C have position vectors;  $\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix}$  respectively.

The perpendicular drawn from point D to the plane containing; A, B and C meets the plane at E. Find the;

- a) Cartesian equation of the plane containing A, B and C.  
 b) Position vector of the point E  
 c) Vector equation of the straight line; DE.

192. A plane  $\pi$  passes through point; A(3, 1, 4).

Find the cartesian equation of  $\pi$  that passes through the intersection of two planes:

$$x + 2y + 3z = 1 \text{ and } 2x - y + z = -3.$$

$$(\text{Ans: } x - 2y - z + 3 = 0).$$

- YouTube Channel: **JK Maths**
- WhatsApp Channel: **TR. JK (A' Level Maths)**

193. A point  $P(x_0, y_0, z_0)$  is  $d_0$  units close to the plane;  $ax + by + cz + d = 0$ .

a) Find the co – ordinates of the foot of the perpendicular drawn from point P to the plane surface, in terms of a scalar,  $\lambda$ .

b) Hence, show that;  $d_0 = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$  units.

194. The equations of the two planes  $\pi_1$  and  $\pi_2$  are;

$$\mathbf{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \text{ and } \mathbf{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} + \rho \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}.$$

a) Find the line  $l$  of intersection of  $\pi_1$  and  $\pi_2$

b) Find the cosine of the acute angle between  $\pi_1$  and  $\pi_2$

c) Show that the length of the perpendicular from the point;  $(1, 5, 1)$  to  $l$  is  $\sqrt{2}$  units.

d) Calculate the angle between  $\pi_1$  and  $\pi_2$ .

195. Find the parametric equations of the line of intersection of planes  $\pi_1$  and  $\pi_2$

$$\pi_1: x + 2y - 3z = 4 \text{ and } \pi_2: 3x - 4y + z = -3$$

196. Find the distance between planes  $\pi_1$  and  $\pi_2$

$$\pi_1: x + y - z + 2 = 0$$

$$\pi_2: x + y - z + 6 = 0$$

197. Find the shortest distance from the origin to the plane;  $3x - 4y - z + 26 = 0$

198. Find the perpendicular distance of the point  $(2, 1, 0)$  from the plane;  $-x - 5y + 2z = 9$

199. Show that the point;  $(1, 2, -3)$  lie on the plane;  $2x + y - z = 7$ .

200. Find the perpendicular distance from the plane;  $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 4$  to the origin.

201. Show that;  $P(1, 2, -1)$  and  $Q(3, 6, 1)$  lie on opposite sides of the plane;  $x + y + 4z = 5$ .

202. Prove that;  $(0, 0, 0)$  and  $(2, -3, 7)$  lie on the same side of plane;  $2x - 3y + 2z + 8 = 0$ .

**Note:** JK urges every user of this material to keenly analyse each question before attempting it. Clearly, reflect on what your teacher talked about in Vectors and correctly apply it where necessary.

**PROVERBS: 13 : 04**

**END:**