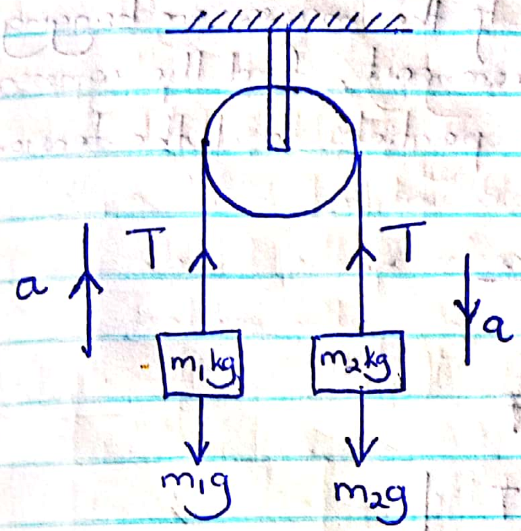
Saturday, 26th June, 2021

CONNECTED PARTICLES

Pulley systems

Single fixed pulley.



If $m_2 > m_1$, then $m_2g > T$ whereas, $T > m_1g$. So, m_2g moves downwards whereas m_1 moves upwards.

Equations of motion

$$m_2g - T = m_2a \quad \text{---(I)}$$

$$T - m_1g = m_1a \quad \text{---(II)}$$

(I) + (II)

$$m_2g - m_1g = m_2a + m_1a$$

$$(m_2 - m_1)g = (m_2 + m_1)a$$

$$a = \left[\frac{(m_2 - m_1)g}{m_2 + m_1}\right] ms^{-2}$$

from (II), $T = m_1a + m_1g$

$$= m_1\frac{(m_2 - m_1)g}{m_2 + m_1} + m_1g$$

$$= \frac{m_1m_2g - m_1^2g + m_1m_2g + m_1^2g}{m_2 + m_1}$$

$$T = \left[\frac{2m_1m_2g}{m_2 + m_1}\right] N$$

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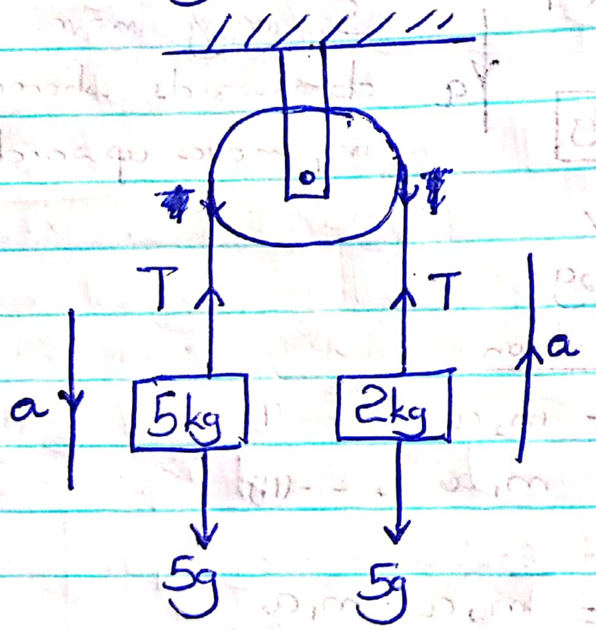
Example 1

Particles of mass 2kg and 5kg are connected by a light inextensible string passing over a smooth fixed pulley. If the masses are hanging freely and released from rest, find the common acceleration of the two particles and the tension in the string.



$$5g - T = 5a \quad \text{---(I)}$$

$$T - 2g = 2a \quad \text{---(II)}$$

(I) + (II)

$$3g = 7a$$

$$a = \frac{3 \times 9.8}{7}$$

$$a = 4.2ms^{-2}$$

$$T = 2a + 2g$$

$$= 2(4.2) + 2(9.8)$$

$$T = 28N$$

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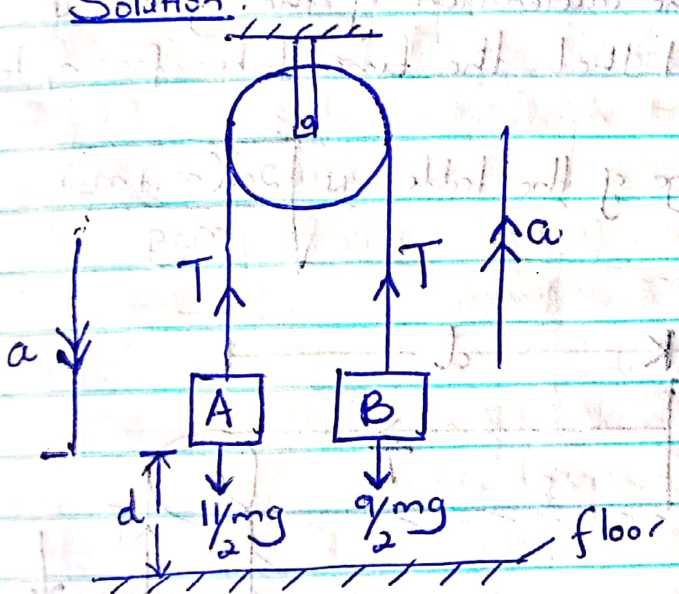
Example 2

Two particles A and B are connected by a light inextensible string passing over a smooth fixed pulley. The masses of A and B are $\frac{11}{2}m$ and $\frac{9}{2}m$ respectively. With A and B hanging vertically, the system is released from rest with particle A, a distance d above the floor. If a time, t elapses before A hits the floor, show that $20d = t^2 g$.

Solution.



$$\frac{11}{2}mg - T = \frac{11}{2}ma \quad \text{---(I)}$$

$$T - \frac{9}{2}mg = \frac{9}{2}ma \quad \text{---(II)}$$

(I) + (II)

$$mg = 10ma$$

$$a = \frac{g}{10}\ ms^{-2}$$

For A to hit the floor, it will have covered displacement d. $\therefore s = d,\ u = 0,\ a = \frac{g}{10}$

From $s = ut + \frac{1}{2}at^2$

$$d = \frac{1}{2}\left(\frac{g}{10}\right)t^2$$

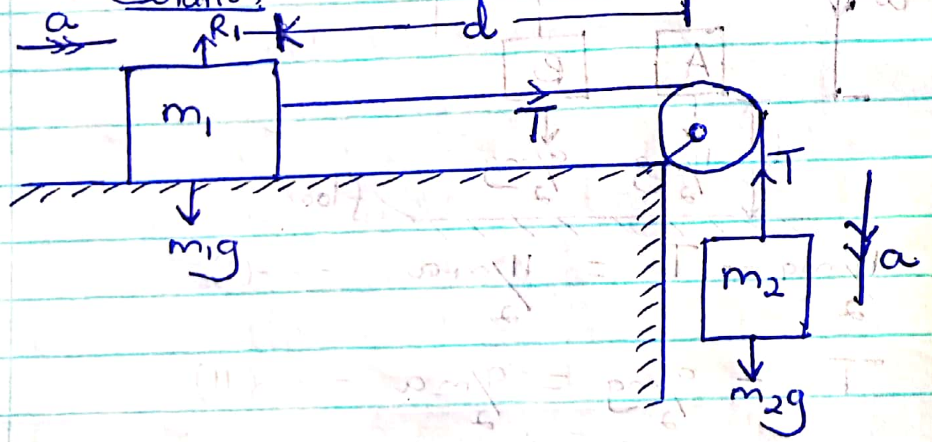
$$d = \frac{gt^2}{20}$$

$$\therefore 20d = gt^2$$

Example 3

A particle of mass m_1 lies on a smooth horizontal table and is connected to a freely hanging particle of mass m_2 by a light inextensible string passing over a smooth fixed pulley situated at the edge of the table. Initially the system is at rest with m_1 a distance d from the edge of the table. Show that the acceleration of the system is $\frac{m_2 g}{m_1+m_2}$ and that the time taken for m_1 to reach the edge of the table is $\sqrt{\frac{2d(m_1+m_2)}{m_2 g}}$

Solution



$$m_2 g - T = m_2 a \quad \text{--- (I)}$$
$$T = m_1 a \quad \text{--- (II)}$$

(I) + (II)

$$m_2 g = (m_2 + m_1) a$$
$$a = \frac{m_2 g}{m_2 + m_1}$$

For m_1 to reach the edge of the table it will have covered a displacement d

$\therefore$ $s = d$, $u = 0$, $a = \frac{m_2 g}{m_2 + m_1}$

From $S = ut + \frac{1}{2}at^2$

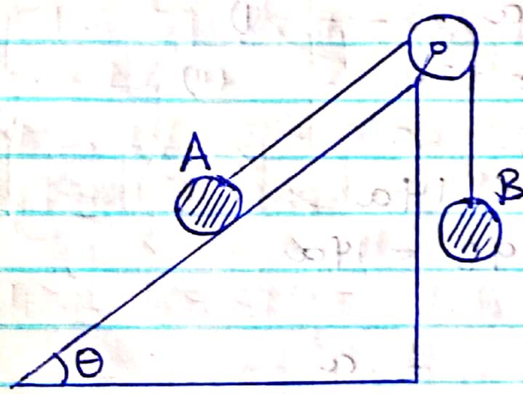
$$d = \frac{1}{2}\left(\frac{m_2 g}{m_2+m_1}\right)t^2$$

$$d = \frac{m_2 g t^2}{2(m_2+m_1)}$$

$$t^2 = \frac{2d(m_2+m_1)}{m_2 g}$$

$$\therefore t = \sqrt{\frac{2d(m_2+m_1)}{m_2 g}}$$

Example 4



The diagram above shows a body A of mass 13kg lying on a smooth ~~rough~~ inclined plane. From A, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley to a body B of mass m kg. The plane makes an angle θ with the horizontal where $\sin\theta = 5/13$. When $m = 1$kg and the system is released from rest, B has an upward acceleration of $a_1 ms^{-2}$. ~~Find a~~. When $m = 11$kg and the system is released from rest, B has a downward acceleration of $a_2 ms^{-2}$. Find a_1 and a_2

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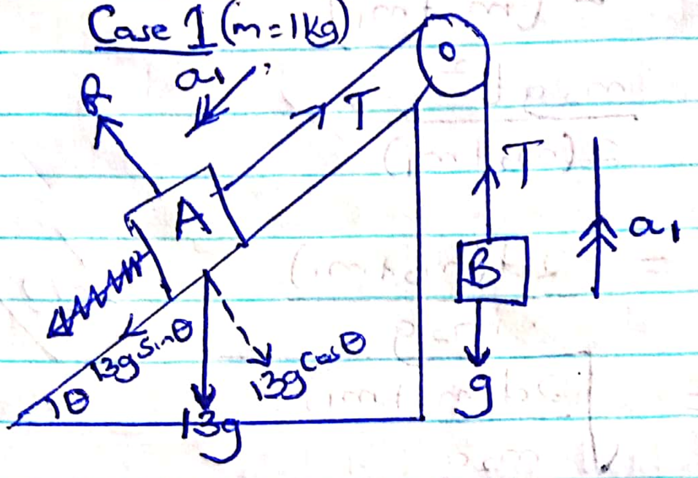
Solution

Case 1 ($m = 1kg$)



Since the plane is smooth, there is no friction. B has an upward acceleration when $m = 1kg$. ~~hence A has~~.

$$T - g = a \quad \text{---(I)}$$

$$13g\sin\theta - T = 13a \quad \text{---(II)}$$

(I) + (II)

$$13g\sin\theta - g = 14a$$

$$13 \times 9.8 \times \frac{5}{13} - 9.8 = 14a$$

$$\frac{39.2}{14} = a$$

$$a = 2.8ms^{-2}$$

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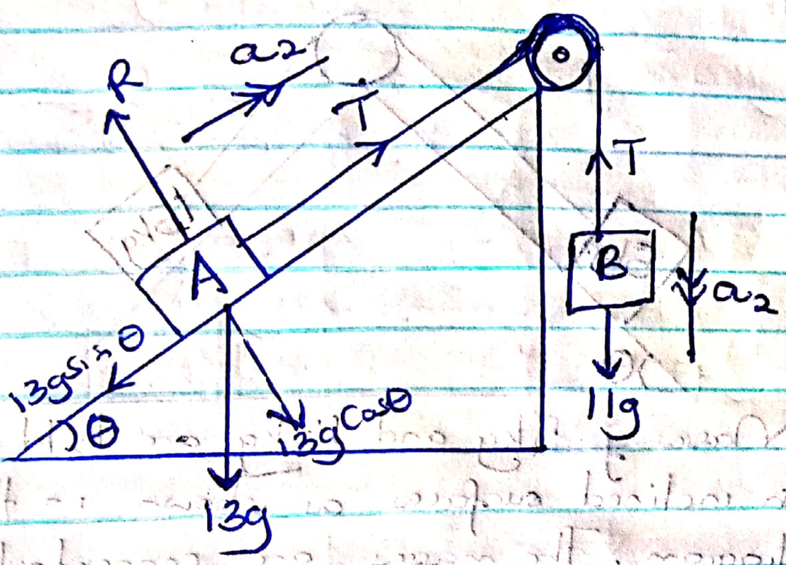
Case 2 ($m = 11kg$)



$11g - T = 11a_2$ ---(I)

$T - 13g\sin\theta = 13a_2$ ---(II)

(I) + (II)

$11g - 13g\sin\theta = 24a_2$

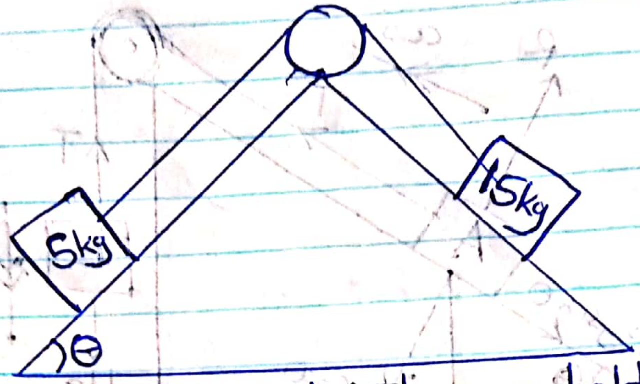
$11 \times 9.8 - 13 \times 9.8 \times 5/13 = 24a_2$

$58.8 = 24a_2$

$a_2 = \frac{58.8}{24}$

$a_2 = 2.45\,ms^{-2}$

Example 5.

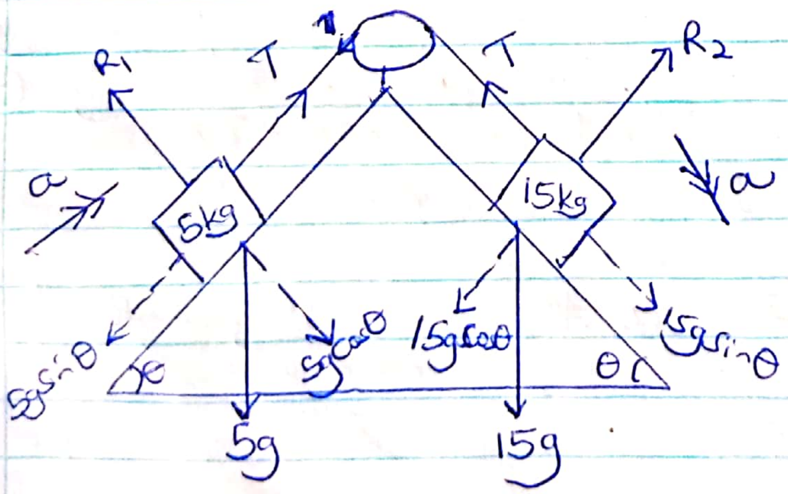


Masses of 5kg and 15kg are held at rest on inclined surfaces as shown in the diagram. The masses are connected by a light, taut, inextensible string passing over a smooth fixed pulley. The inclined surfaces are smooth. The inclination of the plane is such that $\sin\theta = 3/5$. When the system is released from rest, the 15kg mass accelerates down the slope.

a) Find the magnitude of this acceleration and the tension in the string.

b) Calculate the magnitude of the force on the pulley.

Solution (a)



$$15g\sin\theta - T = 15a \quad \text{---(I)}$$

$$T - 5g\sin\theta = 5a \quad \text{---(II)}$$

$$(I) + (II)$$

$$10g\sin\theta = 20a$$

$$a = \frac{10 \times 9.8 \times 3/5}{20}$$

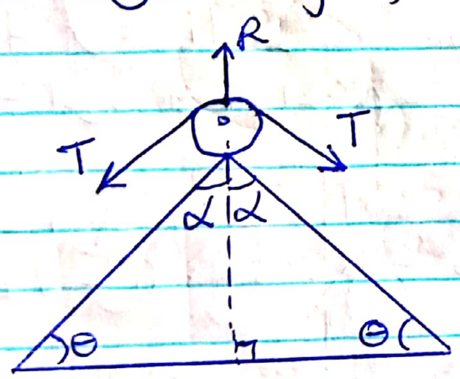
$$a = 2.94\ ms^{-2}.$$

$$T = 5a + 5g\sin\theta.$$

$$= 5(2.94) + 5 \times 9.8 \times 3/5$$

$$T = 44.1\ N$$

b) Let the magnitude of force on the pulley be R.



$$R = T\cos\alpha + T\cos\alpha$$

$$R = 2T\cos\alpha$$

But $\alpha = 90 - \theta.$

$$\therefore R = 2T\cos(90-\theta)$$

$$= 2T\sin\theta$$

$$= 2(44.1) \times 3/5$$

$$R = 52.92\ N$$

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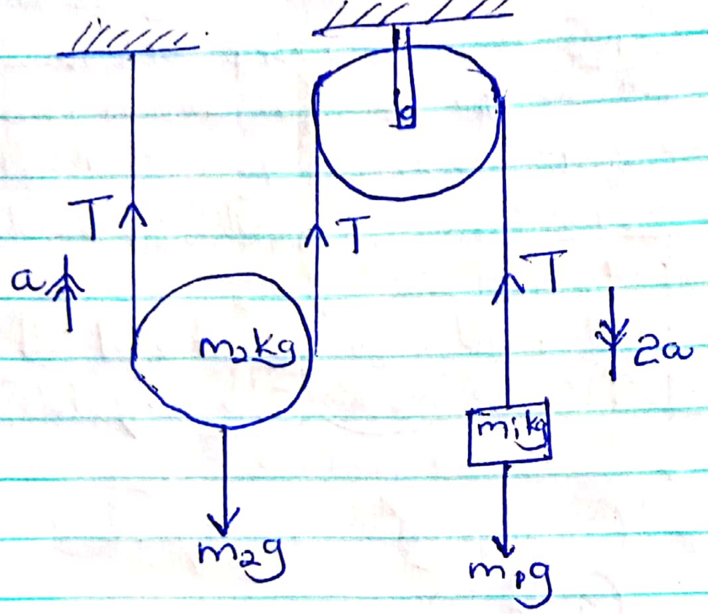
Movable and fixed pulley



$$m_1g - T = m_1(2a)$$

$$m_1g - T = 2m_1a \quad \text{--(I)}$$

$$2T - m_2g = m_2a \quad \text{--(II)}$$

From (I)

$$2m_1g - 2T = 4m_1a \quad \text{--(III)}$$

(II) + (III)

$$2m_1g - m_2g = (4m_1 + m_2)a$$

$$a = \frac{(2m_1 - m_2)g}{4m_1 + m_2}$$

$$T = m_1g - 2m_1a \quad \text{from (I)}$$

$$= m_1g - 2m_1\frac{(2m_1 - m_2)g}{4m_1 + m_2}$$

$$= \frac{4m_1m_1g + m_2m_1g - 4m_1m_1g + 2m_1m_2g}{4m_1 + m_2}$$

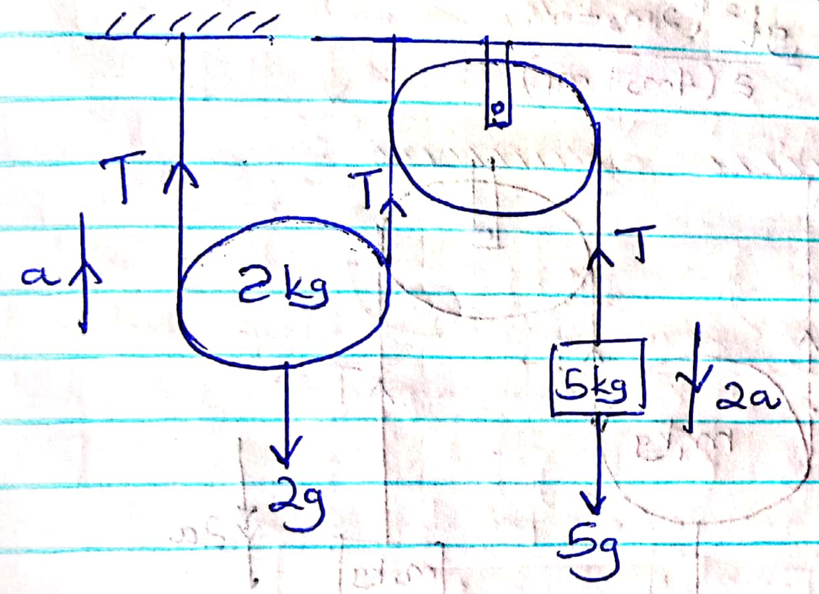
$$T = \frac{3m_1m_2g}{4m_1 + m_2}$$

Example 1:

A string with one end fixed, passes under a movable pulley of mass 2kg, over a fixed pulley and carries a 5kg mass at its other end. Find the acceleration of the movable pulley and the tension in the string.

Solution,

Let the acceleration of the movable pulley be a,



$5g - T = 5(2a)$

$5g - T = 10a$ --- (I)

$2T - 2g = 2a$ --- (II)

From (I)

$10g - 2T = 20a$ --- (III)

(II) + (III)

$8g = 22a$

$a = \frac{8 \times 9.8}{22}$

$a = 3.56\ ms^{-2}$

$T = 5g - 10a$ from (I)

$= 5 \times 9.8 - 10 \times 3.56$

$T = 13.4N$

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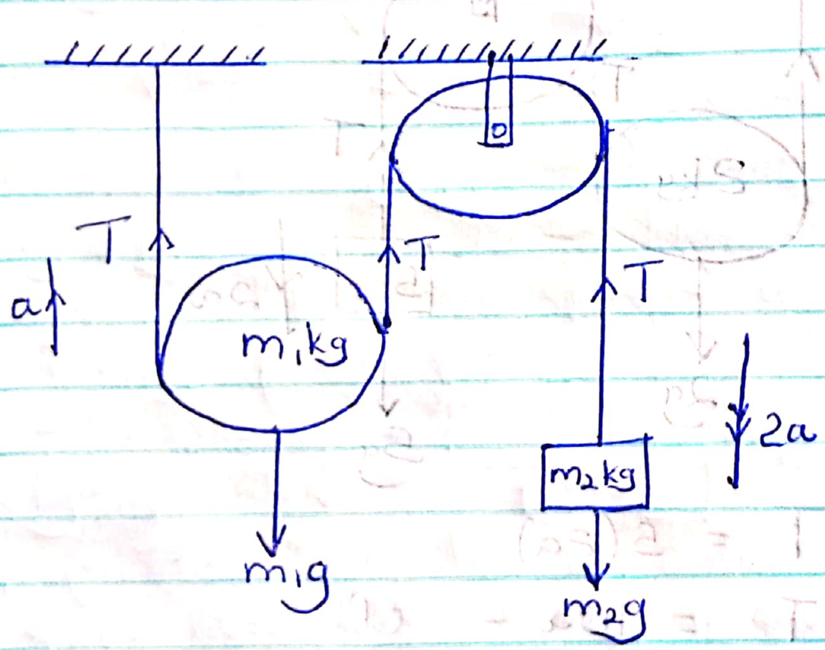
Example 2

A string, with one end fixed, passes under a moveable pulley of mass m_1, over a fixed pulley, and carries a mass, m_2 at its other end. With the system released from rest, show that the tension in the string is $\frac{3m_1m_2g}{4m_2+m_1}$ and that, after the time t, the movable pulley has moved a distance $\frac{gt^2(2m_2-m_1)}{2(4m_2+m_1)}$



$m_2g - T = m_2a$ -- (I)

$2T - m_1g = m_1a$ -- (II)

from (I) ; $2m_2g - 2T = 2m_2a$ -- (III)

(II) + (III)

$2m_2g - m_1g = 2m_2a + m_1a$

$\frac{2m_2g - m_1g}{2m_2 + m_1} = a$

$T = m_2g - m_2a$

$= m_2g - \frac{(2m_2g - m_1g)}{}$

$$m_2g - T = m_2(2a)$$

$$m_2g - T = 2m_2a \quad --(1)$$

$$2T - m_1g = m_1a \quad ---(11)$$

From (1); $2m_2g - 2T = 4m_2a \quad --(111)$

(11) + (111)

$$2m_2g - m_1g = (4m_2 + m_1)a$$

$$a = \frac{2m_2g - m_1g}{4m_2 + m_1}$$

$$T = m_2g - 2m_2a$$

$$= m_2g - 2m_2\left(\frac{2m_2g - m_1g}{4m_2+m_1}\right)$$

$$= \frac{4m_2^2g + m_1m_2g - 4m_2^2g + 2m_2m_1g}{4m_2 + m_1}$$

$$T = \frac{3m_1m_2g}{4m_2 + m_1}$$

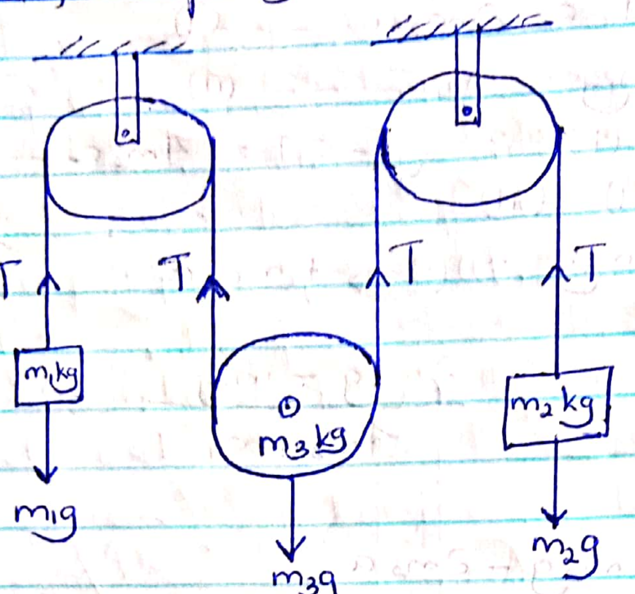
After ts, from $S = ut + \frac{1}{2}at^2$

$u = 0ms^{-1}$, $a = \frac{2m_2g - m_1g}{4m_2+m_1}$

$$S = \frac{1}{2}\left(\frac{2m_2g - m_1g}{4m_2 + m_1}\right)t^2$$

$$S = \frac{gt^2(2m_2 - m_1)}{2(4m_2 + m_1)}$$

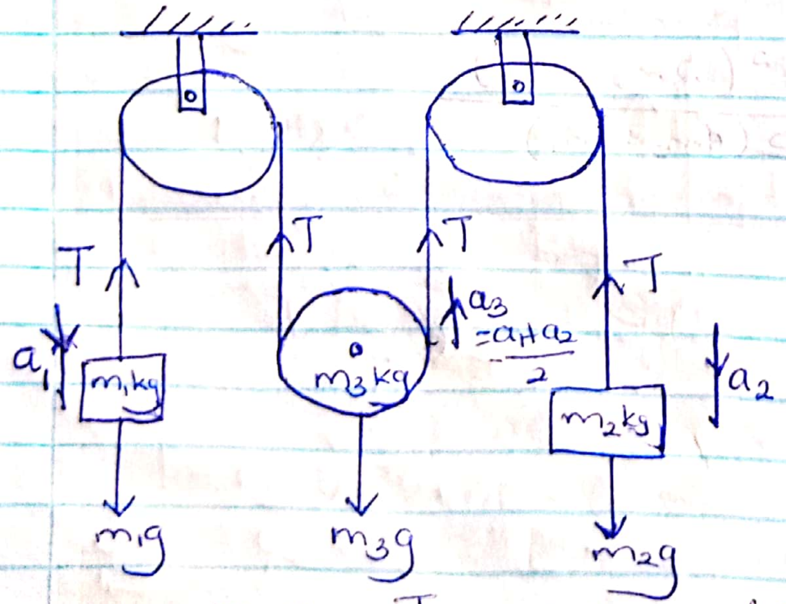
Two fixed pulleys and one moveable pulley.



There are two ways in which the movable pulley may move ie either downwards or upwards. It can move upwards if m_1 and m_2 are moving downwards.

It can move downwards if m_1 and m_2 are moving upwards.

You write the equation of motion considering one of the above two motions of the movable pulley. Let us take the case of the movable pulley ~~coming down~~ moving upwards,



$$m_2g - T = m_2a_2 \quad \text{--- (I)}$$

$$2T - m_3g = m_3\left(\frac{a_1+a_2}{2}\right) \quad \text{--- (II)}$$

$$m_1g - T = m_1a_1 \quad \text{--- (III)}$$

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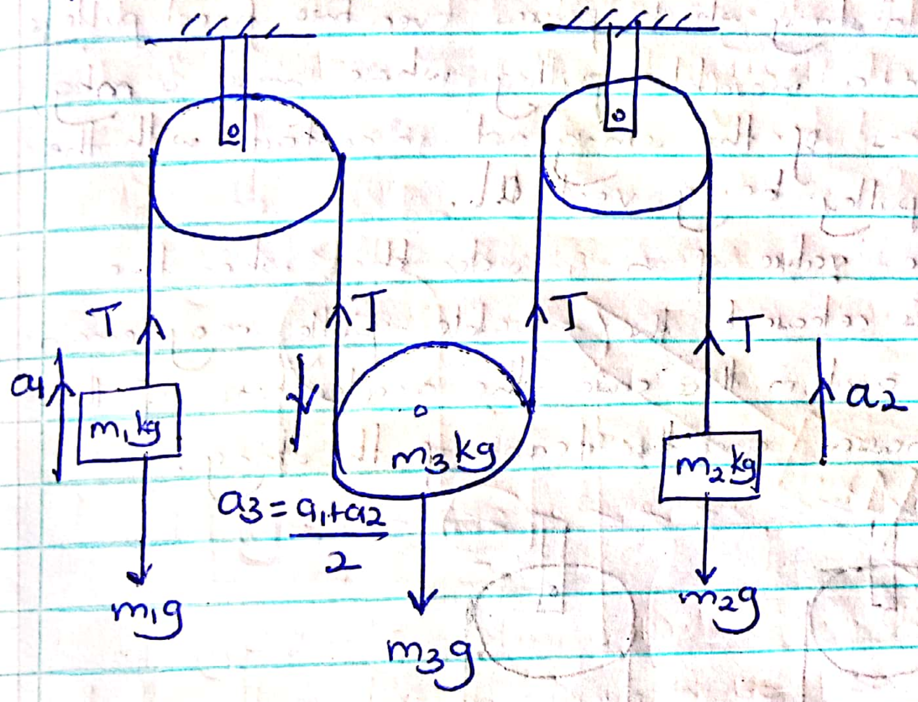
If we take the case of the movable pulley moving downwards



Equations of motion

$$T - m_1 g = m_1 a_1 \quad \text{---(I)}$$

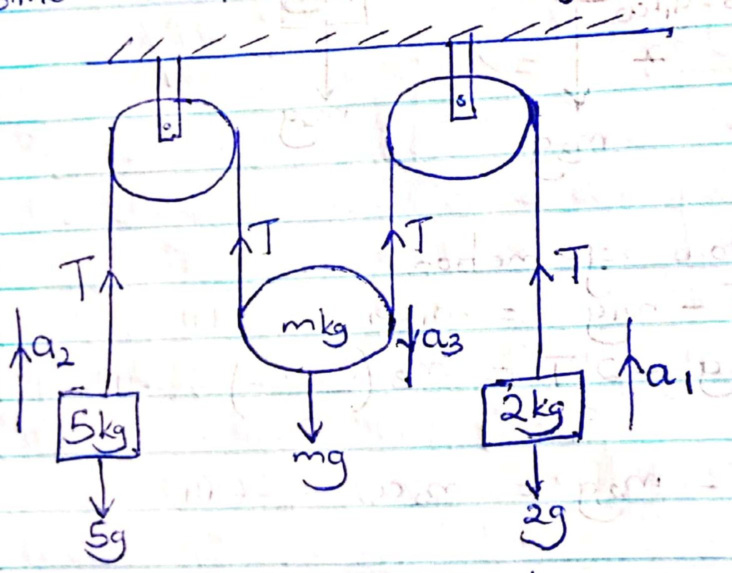
$$m_3 g - 2T = m_3 \left(\frac{a_1 + a_2}{2}\right) \quad \text{---(II)}$$

$$T - m_2 g = m_2 a_2 \quad \text{---(III)}$$

Example 1.

Masses of 5kg and 2kg are suspended from the ends of a string which passes over two fixed pulleys and under a movable pulley whose mass is mkg, the portions of the string not in contact with the movable pulley being vertical.

Find the value of m in order that when the system is released, the movable pulley may remain at rest, and in this case the accelerations of the other masses and the tension of the string.



For the movable pulley to remain at rest as indicated in the question, 2T must balance with mg hence acceleration of movable pulley is zero.

But to know the accelerations of the 5kg and 2kg masses, we need to assume the direction in which the movable pulley moves. let us assume, it was going downwards

Equations of motion

$$T - 2g = 2a_1 \quad \text{---(I)}$$

Since $a_3 = 0$; $mg - 2T = ma_3 = 0$

$$\therefore 2T = mg \quad \text{---(II)}$$

$$T - 5g = 5a_2 \quad \text{---(III)}$$

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Recall ; $a_3 = \frac{a_1+a_2}{2}$

$\therefore\ 0 = \frac{a_1+a_2}{2}$

$a_1 = -a_2$

Solving (I) and (III)

$$T - 2g = 2a_1 \quad \text{---(I)}$$
$$-\ (T - 5g = 5a_2) \quad \text{---(III)}$$
$$3g = 2a_1 - 5a_2$$

$$3 \times 9.8 = 2(-a_2) - 5a_2$$
$$3 \times 9.8 = -7a_2$$
$$a_2 = \frac{3 \times 9.8}{-7}$$
$$a_2 = -4.2 ms^{-2}$$

$$a_1 = -(-4.2) = 4.2 ms^{-2}$$

$$T = 2a_1 + 2g$$
$$= 2(4.2) + 2(9.8)$$
$$T = 28N$$

From $2T = mg$

$$m = \frac{2T}{g}$$
$$m = \frac{2(28)}{9.8}$$
$$m = 5.71 kg$$

$\therefore$ m is 5.71 kg when the 5kg mass moves at an acceleration of $4.2 ms^{-2}$ downwards and the 2kg moves at acceleration $4.2 ms^{-2}$ upwards.

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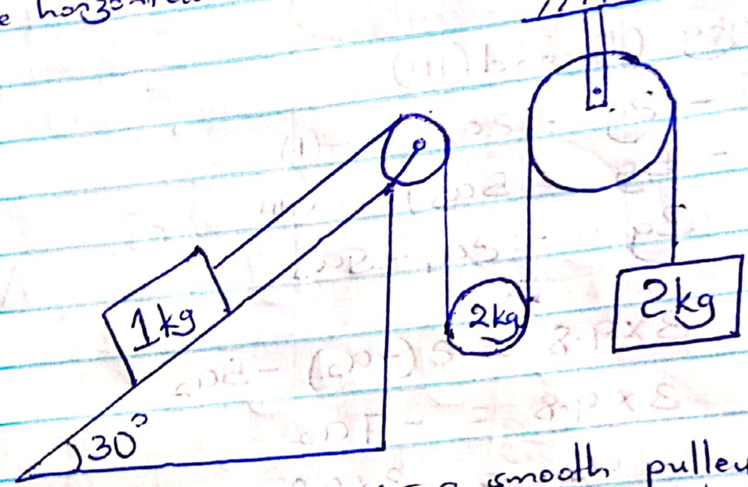
Example 2

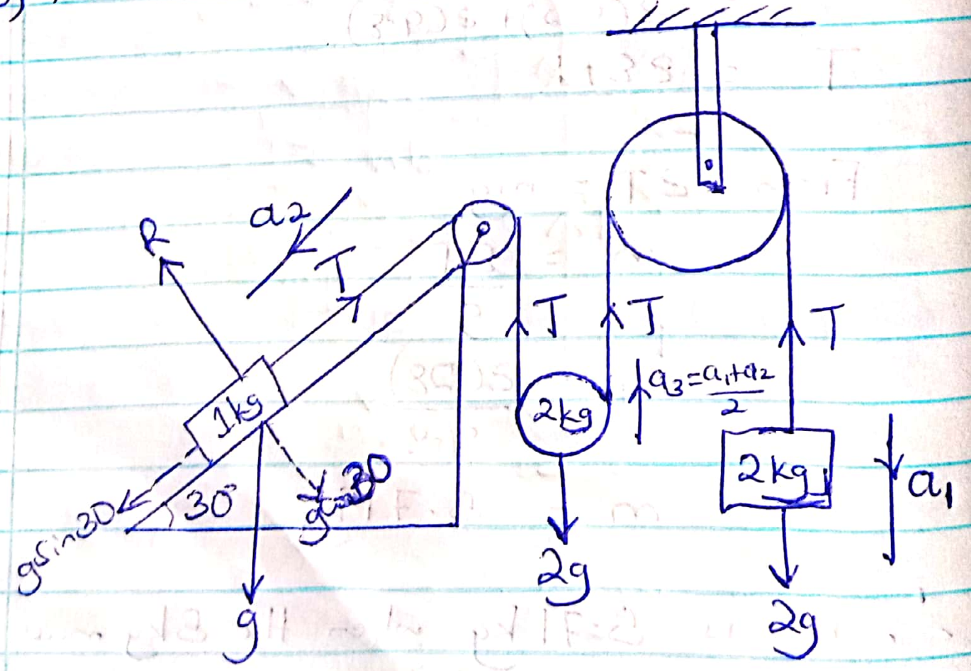
The figure below shows one end of a light inextensible string attached to a mass of 1 kg which rests on a smooth plane inclined at 30° the horizontal.



The string passes over a smooth pulley at the edge of the plane, under a second movable pulley of mass 2kg and over a third fixed pulley, and has a mass of 2kg attached to the other end. Find the:

a) acceleration of the masses and the moveable pulley

b) tension in the string.



Since the plane is smooth, there is no fiction, Let us take the movable pulley to be going upwards and the 2 other masses downwards

Equations of motion

$$2g - T = 2a_1 \quad --(I)$$

$$2T - 2g = 2\left(\frac{a_1 + a_2}{2}\right) \quad --(II)$$

$$g\sin 30 - T = a_2 \quad ---(III)$$

(I) and (III)

$$\begin{aligned} 2g - T &= 2a_1 \\ -(g\sin 30 - T &= a_2) \\ \hline 2g - g\sin 30 &= 2a_1 - a_2 \quad --(IV) \end{aligned}$$

(I) and (II)

$$\begin{aligned} (2g - T &= 2a_1)2 \\ \Rightarrow 4g - 2T &= 4a_1 \\ +(2T - 2g &= 2\left(\frac{a_1 + a_2}{2}\right)) \\ \hline 4g - 2g &= 4a_1 + 2\left(\frac{a_1 + a_2}{2}\right) \quad --(V) \end{aligned}$$

From (IV) $2g - \frac{1}{2}g = 2a_1 - a_2$

$$\frac{3}{2}g = 2a_1 - a_2$$

$$3g = 4a_1 - 2a_2 \quad ---(VI)$$

From (V) $2g = 4a_1 + a_1 + a_2$

$$2g = 5a_1 + a_2 \quad ---(VII)$$

Solving (VI) and (VII)

$$\begin{aligned} 3g &= 4a_1 - 2a_2 \\ + 4g &= 10a_1 + 2a_2 \\ \hline 7g &= 14a_1 \\ a_1 &= \frac{7 \times 9.8}{14} = 4.9\,ms^{-2} \end{aligned}$$

$$a_2 = 2g - 5a_1 = 2(9.8) - 5(4.9)$$

$$a_2 = -4.9\,ms^{-2}$$

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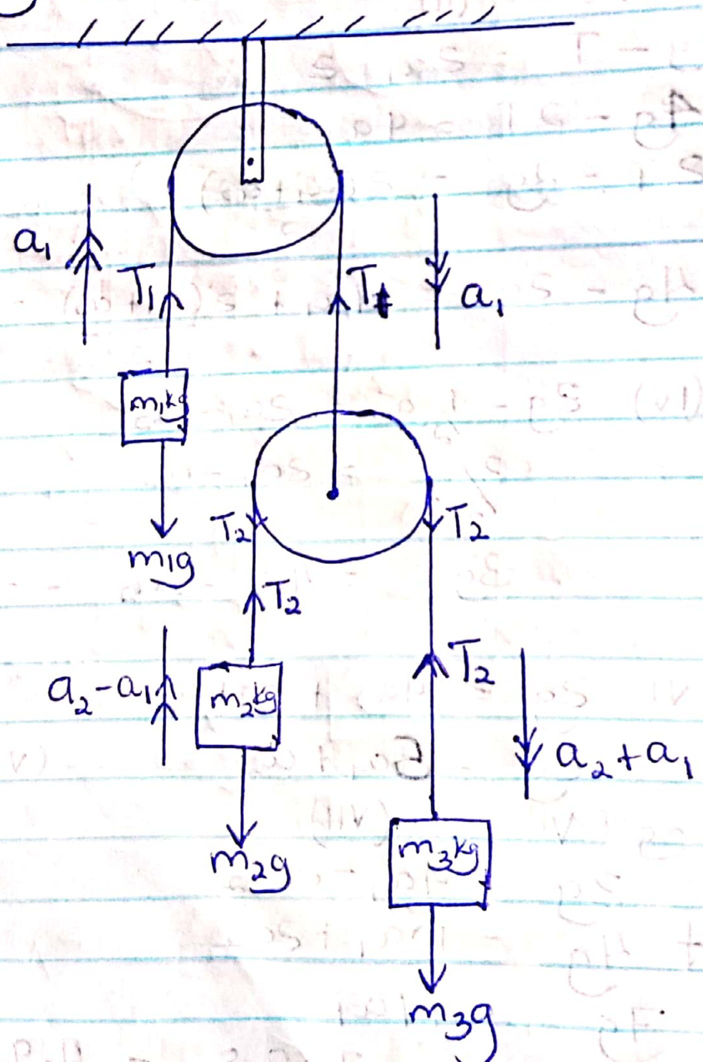
$$a_3 = \frac{a_1 + a_2}{2} = \frac{4.9 - 4.9}{2} = 0\,ms^{-2}$$

$\therefore$ Acceleration of the 1kg mass is $4.9\,ms^{-2}$ up the slope ; acceleration of the 2kg mass is $4.9\,ms^{-2}$ upwards and acceleration of the movable pulley is $0\,ms^{-2}$, ~~initially and it is at rest~~.

A fixed pulley with a string suspending a mass at one end and the other end having a light pulley.



Lets take the m_1 kg mass to be moving upwards with an acceleration $a_1\,ms^{-2}$, the string comes downwards to the light pulley at the other end at $a_1\,ms^{-2}$.

Lets say the light pulley at the other end has another string with masses at both ends which move with acceleration $a_2 ms^{-2}$ upwards or downwards.

The heavier of the two masses attached to the string passing over the light pulley moves at acceleration $a_2 ms^{-2}$ downwards whereas the lighter one moves at an acceleration $a_2 ms^{-2}$ upwards.

Now considering the whole system, of both pulleys and all masses, the heavier mass attached on the one end of the string passing through the light pulley will move with an acceleration $(a_1+a_2) ms^{-2}$ downwards whereas the lighter one will move at an acceleration $(a_2-a_1) ms^{-2}$ upwards.

So from our illustration, let's take $m_3$kg to be greater than $m_2$kg ; m_3 moves downward with acceleration $(a_2+a_1) ms^{-2}$ whereas m_2 moves upward with acceleration $(a_2-a_1) ms^{-2}$.

Equation of motion.

$$T_1 - m_1 g = m_1 a_1 \quad \text{-- (I)}$$

$2T_2 - T_1 = 0 \times a_1$ since the light pulley is assumed to have 0 kg mass

$$\therefore 2T_2 = T_1 \quad \text{-- (II)}$$

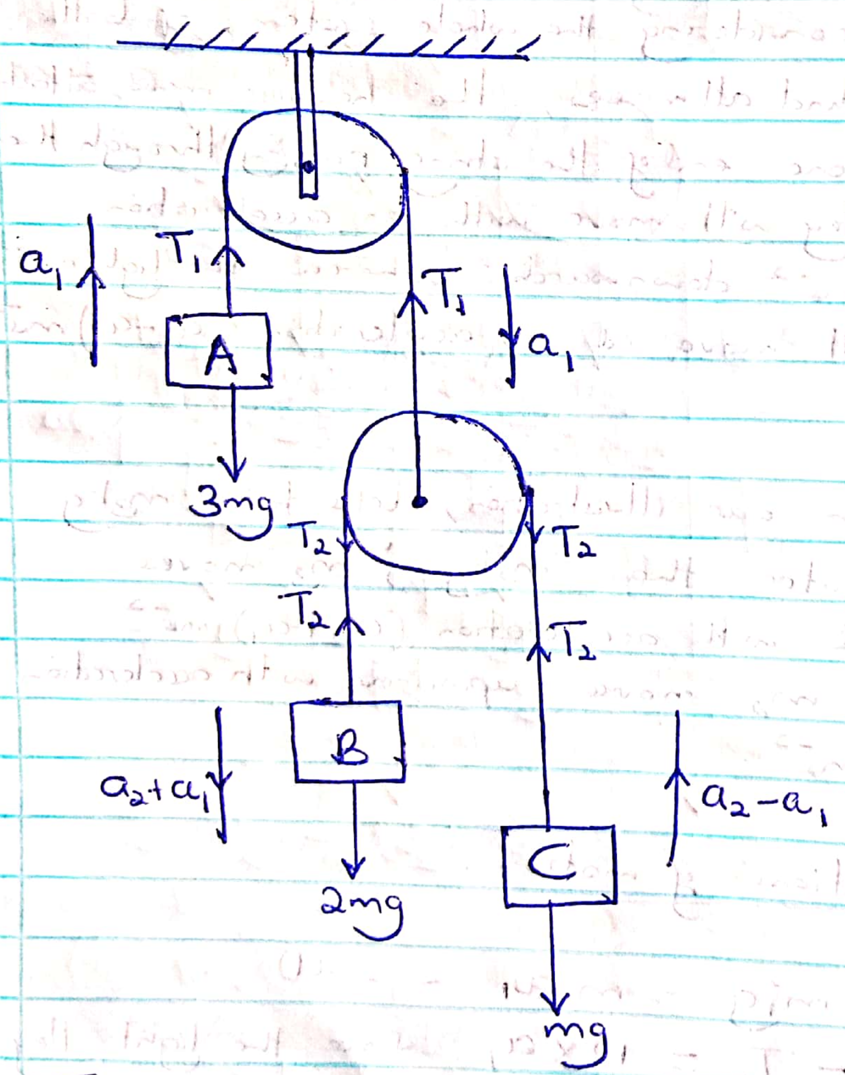
$$T_2 - m_2 g = m_2 (a_2 - a_1) \quad \text{-- (III)}$$

$$m_3 g - T_2 = m_3 (a_2 + a_1) \quad \text{-- (IV)}$$

Example

A string, carrying a particle A at one end, passes over a fixed pulley and has a light pulley attached to its other end. Over this light pulley runs another string carrying particle B at one end and particle C at the other. The masses of A, B and C are 3m, 2m and m respectively. Find the acceleration of A and the tensions in the strings.

Solution:



For A;

$$T_1 - 3mg = 3ma_1 \quad --(I)$$

For light pulley;

$$2T_2 - T_1 = 0 \times a_1$$

$$2T_2 = T_1 \quad --(II)$$

For B;

$$2mg - T_2 = 2m(a_2+a_1) \quad ---(III)$$

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For C;

$T_2 - mg = m(a_2 - a_1)$ ---(IV)

From (III) and (IV)

$$\begin{aligned} 2mg - T_2 &= 2ma_2 + 2ma_1 \\ + \quad (T_2 - mg &= ma_2 - ma_1) \\ \hline mg &= 3ma_2 + ma_1 \quad \text{--- (V)} \end{aligned}$$

From (I) and (II)

$T_1 - 3mg = 3ma_1$ ---(I)

$2T_2 = T_1$ ---(II)

Substitute for T_1

we get $2T_2 - 3mg = 3ma_1$ ---(VI)

$2(III) = 4mg - 2T_2 = 4ma_2 + 4ma_1$ ---(VII)

(VI) + (VII)

$mg = 7ma_1 + 4ma_2$ ---(VIII)

Solving V and VIII simultaneously

$7ma_1 + 4ma_2 = 3ma_2 + ma_1$

$7a_1 + 4a_2 = 3a_2 + a_1$

$a_2 = -6a_1$

Substituting in (V)

$mg = 3ma_2 + ma_1$

$g = 3a_2 + a_1$

$g = 3(-6a_1) + a_1$

$g = -17a_1$

$a_1 = \frac{-g}{17}\ ms^{-2}$

$a_2 = -6\left(\frac{-g}{17}\right) = \frac{6g}{17}\ ms^{-2}$

$T_1 = 3m\left(\frac{-g}{17}\right) + 3mg$ from (I)

$T_1 = \frac{48mg}{17}$; $T_2 = \frac{1}{2}T_1 = \frac{24}{17}mg\,N$

$\therefore$ The acceleration of A is $\frac{g}{17}\ ms^{-2}$ downwards and the tension in string are $\frac{48}{17}mgN$ and $\frac{24}{17}mgN$

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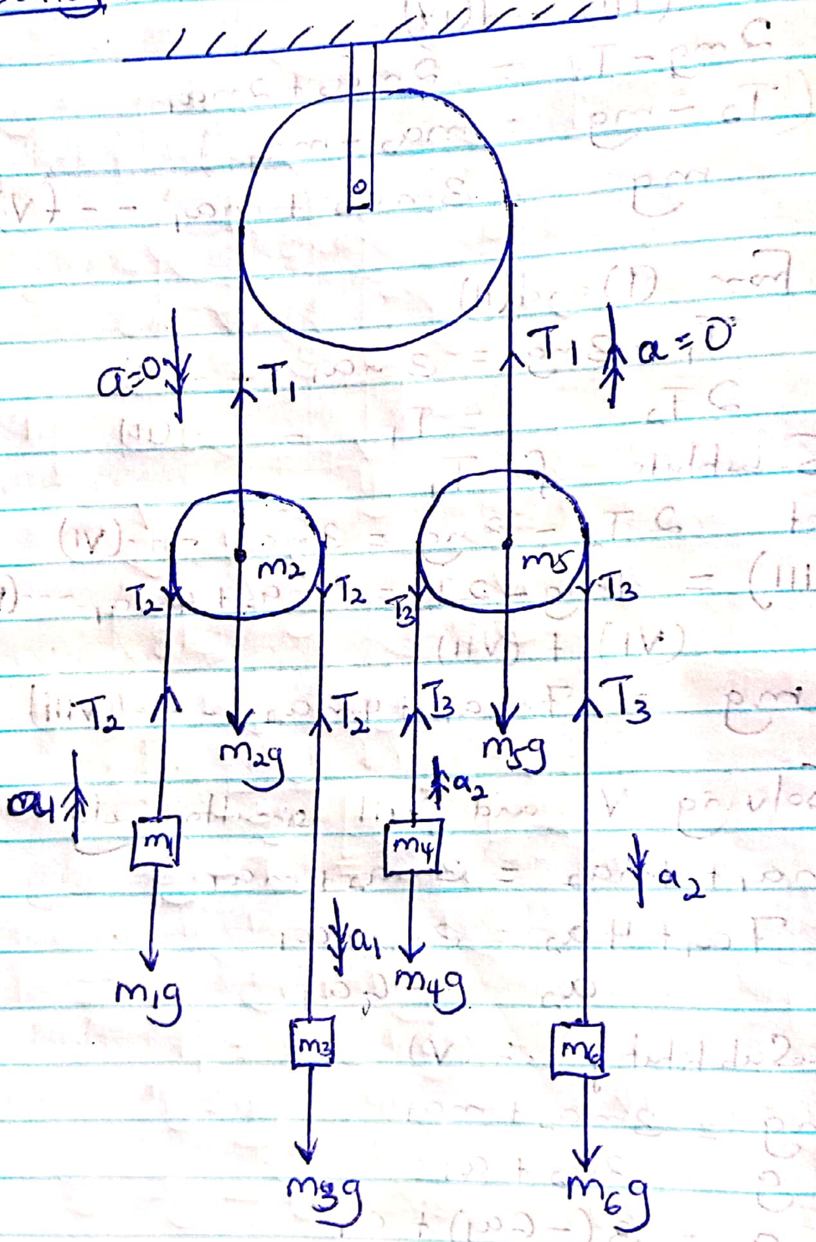
A fixed pulley, with two pulleys both ends of the string passing through the fixed pulley.



If $m_6 > m_4$; then m_6 moves ~~upwards~~ down

Similarly if $m_3 > m_1$, then m_3 moves ~~upwards~~ down

In actual sense m_2 and m_5 do not move, so acceleration of the string passing through the fixed pulley is 0. But this is a string on a pulley, we need to give it direction of movement.

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Equation of motion

For m_2 kg pulley ; $T_1 = 2T_2 + m_2 g$

$$T_2 - m_1 g = m_1 a_1$$

$$m_3 g - T_2 = m_3 a_1$$

For m_5 kg pulley ; $T_1 = 2T_3 + m_5 g$

$$T_3 - m_4 g = m_4 a_2$$

$$m_6 g - T_3 = m_6 a_2 .$$

Example

Two pulleys of masses 12kg and 8kg, are connected by a fine string hanging over a smooth fixed pulley. Over the former is hung a fine string with masses 3kg and 6kg at its ends, and over the latter a fine string with masses 4kg and x kg. Determine x so that the string over the fixed pulley remains stationary, and find the tension in it.

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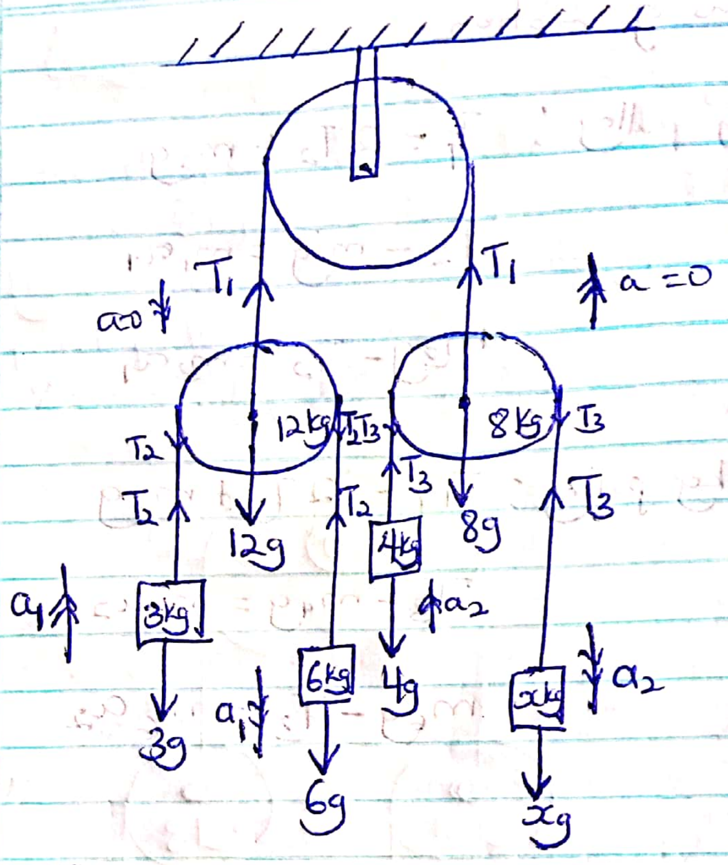


For the 12kg pulley:

$$T_1 = 2T_2 + 12g \quad ---(\text{I})$$

$$T_2 - 3g = 3a_1 \quad ---(\text{II})$$

$$6g - T_2 = 6a_1 \quad ---(\text{III})$$

For the 8kg pulley:

$$T_1 = 2T_3 + 8g \quad ---(\text{IV})$$

$$T_3 - 4g = 4a_2 \quad ---(\text{V})$$

$$xg - T_3 = xa_2 \quad ---(\text{VI})$$

Adding (II) and (III)

$$3g = 9a_1$$

$$a_1 = g/3$$

From (II) $T_2 = 3a_1 + 3g$

$$= 3(g/3) + 3g$$

$$T_2 = 4g$$

$T_1 = 2T_2 + 12g$

$T_1 = 2(4g) + 12g$

$T_1 = 20g$

From (IV)

$20g = 2T_3 + 8g$

$2T_3 = 12g$

$T_3 = 6g$

From (V)

$6g - 4g = 4a_2$

$\frac{2g}{4} = a_2$

$a_2 = g/2$

From (VI)

$xg - 6g = x(g/2)$

$2xg - 12g = xg$

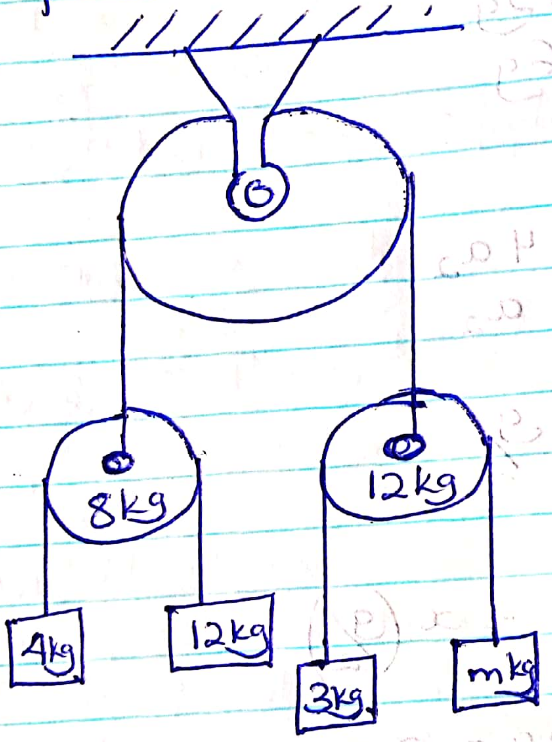
$xg = 12g$

$x = 12\text{ kg}.$

$\therefore$ The tension in the fixed pulley is 20g N or 196N when and $x = 12$ kg

Assignment.

Qn The diagram below shows two pulleys of masses 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley



The acceleration of 4kg and 12kg masses are $g/2$ upwards and $g/2$ downwards respectively.

The acceleration of the 3kg and mkg masses are $g/3$ upwards and $g/3$ downwards respectively.

The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the:

a) tension in the strings. (09 marks)

b) value of m. (03 marks).

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