SENIOR FIVE TERM 1

TOPIC 3/6: LINEAR MOTION

Competency: The learner investigates the effect of force on the motion of bodies on land, water and air, and devises safety precautions for users of automobiles.

Motion in straight line

Distance and displacement

Distance is a length between 2 fixe points

Displacement is the distance covered in a specific direction Speed and velocity

Speed is the rate of change of distance with time

Velocity is the rate of change of displacement with time

Average speed = $\frac{total\ distance}{total\ time\ taken}$

Average velocity = $\frac{total\ displacement}{total\ time\ taken}$

Example 1

Find the distance travelled in 5s by a body moving with a constant speed of 3.2ms^{-1}

Solution

Average speed = $\frac{total\ distance}{total\ time\ taken}$

3.2 = $\frac{total\ distance}{5}$

Distance = 3.2 x 5 = 16m

Example 2

John ran 1500m in 3minutes and 33s, find his average speed.

Average speed = $\frac{total\ distance}{total\ time\ taken}$

speed = $\frac{1500}{(3\ x\ 60+33)}$

= 7.04ms^{-1}

Acceleration

It is the rate of change of velocity

Acceleration =$\frac{change\ in\ velocity}{total\ time\ taken}$

a = $\frac{v-u}{t}$

where v = final velocity,

u = initial velocity,

t = time

Uniform acceleration

This is the constant rate of change of velocity with time

Equations of uniform acceleration

1st equation

Suppose a body moving in a straight line with uniform acceleration a, increases its velocity from u to v in time t, then from the definition of acceleration

a = $\frac{v-u}{t}$

at = v -u

v = u + at 1

2nd equation

Suppose an object with velocity u moves with uniform acceleration a in time t and attains a velocity v, the distance s travelled by the object is given by:

s = average velocity x time

s = $\left(\frac{v+u}{2}\right)t$ but v = u + at

s = $\left(\frac{u+u+at}{2}\right)t$

s = $\left(\frac{2ut+at^2}{2}\right)$

s = ut + $\boldsymbol{\frac{1}{2}at^2}$ 2

3rd equation

s = average velocity x time

s = $\left(\frac{v+u}{2}\right)t$ but t = $\frac{v-u}{a}$

s = $\left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$

s = $\left(\frac{v^2-u^2}{2a}\right)$

$\boldsymbol{v^2 = u^2 - 2as}$ 3

Example 3

A car is initially at rest at a point O. The car moves from O in a straight line with an acceleration of 4ms^{-2}. Find how far the car

(i) is from O after 2s

From **s = ut +** $\boldsymbol{\frac{1}{2}at^2}$**;**

s = 0 x 2 + $\frac{1}{2}x\ 4\ x\ 2^2$ **=** 8m

(ii) is from O after 3s

s = 0 x 2 + $\frac{1}{2}x\ 4\ x\ 3^2$ **= 1**8m

(iii) distance travelled in the third second = 18 – 8 = 10m

Example 4

A body at O moving with a velocity 10ms^{-2} decelerates at 2ms^{-2}.

(i) find the displacement of the body from O after 7s

From s = ut + $\frac{1}{2}at^2$

s = 10 x 7 + $\frac{1}{2}x - 2\ x\ 7^2$ = 21m

(ii) how far from O does the body come to rest and how long does it takes.

s = $\left(\frac{v^2-u^2}{2a}\right) = \frac{0^2-10^2}{2\ x -2}$ = 25m

t = $\frac{v-u}{a} = \frac{0-10}{-2}$ = 5s

Example 5

A taxi approaching a stage runs two successive half kilometres in 16s and 20s respectively. Assuming the retardation is uniform, find

(i) Initial speed of the taxi

s = ut + $\frac{1}{2}at^2$

For the first half kilometre or 500m

$500 = 16u + \frac{1}{2}a\,(16)^2$ .... (i)

For the kilometre or 1000m

$1000 = 36u + \frac{1}{2}a\,(36)^2$............. (ii)

From eqn. (i) and eqn. (ii)

$a = \frac{25}{72}$ and u = 34.028ms^{-1}

(ii) the further distance, the taxi runs before stopping

$s = \left(\frac{v^2 - u^2}{2a}\right) = s = \left(\frac{0^2 - (34.028)^2}{2\left(\frac{25}{72}\right)}\right) = 1667.3\text{m}$

Extra distance = 1667.3 – 1000

= 667.3m

Example 6

An overloaded taxi travelling at constant velocity of 90km/h overtakes a stationary traffic police car. 2s later, the police car sets in pursuit, accelerating at a uniform rate of 6ms^{-2}. How far does the traffic car travel before catching up with the taxi?

Solution

t_1 = time taken by the taxi

t_2 = time taken by the police car

$t_1 = 2 + t_2$

Speed of the taxi in m/s

90km/h = $\frac{90 \, x \, 1000}{3600}$ = 25ms^{-1}

$s = ut + \frac{1}{2}at^2$

$s_T = 25t_1$

$s_C = 0 \times t_2 + \frac{1}{2} x \; 6 \; x \; {t_2}^2 = 3\,{t_2}^2$

For the car to catch taxi; $s_T = s_C$

$25t_1 = 3\,{t_2}^2$

$25(2 + t_2) = 3\,{t_2}^2$

$t_2 = 10$s or $t_2 = \frac{4}{3}$s

The car leaves 2s later then 10s is the correct time since it gives positive distance

$s_C = 3\,{t_2}^2 = 3 \; x \; 10^2 = 300$m

Example 7

A lorry starts from a point A and moves along a straight horizontal road with a constant acceleration of 2ms^{-2}. At the same time a car moving with a speed of 20ms^{-1} and a constant acceleration of 3ms^{-1} is 400m behind the point A and moving in the same direction as the lorry. find:

(a) how far from A the car overtakes the lorry.

Solution

A car over takes the lorry; both move in the same time, t

$s = ut + \frac{1}{2}at^2$

distance moved by the car =

400 + distance moved by the lorry

$20t + \frac{1}{2} \; x \; 3x \; t^2 = 400 + \frac{1}{2} \; x \; 2 \; x \; t^2$

$t^2 + 40t - 800 = 0$;

t = 14.64s or t = -54.64s

Hence t = 14.64s

$s_L = \frac{1}{2} \; x \; 2 \; x \; (14.64)^2 = 214.33$m

Hence distance moved by lorry

= 214.33m

(b) the speed of the lorry when it is being overtaken

v = u + at

= 0 + 2 x 14.64 = 29.28ms^{-1}

Example 8

The seed of a taxi decreases from 90kmh^{-1} to 18kmh^{-1} in a distance of 120 metres. Find the speed of the taxi when it had covered a distance of 50metres. (05marks)

Solution

Given u = 90kmh^{-1},

v = 18kmh^{-1},

s = 120m = 0.12km

Using $v^2 = u^2 + 2as$

$18^2 = 90^2 + 2a(0.12)$

a = -32400kmh^{-2}

When s = 50m = 0.05km, u = 90kmh^{-1},

a = -32400kmh^{-2}

Using $v^2 = u^2 + 2as$

$v^2 = 90^2 – 2 \times 32400 \times 0.05 = 4860$

v = $\sqrt{4860} = 69.71kmh^{-1}$

Example 9

(a) Show that the final velocity v of a body which starts with an initial velocity u and moves with uniform acceleration a consequently covering a distance x, is given by v = $[u^2 + 2ax]^{\frac{1}{2}}$

Solution

x = average velocity x time

x = $\left(\frac{v+u}{2}\right) t$ but t = $\frac{v-u}{a}$

x = $\left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$ = $\left(\frac{v^2 - u^2}{2a}\right)$

$\boldsymbol{v^2 = u^2 + 2ax}$

v = $[u^2 + 2ax]^{\frac{1}{2}}$

(b) Find the value of x in (a) if v = 300m/s, u = 10m/s and a = 5m/s

Solution

30 = $[10^2 + 2x\ 5x]^{\frac{1}{2}}$

900 = 100 + 10x

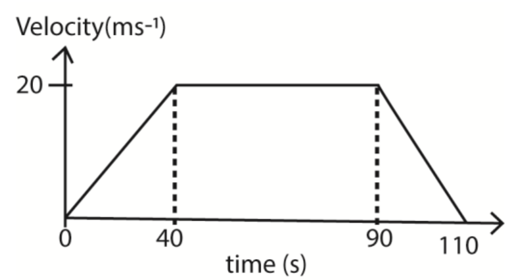
x =80m

Velocity-time graphs

Example 10

A car started from rest and attained a velocity of 20m/s in 40s. It then maintained the velocity attained for 50s. After that it was brought to rest by a constant breaking force in 20s.

(i) Draw a velocity-time graph for the motion



(ii) using the graph, find the total distance travelled by the car

Total distance = total area under the graph

$= \frac{1}{2}bh + lw + \frac{1}{2}bh$

$= \frac{1}{2}x\ 40\ x\ 20 + 50x\ 20 + \frac{1}{2}x\ 20\ x\ 20$

= 1600m

Method II (area of a trapezium)

A = $\frac{1}{2}h(a + b) = \frac{1}{2}x\ 20(50 + 110)$

= 1600m

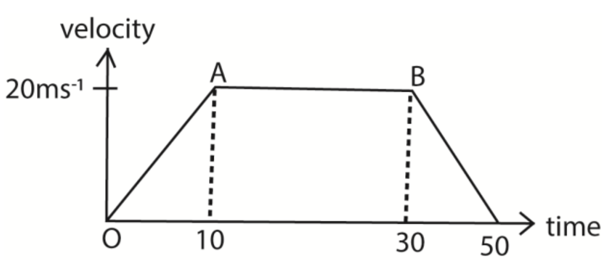
(iii) what is the acceleration of the car?

a= $\frac{v-u}{t} = \frac{20-0}{40} = 0.5ms^{-2}$

Example 11

A car from rest accelerates steadily to 10s up to a velocity f 20ms. It continues with uniform velocity for further 20s and then decelerates so that it stops in 20s.

(a) Draw a velocity-time graph to represent the motion



(b) Calculate

(i) acceleration

a= $\frac{v-u}{t} = \frac{20-0}{10} = 2ms^{-2}$

(ii) deceleration

a= $\frac{v-u}{t} = \frac{0-20}{20} = -1ms^{-2}$

(c) Distance travelled

Method I

Distance = area under the graph

A = $\frac{1}{2}x\ 10\ x\ 20 + 20x\ 20 + \frac{1}{2}x\ 20\ x\ 20$

= 700m

Method II (area of a trapezium)

A = $\frac{1}{2}\ x\ 20(50 + 20)$

= 700m

(d) Average speed

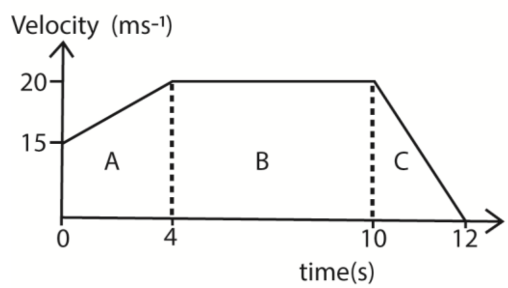
Average speed = $\frac{distance}{time}$

$= \frac{700}{50}$

$= 14m/s$

Example 12

The graph below shows the motion in the body.



(a) Describe the motion of the body
A body with initial velocity of 15m/s accelerates steadily to a velocity of 20m/s in 4s, it then continues with a uniform velocity for 6s and brought to rest in 2s.

(b) Calculate the total distance travelled

Distance

= 4 x 15 + $\frac{1}{2}x\ 4\ \ x\ 5 + 20\ x\ 6 + \frac{1}{2}x\ 20\ \ x\ 2$ = 210m

Revision exercise 1

1. P, Q and R are points on a straight road such that PQ = 20m and QR = 55m. A cyclist moving with uniform acceleration passes O and then notices that it takes him 10s and 15s to travel between P and Q and Q and R respectively. find the acceleration [a = $\frac{2}{15}ms^{-2}$]
2. A car travels from Kampala to Jinja and back. It takes average speed on the return journey is 4km/h greater than that on the outward journey and it takes 12 minutes less. Given that Kampala and Jinja are 80km apart, find the average speed on the outward journey.[30.05kmh]
3. Car A traveling at 35ms^{-1} along a straight horizontal road, accelerates uniformly at 0, 4ms^{-2}. At the same time, another car B moving at 44ms^{-1} and accelerating uniformly at 0.5ms^{-2} is 200m behind A
   (i) Find the time taken before car B over takes car A. [20s]
   (ii) speed with which B over takes A. [55m/s]
4. A car is being driven along a road at 72kmh^{-1} notices a fallen tree on the road 800m ahead and suddenly reduces the speed to 36kh^{-1} by applying brakes. For how long were the brakes applied [53.33s]
5. A train starts from station a with a uniform acceleration of 0.2ms^{-2} for 2 minutes and attains a maximum speed and moves uniformly for 15 minutes. it is then brought to rest at constant retardation of 5/3ms^{-2} at station B. find the distance between A and B. [23212.8m]
6. A motorcycle decelerated uniformly from 20kmh^{-1} to 8kmh^{-1} in travelling 896m. find the rate of deceleration in ms2 [0.0145ms^{-2}]
7. A body moves with a uniform acceleration and covers a distance of 27m in 3s; it then moves with a uniform velocity and covers a distance of 60m in 5s. Find the initial velocity and acceleration of the body. [6ms^{-1}, 2ms^{-2}]
8. A particle is projected away from an origin O with initial velocity of 0.25ms^{-1}. The particle travels in a straight line and accelerates at 1.5ms^{-2}. find
   (i) how far the particle is from O after 4s [7.5m]
   (ii) the distance travelled by the particle during the fourth second after projection. [5.5m]
9. A taxi which is moving with a uniform acceleration is observed to take 20s and 30s to travel successive 400m. find
   (i) initial speed of the taxi. [$\frac{68}{3}ms^{-1}$]
   (ii) the further distance it covers before stopping [163.3m]
10. Two cyclist A and B are 36m apart on a straight road. Cyclist B starts from rest with an acceleration of 6ms^{-2} while A is in pursuit of B with velocity of 20ms^{-1} and acceleration of 4ms^{-1}. Find the time taken when A overtakes B [1.3466s]

Relative velocity

This is the velocity a body would have as seen by an observer on another body. Suppose A and B are two moving bodies, the velocity of A relative to B is the velocity of A as it appears to observer on B

It is denoted by $V_{AB} = V_A - V_B$

Note that $V_{AB} \neq V_{BA}$ since $V_{BA} = V_B - V_A$.

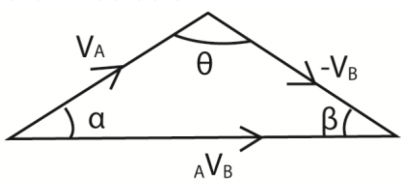
Numerical calculations

There are two methods used in calculations

- Geometric method and
- Vector method

(i) Geometric method.

If V_A and V_B are not given in vector form and the velocity of A relative to B is required, then we can reverse the velocity of B such that $V_{AB} = V_A + (-V_B)$ and the vector triangle is drawn as below.



$V_{AB}^2 = V_A^2 + V_B^2 - 2V_A \times V_B \cos\theta$

and

$$\frac{V_{AB}}{\sin\theta} = \frac{V_B}{\sin\alpha} = \frac{V_A}{\sin\beta}$$

(ii) Vector method

Find components of velocity for each separately

$\therefore V_{AB} = V_A - V_B$

Example 13

Particle A is moving due north at $30ms^{-1}$ and particle B is moving due south at $20ms^{-1}$. Find the velocity of A relative to B.

Solution

$\uparrow V_A = 30ms^{-1}$ and $\downarrow V_B = 20ms^{-1}$

$V_{AB} = V_A - V_B$

$V_{AB} = \begin{pmatrix} 0 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ -20 \end{pmatrix} = \begin{pmatrix} 0 \\ 50 \end{pmatrix}$

$|V_{AB}| = \sqrt{0^2 + 50^2} = 50ms^{-1}$ due north

Example 14

A particle A has a velocity $(4i + 6j - 5k)ms^{-1}$ while B has a velocity of $(-10i - 2j + 6k)ms^{-1}$. Find the velocity of A relative to B.

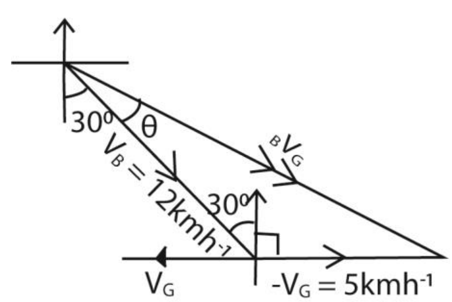
$V_{AB} = V_A - V_B$

$$V_{AB} = \begin{pmatrix} 4 \\ 6 \\ -5 \end{pmatrix} - \begin{pmatrix} -10 \\ -2 \\ 6 \end{pmatrix} = \begin{pmatrix} 14 \\ 8 \\ -11 \end{pmatrix} ms^{-1}$$

Example 15

A girl walks at $5kmh^{-1}$ due west and a boy runs $12kmh^{-1}$ at a bearing of 150^0. Find the velocity of the boy relative to the girl

Method I (geometrical)



$V_{BG}^2 = V_B^2 + V_G^2 - 2V_B \times V_G \cos 120^0$

$V_{BG} = \sqrt{5^2 + 12^2 - 2 \times 5 \times 12 \cos 120^0}$

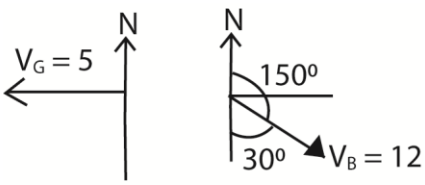
$= 15.13ms^{-1}$

$\frac{5}{sin\theta} = \frac{15.13}{\sin 120}$

$\theta = 16.63^0$

The relative velocity is $15.13ms^{-1}$ at $S46.63^0E$

Method II (Vector)

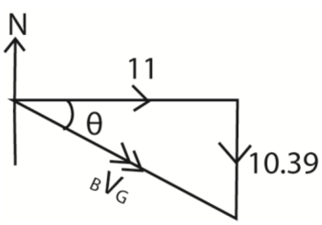


$V_{BG} = V_B - V_G$

$= \begin{pmatrix} 12sin30 \\ -12cos30 \end{pmatrix} - \begin{pmatrix} -5 \\ 0 \end{pmatrix} = \begin{pmatrix} 11 \\ -10.39 \end{pmatrix}$

$|V_{BG}| = \sqrt{11^2 + (-10.39)^2}$

$= 15.13ms^{-1}$

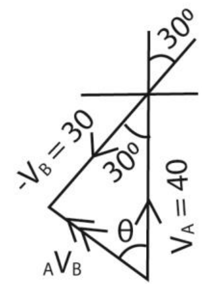


$\theta = \tan^{-1}\frac{10.39}{11} = 43.40$

The relative velocity is $15.13ms^{-1}$ at $S46.63^0E$

Example 16

Plane A is flying due north at $40kmh^{-1}$ while plane B is flying in the direction $N30^0E$ at $30kmh^{-1}$. Find the velocity of A relative B



$V_{AB}^2 = V_A^2 + V_B^2 - 2V_A \times V_B \cos30^0$

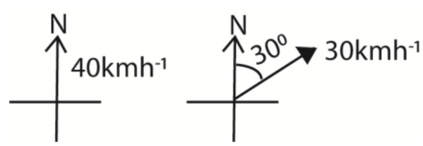
$V_{BG} = \sqrt{40^2 + 30^2 - 2 \ x \ 40 \ x \ 30 \ cos30^0}$

$= 20.53kmh^{-1}$

$\frac{30}{sin\theta} = \frac{20.53}{\sin 30}$; $\theta = 46.94^0$

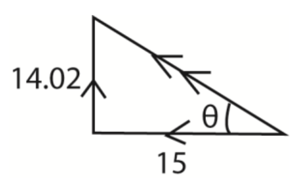
The relative velocity is $20.53kmh^{-1}$ at $N46.9^0W$

Method II (vectors)



$V_{AB} = V_A - V_B = \begin{pmatrix} 0 \\ 40 \end{pmatrix} - \begin{pmatrix} 30sin30 \\ 30cos30 \end{pmatrix} = \begin{pmatrix} -15 \\ 14.02 \end{pmatrix}$

$|V_{BG}| = \sqrt{(-15)^2 + (14.02)^2} = 20.53kmh^{-1}$



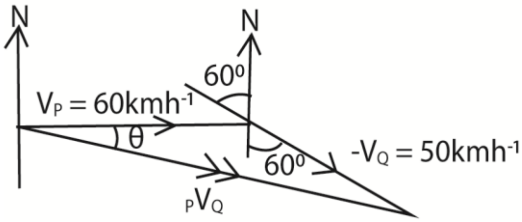
$\theta = \tan^{-1}\frac{14.02}{15} = 43.1^0$

The relative velocity is $20.53kmh^{-1}$ at $N43.1^0W$

Example 17

Ship P is steering $60kmh^{-1}$ due east while ship Q is steering in the direction $N60^0W$ at $50kmh^{-1}$. Find the velocity of P relative to Q.

Method I (Geometrical)



$V_{PQ}^2 = V_P^2 + V_Q^2 - 2V_P \times V_Q \cos150^0$

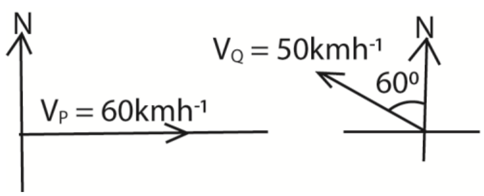
$V_{BG} = \sqrt{60^2 + 50^2 - 2 \times 60 \times 50 \cos150^0}$

$= 106.28kmh^{-1}$

$\frac{50}{\sin\theta} = \frac{106.28}{\sin 30}$; $\theta = 13.6^0$

The relative velocity is $106.28kmh^{-1}$ at $S76.4^0E$

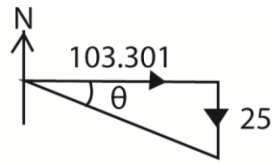
Method II (Vector)



$V_{PQ} = V_P - V_Q$

$= \begin{pmatrix}60\\0\end{pmatrix} - \begin{pmatrix}-50\sin60\\50\cos60\end{pmatrix} = \begin{pmatrix}103.301\\-25\end{pmatrix}$

$|V_{BG}| = \sqrt{(103.301)^2 + (-25)^2}$ $=106.3kmh^{-1}$



$\theta = \tan^{-1}\frac{25}{103.301} = 13.6^0$

Direction $S(90^0 - 13.6^0)E$

The relative velocity is $106.28kmh^{-1}$ at $S76.4^0E$

Finding true velocity

Example 18

To a cyclist riding due north at $40kmh^{-1}$, a steady wind appears to blow from west at $30kmh^{-1}$. Find the true velocity of the wind

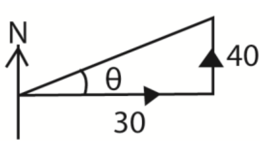
Solution

$V_{WC} = V_W - V_C$

$\begin{pmatrix}30\\0\end{pmatrix} = V_W - \begin{pmatrix}0\\40\end{pmatrix}$

$V_W = \begin{pmatrix}30\\40\end{pmatrix}$

$|V_W| = \sqrt{(30)^2 + (40)^2}$ = $50kmh^{-1}$



$\theta = \tan^{-1}\frac{40}{30} = 53.13^0$

Direction $N(90 - 15.13)^0E = N36.87^0E$

Example 19

To a motorist travelling due north at $40kmh^{-1}$, a steady wind appears to blow from $N60^0E$ at $50kmh^{-1}$.

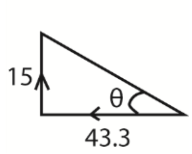
(a) find the true velocity of the wind

$V_{WM} = V_W - V_M$

$\begin{pmatrix}-50\sin60\\-50\cos60\end{pmatrix} = V_W - \begin{pmatrix}0\\40\end{pmatrix}$

$V_W = \begin{pmatrix}-43.3\\15\end{pmatrix}$

$|V_W| = \sqrt{(-43.3)^2 + (15)^2}$ = $46kmh^{-1}$



θ = $\tan^{-1}\frac{15}{43.3}$ = 19^0

Direction: $N71^0W$

(b) If the wind velocity and direction remain constant but the speed of the motorist is increases, find his speed when the wind appears to be blowing from the direction $N45^0E$.

$V_{WM} = V_W - V_M$

$\begin{pmatrix}-bsin45\\-bcos45\end{pmatrix} = \begin{pmatrix}46\sin71\\-46cos71\end{pmatrix} - \begin{pmatrix}0\\a\end{pmatrix}$

i components : -bsin45 = 46sin71

b = 61.5096

j components: -bcos45 = -46cos71 – a

a = 61.51cos45 - 46cos71

a = 28.51kmh^{-1}

Example 20

To a man travelling due north at 10kmh^{-1}, a steady wind appears to blow from East. When he travels in the direction $N60^0W$ at 8kmh^{-1}, it appears to come from south. Find the velocity of the wind.

$V_{WM} = V_W - V_M$

$\begin{pmatrix}-a\\0\end{pmatrix} = V_W - \begin{pmatrix}0\\10\end{pmatrix}$

$V_W = \begin{pmatrix}-a\\10\end{pmatrix}$ ……………………………(i)

Also

$V_{WM} = V_W - V_M$

$\begin{pmatrix}0\\b\end{pmatrix} = V_W - \begin{pmatrix}-8sin60\\8cos60\end{pmatrix}$

$V_W = \begin{pmatrix}-8sin60\\b+8cos60\end{pmatrix}$ ……………………………(i)

(i) and (ii)

$\begin{pmatrix}-a\\10\end{pmatrix} = \begin{pmatrix}-8sin60\\b+8cos60\end{pmatrix}$

a = 8sin60 = $4\sqrt{3}$

10 = b + 8cos60

b = 6

$V_W = \begin{pmatrix}-a\\10\end{pmatrix} = \begin{pmatrix}-4\sqrt{3}\\10\end{pmatrix}$

$|V_W| = \sqrt{(-4\sqrt{3})^2 + 10^2}$ = 12.17kmh^{-1}

$\theta = \tan^{-1}\left(\frac{10}{4\sqrt{3}}\right)$ = 55.3^0

Direction: $N(90 - 53.3)^0W = N34.7^0W$

Example 21

To a cyclist riding due north at 40kmh^{-1}, a steady wind appears to blow eastwards. On reducing his speed to 30kmh^{-1} but moving in the same direction, the wind appears to come from southwest. Find the velocity of the wind

$V_{WC} = V_W - V_C$

$\begin{pmatrix}a\\0\end{pmatrix} = V_W - \begin{pmatrix}0\\40\end{pmatrix}$

$V_W = \begin{pmatrix}a\\40\end{pmatrix}$ ……………………………(i)

Also

$V_{WC} = V_W - V_C$

$\begin{pmatrix}-bsin45\\bcos45\end{pmatrix} = V_W - \begin{pmatrix}0\\30\end{pmatrix}$ …….. (ii)

$V_W = \begin{pmatrix}-bsin45\\30+bcos45\end{pmatrix}$ ……………………………(i)

(i) and (ii)

$\begin{pmatrix} a \\ 40 \end{pmatrix} = \begin{pmatrix} -bsin45 \\ 30 + bcos45 \end{pmatrix}$

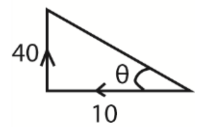
40 = 30 + bcos45

b = $10\sqrt{2}$

a =- bsin45 = $10\sqrt{2}$sin45 = -10

$V_W = \begin{pmatrix} a \\ 40 \end{pmatrix} = \begin{pmatrix} -10 \\ 40 \end{pmatrix}$

$|V_W| = \sqrt{10^2 + 40^2}$ = 41.23kmh^{-1}



$\theta = \tan^{-1}\left(\frac{40}{-10}\right)$= 75.96^0

Direction: N(90 -76)^{0}E= N14^{0}W

Exercise 2

1. Car A moving Eastward at 20ms^{-1} and car B is moving Northward at 10ms^{-1}. Find the
   (i) velocity of A relative to B [$10\sqrt{5}ms^{-1}$
   (ii) velocity of B relative to A [$10\sqrt{5}ms^{-1}$
2. A yatch and a trawler leave a harbour at 8am. The yatch travels due west at 10kmh-1 and trawler due east at 20kmh^{-1}
   (i) what is the velocity of the trawler relative to yatch [30kmh^{-1} east]
   (ii) how far apart are the boats at 9.30am [45km]
3. At 10.30am a car travelling at 25ms^{-1} due east overtakes a motor bike travelling at 10ms^{-1} due east. What is the velocity of the car relative to the motor bike and how far apart are the vehicle at 10.30am. [15ms^{-1} east, 900m]
4. Bird A has a velocity of (7i + 3j + 10k)ms^{-1} while bird B has a velocity (6i – 17k)ms^{-1}. Find the velocity of B relative to A [(-i -3j – 27k0ms^{-1}]
5. Joe rides his horse with a velocity $\begin{pmatrix} 5 \\ 24 \end{pmatrix}$kmh^{-1} while Jill is riding her horse with velocity $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$kmh^{-1}
   (i) Find Joe's velocity as seen by Jill $\left[\begin{pmatrix} 5 \\ 24 \end{pmatrix} \text{kmh}^{-1}\right]$
   (ii) What is Jill's velocity as seen by Joe. $\left[\begin{pmatrix} 0 \\ -124 \end{pmatrix} \text{kmh}^{-1}\right]$
6. In EPL football match, a ball is moving at 5ms^{-1}in the direction of N450E and the player is running due north at 8ms^{-1}. Find the velocity of the ball relative to the player. [5.69ms^{-1} at S38.38^{0}E]
7. An aircraft is flying at 250kmh^{-1} in direction N60^{0}E and a second aircraft is flying at 200kmh^{-1} in the direction N20^{0}W. Find the velocity of the first aircraft as seen by the pilot of the second aircraft. [292kmh^{-1} at S77.9^{0}E]
8. A ship is sailing southeast at 20kmh^{-1} and a second ship is sailing due west at 25kmh^{-1}. Find the magnitude and direction of the velocity of the first ship relative to the second. [41.62kmh^{-1} at S70.13^{0}E]
9. What is the velocity of a cruiser moving at 20kmh^{-1} due to north as seen by an

observer on a liner moving at 15kmh^{-1} in the direction N30^{0}W [10.3kmh^{-1} at N46.9^{0}E]

10. A car is being driven at 20ms-1 on a bearing of 0400. Wind is blowing from 3000 with speed of 10ms-1. Find the velocity of the wind as experienced by the driver of the car.
    [48.13ms^{-1} at S18.13^{0}W]
11. An aircraft is moving at 250kmh^{-1} in direction N60^{0}E. The second aircraft is moving at 200kmh^{-1} in a direction N20^{0}W. Find the velocity of the first aircraft as seen by the pilot of the second aircraft. [292kmh^{-1} at S77.9^{0}E]
12. To a pigeon flying with velocity of (-2i + 3j + k)ms^{-1}, a hawk appears to have a velocity of (i – 5j -10k)ms^{-1}.Find the true velocity of the hawk[(-i -2j – 9k)ms^{-1}]
13. To a cyclist riding at 3ms^{-1} due east, the wind appears to come from the south with the speed $3\sqrt{3}ms^{-1}$. Find the true speed and direction of the wind. [6ms^{-1} from S30^{0}W]
14. To the pilot of an aircraft A travelling at300kmh^{-1} due south, it appears that an aircraft B is travelling at 600kmh^{-1} in a direction N60^{0}W. Find the true speed and direction of the aircraft B. [520kmh^{-1} west]
15. Jane is riding her horse at 5kmh^{-1} due north and sees Suzan riding her horse apparently with velocity 4kmh^{-1}, N60^{0}E. Find Suzan's true velocity. [7.81kmh^{-1} N26.3^{0}E]
16. A eagle flying at 8ms^{-1} on a bearing of 240^0 sees a chick apparently running at 5ms^{-1} on bearing300^0. Find true velocity of the chick. [11.4ms^{-1} at 262.4^0]
17. A train is travelling at 80kmh^{-1} in direction N15^{0}E. A passenger on the train observes a plane apparently moving at 125kmh^{-1} in the direction N50^{0}E. Find the true velocity of the plane. [196kmh^{-1} N36.5^{0}E]
18. To an athlete jogging at 12kmh^{-1} on a bearing of N10^{0}E, the wind seems to come from a direction N20^{0}W at 15kmh^{-1}. Find the true velocity of the wind. [7.57kmh^{-1} N72.5^{0}W]
19. To a passenger on a boat which is travelling at 20kmh^{-1} on a bearing 230^0, the wind seems to be blowing from 250^0 as 12kmh^{-1}. Find the true velocity of the wind [9.64kmh^{-1} N24.8^{0}E]
20. On a particular day wind is blowing N30^{0}E at a velocity of 4ms^{-1} and a motorist is driving at 40ms^{-1} in the direction of S60^{0}E.
    (a) Find the velocity of the wind relative to the motorist. [40.2ms^{-1} at N54.28^{0}W]
    (b) If the motorist changes the direction maintaining his speed and the wind appears to blow due east. What is the new direction of the motorist [N85.03^{0}W]
21. A, B and C are three aircrafts. A has velocity (200i + 170j)ms^{-1}. To the pilot of A it appears that B has velocity (50i – 270j)ms^{-1}. To the pilot of B it appears that C has a velocity (50i + 170j)ms^{-1}. Find the velocities of B and C [(250i - 100j)ms^{-1}, (300i + 70j)ms^{-1}]
22. To a bird flying due east at 10ms^{-1}, the wind seems to come from south. When the bird alters its direction of flight to N30^{0}E without altering its speed, the wind seems to come from the north-

west. Find the true velocity of wind. [$10.6ms^{-1}$ from $S69.9^0W$]

23. To an observer on a trawler moving at $12kmh^{-1}$ in the direction $S30^0W$, the wind appears to come from N600W. To an observer on a ferry moving at $15kmh^{-1}$ in a direction $S80^0E$, the wind appears to come from the north. Find the true velocity of the wind. [$26.8kmh^{-1}$ $N33.4^0W$]

Relative position

Consider two bodies A and B moving with V_A and V_B from points with position vectors OA and OB respectively.

Position of A at time t is $R_{A(t=t)} = R_{A(t=0)} + t \times V_A$

Position of A at time t is $R_{Bt=t)} = R_{B(t=0)} + t \times V_B$

Position of A relative to B at time t is $R_{ABt=t)} = R_{A(t=0)} - R_{B(t=0)}$

$R_{ABt=t)} = (R_{A(t=0)} + t \times V_A) - (R_{B(t=0)} + t \times V_B)$

$R_{ABt=t)} = (R_{A(t=0)} - R_{B(t=0)}) + (V_{AB})t$

Example 22

The velocities of ships P and Q are $(i + 6j)kmh^{-1}$ and $(-i + 3j)kmh^{-1}$. At a certain instant, the displacement between the two ship is $(7i + 4j)$km. Find the

(a) Relative velocity of ship P to Q

$V_{PQ} = V_P - V_Q$

$V_{PQ} = \begin{pmatrix}1\\6\end{pmatrix} - \begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} kmh^{-1}$

(b) Magnitude of displacement between ships P and Q after 2hours.

$R_{PQt=t)} = (R_{P(t=0)} - R_{Q(t=0)}) + (V_{PQ})t$

$R_{PQt=t)} = \begin{pmatrix}7\\4\end{pmatrix} + t\begin{pmatrix}2\\3\end{pmatrix}$

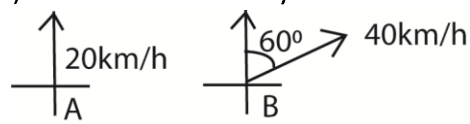
$R_{PQt=2)} = \begin{pmatrix}7\\4\end{pmatrix} + 2x\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}11\\10\end{pmatrix} km$

$|R_{PQ}| = \sqrt{11^2 + 10^2} = 14.87km$

Example 23

Two ship A and B move simultaneously with velocities $20kmh^{-1}$ and $40kmh^{-1}$. Ship A moves in the northern direction while ship B moves in $N60^0E$. Initially ship B is 10km due west of A. Determine

(a) the relative velocity of A to B.



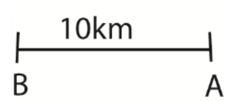
$V_{AB} = V_A - V_B$

$V_{AB} = \begin{pmatrix}0\\20\end{pmatrix} - \begin{pmatrix}40sin60\\40cos60\end{pmatrix}$

$= \begin{pmatrix}-34.641\\0\end{pmatrix}$

$|V_{AB}| = \sqrt{(-34.641)^2 + 0^2} = 34.641$

(b) the position of A relative to B at any time t



$R_{AB(t=t)} = (R_{A(t=0)} - R_{B(t=0)}) + (V_{AB})t$

$R_{AB(t=t)} = \left[\begin{pmatrix}10\\0\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right] + t\begin{pmatrix}-34.641\\0\end{pmatrix}$

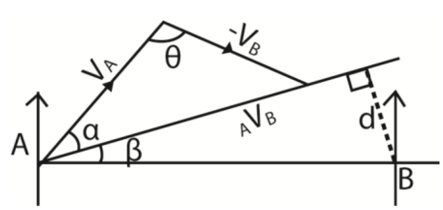
$R_{AB(t=t)} = \begin{pmatrix}10 - 34.641t\\0\end{pmatrix} km$

Shortest Distance and time of closest approach or (Shortest distance and time of shortest distance)

When two particles are moving simultaneously with specific velocities, time will come when they are closest to each other **without** colliding.

There are three methods used for the distance and time of closest approach, i.e. Geometrical, vector and differential method.

1. vector method
   Consider particle A and B moving with velocities VA and VB from point with position vector OA and OB respectively.
   For minimum distance to be attained then $V_{AB}.R_{AB(t=t)} = 0$. This gives the time.
   Then **shortest distance,**
   d = $\left|R_{AB(t=t)}\right|$
2. Differential method
   The minimum distance is reached when $\frac{d}{dt}\left|R_{AB(t=t)}\right|^2 = 0$. This gives time
   Then **shortest distance,**
   d = $\left|R_{AB(t=t)}\right|$
3. Geometrical method
   If V_A and V_B are not given in vector form, then the velocity of B is reversed such that $V_{AB} = V_A + (-V_B)$ and the vector triangle is drawn as below.
   The shortest distance, d will be perpendicular to V_{AB}



$V_{AB}^2 = V_A^2 + V_B^2 - 2_AV_B\cos\theta$

$\frac{V_{AB}}{\sin\theta} = \frac{V_B}{\sin\alpha}$

Shortest distance, d =ABsinβ

Time to the shortest distance, t = $\frac{AB\cos\beta}{V_{AB}}$

Example 24

A particle P starts from rest from a point with position vector (2j + 2k)m with a velocity (j + k)ms^{-1}. A second particle Q starts at the same time from a point whose position vector is (–11i – 2j – 7k)m with a velocity of (2i + j + 2k)ms^{-1}. Find

(i) the shortest distance between the particles

(ii) how far each has travelled by this time.

Method 1

$$V_{PQ} = V_P - V_Q = \begin{pmatrix}0\\1\\1\end{pmatrix} - \begin{pmatrix}2\\1\\2\end{pmatrix} = \begin{pmatrix}-2\\0\\-1\end{pmatrix}$$

$$R_{PQ(t=t)} = \left(R_{P(t=0)} - R_{Q(t=0)}\right) + \left(V_{PQ}\right)t$$

$$R_{PQ(t=t)} = \left[\begin{pmatrix}0\\2\\2\end{pmatrix} - \begin{pmatrix}-11\\-2\\-7\end{pmatrix}\right] + \begin{pmatrix}-2\\0\\-1\end{pmatrix}t$$

$$R_{PQ(t=t)} = \begin{pmatrix}11-2t\\4\\9-t\end{pmatrix}$$

For minimum distance: $V_{AB}.R_{AB(t=t)} = 0$.

$$\begin{pmatrix}-2\\0\\-1\end{pmatrix}.\begin{pmatrix}11-2t\\4\\9-t\end{pmatrix} = 0$$

-22 + 4t + 0 -9 + t = 0

t = $\frac{31}{5} = 6.2s$

(i) Then **shortest distance,**

d = $\left|R_{PQ(t=t)}\right|$

$$R_{PQ(t=6.2)} = \begin{pmatrix}11-2t\\4\\9-t\end{pmatrix}$$

$$R_{PQ(t=6.2)} = \begin{pmatrix}11-2x\ 6.2\\4\\9-6.2\end{pmatrix} = \begin{pmatrix}-1.4\\4\\2.8\end{pmatrix}$$

$\left|R_{PQ(t=6.2)}\right| = \sqrt{(-1.4)^2 + 4^2 + 2.8^2}$ = 5.08m

(ii) How far each has travelled

$R_{P(t=t)} = R_{P(t=0)} + (V_P)t$

$R_{P(t=6.2)} = \begin{pmatrix}0\\2\\2\end{pmatrix} + 6.2\,x\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\8.2\\8.2\end{pmatrix}$

$\left|R_{P(t=6.2)}\right| = \sqrt{0^2 + 8.2^2 + 8.2^2} = 11.6m$

$R_{Q(t=6.2)} = \begin{pmatrix}-11\\-2\\-7\end{pmatrix} + 6.2\,x\begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}1.4\\4.2\\5.4\end{pmatrix}$

$\left|R_{Q(t=6.2)}\right| = \sqrt{1.4^2 + 4.2^2 + 5.4^2} = 6.8m$

Method II

$\frac{d}{dt}\left|R_{AB(t=t)}\right|^2 = 0.$

$\left|R_{PQ(t=t)}\right|^2 = \begin{pmatrix}11-2t\\4\\9-t\end{pmatrix}^2$

$\left|R_{PQ(t=t)}\right|^2 = (11-2t)^2 + 4^2 + (9-t)^2$

$\left|R_{PQ(t=t)}\right|^2 = 218 - 62t + 5t^2$

$\frac{d}{dt}\left|R_{PQ(t=t)}\right|^2 = \frac{d}{dt}(218 - 62t + 5t^2)$

$\frac{d}{dt}\left|R_{PQ(t=t)}\right|^2 = -62 + 10t = 0$

t = 6.2s

$R_{PQ(t=6.2)} = \begin{pmatrix}11-2x\,6.2\\4\\9-6.2\end{pmatrix} = \begin{pmatrix}-1.4\\4\\2.8\end{pmatrix}$

$\left|R_{PQ(t=6.2)}\right| = \sqrt{(-1.4)^2 + 4^2 + 2.8^2}$ = 5.08m

Example 25

At 12 noon the position vectors r and velocity vectors v of ship A and ship B are as follows

r_A = (-9i + 6j)km, v_A = (3i + 12j) kmh^{-1} and r_B = (16i + 6j), v_B = (-9i+ 3j)kmh^{-1} respectively

(i) Find how far apart the ships are at noon

$R_{AB(t=0)} = \left(R_{A(t=0)} - R_{B(t=0)}\right)$

$R_{AB(t=0)} = \begin{pmatrix}-9\\6\end{pmatrix} - \begin{pmatrix}16\\6\end{pmatrix} = \begin{pmatrix}-25\\0\end{pmatrix}$

$\left|R_{AB(t=0)}\right| = \sqrt{(-25)^2 + 0^2}$

$= 25km\ apart$

(ii) Assuming velocities do not change, find the least distance between the ships in the subsequent motion

$V_{AB} = V_A - V_B = \begin{pmatrix}3\\12\end{pmatrix} - \begin{pmatrix}-9\\3\end{pmatrix} = \begin{pmatrix}12\\9\end{pmatrix}$

$R_{AB(t=t)} = \left(R_{A(t=0)} - R_{B(t=0)}\right) + (V_{AB})t$

$R_{AB(t=t)} = \begin{pmatrix}-25\\0\end{pmatrix} + \begin{pmatrix}12\\9\end{pmatrix}t$

$= \begin{pmatrix}-25+12t\\9t\end{pmatrix}km$

For minimum distance: $V_{AB}.R_{AB(t=t)} = 0.$

$\begin{pmatrix}12\\9\end{pmatrix}.\begin{pmatrix}-25+12t\\9t\end{pmatrix} = 0$

-300 + 144t + 81t = 0

t = $\frac{4}{3}hours$

Shortest distance, d = $\left|R_{PQ(t=\frac{4}{3})}\right|$

$R_{AB(t=\frac{4}{3})} = \begin{pmatrix}-25+12\,x\,\frac{4}{3}\\9x\,\frac{4}{3}\end{pmatrix} = \begin{pmatrix}-9\\12\end{pmatrix}km$

$\left|R_{AB(t=\frac{4}{3})}\right| = \sqrt{(-9)^2 + 12^2}$ = 15km

(iii) Find when their distance of closest approach occurs and the potion vectors of A and B

It occurs at 12.00 + $\frac{4}{3}$ x 60 = 1.20pm

how far each travelled

$R_{A(t=t)} = R_{A(t=0)} + V_A t$

$R_{A(t=\frac{4}{3})} = \begin{pmatrix}-9\\6\end{pmatrix} + \begin{pmatrix}3\\12\end{pmatrix} x \frac{4}{3}$

$= \begin{pmatrix}-5\\22\end{pmatrix} km$

$R_{B(t=t)} = R_{B(t=0)} + V_B t$

$R_{B(t=\frac{4}{3})} = \begin{pmatrix}16\\6\end{pmatrix} + \begin{pmatrix}-9\\3\end{pmatrix} x \frac{4}{3} = \begin{pmatrix}4\\10\end{pmatrix} km$

Example 26

At a certain time, the position vectors r and velocity vectors v of ship A and ship B are as follows

r_A = (20j)km V_A = (9i -2j)kmh^{-1} at 14.00hrs

r_B = (i + 4j)km V_B = (4i + 8j)kmh^{-1} at 15.00hrs

Assuming velocities do not change, find

(a) the position vector of A at 15.00hrs

$R_{A(t=t)} = R_{A(t=0)} + V_A t$

At 16.00hrs: $R_{A(t=1)} = \begin{pmatrix}0\\20\end{pmatrix} + \begin{pmatrix}9\\-2\end{pmatrix} x\ 1$

$= \begin{pmatrix}9\\18\end{pmatrix} km$

(b) the least distance between A and B in the subsequent motion

$R_{AB(t=t)} = \left(R_{A(t=0)} - R_{B(t=0)}\right) + (V_{AB})t$

$R_{AB(t=t)} = \left[\begin{pmatrix}9\\18\end{pmatrix} - \begin{pmatrix}1\\4\end{pmatrix}\right] + \left[\begin{pmatrix}9\\-2\end{pmatrix} - \begin{pmatrix}4\\8\end{pmatrix}\right]$t

$R_{AB(t=t)} = \begin{pmatrix}8+5t\\14-10t\end{pmatrix}$

For minimum distance: $V_{AB}.R_{AB(t=t)} = 0.$

$\begin{pmatrix}5\\-10\end{pmatrix} . \begin{pmatrix}8+5t\\14-10t\end{pmatrix}$ = 0

t = $\frac{4}{5}$ = 0.8hrs

(c) time at which this least separation occurs.

15.00 + 0.8 x 60 = 15.48hrs

Shortest distance, d = $\left|R_{PQ(t=0.8)}\right|$

$R_{AB(t=0.8)} = \begin{pmatrix}8+5\ x\ 0.8\\14-10\ x\ 0.8\end{pmatrix} = \begin{pmatrix}12\\6\end{pmatrix} km$

$R_{AB(t=0.8)} = \sqrt{12^2+6^2}$ = 13.42km

Example 27

At a certain time, the position vectors r and velocity vectors v of ship A and ship B are as follows

r_A = (-2i + 3j)km,

v_A =(12i -4j)kmh-1 at 11.45am

r_B = (8i + 7j)km

v_B = (2i – 14j)kmh-1 at 12.00 noon

Assuming velocities do not change, find

(a) The least distance between A and B in the subsequent motion

OA = $\begin{pmatrix}-2\\3\end{pmatrix}$ and v_A = $\begin{pmatrix}12\\-4\end{pmatrix} kmh^{-1}$

$R_{A(t=t)} = R_{A(t=0)} + V_A t$

12.00: $R_{A\left(t=\frac{1}{4}\right)} = \begin{pmatrix}-2\\3\end{pmatrix} + \frac{1}{4}\begin{pmatrix}12\\-4\end{pmatrix}$

$= \begin{pmatrix}1\\2\end{pmatrix} km$

$V_{AB} = V_A - V_B = \begin{pmatrix}12\\-4\end{pmatrix} - \begin{pmatrix}2\\-14\end{pmatrix}$

$= \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

$R_{AB(t=t)} = \left(R_{A(t=0)} - R_{B(t=0)}\right) + (V_{AB})t$

$R_{AB(t=t)} = \left[\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ 7 \end{pmatrix}\right] + \begin{pmatrix} 10 \\ 10 \end{pmatrix} t$

$R_{AB(t=t)} = \begin{pmatrix} -7 + 10t \\ -5 + 10t \end{pmatrix}$

For minimum distance: $V_{AB}.R_{AB(t=t)} = 0.$

$\begin{pmatrix} 10 \\ 10 \end{pmatrix} . \begin{pmatrix} -7 + 10t \\ -5 + 10t \end{pmatrix} = 0$

t = 0.6hrs

$R_{AB(t=0.6)} = \begin{pmatrix} -7 + 10 \; x \; 0.6 \\ -5 + 10 \; x \; 0.6 \end{pmatrix}$ $= \begin{pmatrix} -1 \\ 1 \end{pmatrix} km$

$\left|R_{AB(t=0.6)}\right| = \sqrt{(-1)^2 + 1^2}$ =1.4142km

(b) length of time for which A is within range, if ship B has guns within a range of up to 2km

$R_{AB(t=t)} = \begin{pmatrix} -7 + 10t \\ -5 + 10t \end{pmatrix}$

$\left|R_{AB(t=t)}\right| = 2km$

$\begin{pmatrix} -7 + 10t \\ -5 + 10t \end{pmatrix}^2 = 2^2$

$(-7 + 10t)^2 + (-5 + 10t)^2 = 4$

$100t^2 - 12t + 35 = 0$

t = 0.7hrs or t = 0.5hrs

Time for which they are in range

= 0.7 – 0.5 = 0.2h

Example 28

At 10am, ship A moves with a constant velocity (4i+ 20j) kmh^{-1} and ship B due north of A moves with a constant velocity (-3i – 4j) kmh^{-1}.

(a) Find the velocity of A relative to B

$V_{AB} = V_A - V_B = \begin{pmatrix} 4 \\ 20 \end{pmatrix} - \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 24 \end{pmatrix} kmh^{-1}$

(b) If the shortest distance between the two ships is 4.2km. Find the
   (i) time to the nearest minute when they are closest together
   (ii) original distance apart at 10am
   (iii) the bearing of B from A when they are closest together.

Solution

(i) Let a km be the distance apart at 10am

$R_{AB(t=t)} = \left(R_{A(t=0)} - R_{B(t=0)}\right) + (V_{AB})t$

$R_{AB(t=t)} = \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ a \end{pmatrix}\right] + \begin{pmatrix} 7 \\ 24 \end{pmatrix} t = \begin{pmatrix} 7t \\ -a + 24t \end{pmatrix} km$

$R_{AB(t=t)} = \begin{pmatrix} 7t \\ -a + 24t \end{pmatrix} km$

$\left|R_{AB(t=t)}\right| = 4.2km$

$\begin{pmatrix} 7t \\ -a + 24t \end{pmatrix}^2 = (4.2)^2$

$(7t)^2 + (-a + 24t)^2 = 17.64$

$625t^2 - 48at + a^2 = 17.64$ ………….(i)

For minimum distance: $V_{AB}.R_{AB(t=t)} = 0.$

$\begin{pmatrix} 7 \\ 24 \end{pmatrix} . \begin{pmatrix} 7t \\ -a + 24t \end{pmatrix} = 0$

49t – 24a + 576t = 0

$a = \frac{625t}{24}$ ………………………………….(ii)

Substituting for a in eqn. (i)

625t^2 – 48$\left(\frac{625t}{24}\right)$t + $\left(\frac{625t}{24}\right)^2$ = 17.64

53.1684t^2 = 17.67

t = $\pm 0.57h = 0.576\ x\ 60 = 35 minute$

(ii) a = $\frac{625t}{24}$ = $\frac{625\ x\ 0.576}{24}$ = 15km

(iii) $R_{AB(t=0.576)} = \begin{pmatrix} 7x\ 0.576 \\ -15 + 24\ x\ 0.576 \end{pmatrix} km$

$R_{AB(t=0.576)} = \begin{pmatrix} 4.032 \\ -1.176 \end{pmatrix}$

$\theta = \tan^{-1}\left(\frac{1.176}{4.032}\right) = 16.3^0$

$Direction: E16.3^0S$

(c) length of time for which A is within range, if the visibility of ship B is within 12km

$R_{AB(t=t)} = \begin{pmatrix} 7t \\ -a + 24t \end{pmatrix}$

$= \begin{pmatrix} 7t \\ -15 + 24t \end{pmatrix} km$

$|R_{AB(t=t)}| = 12km$

$(7t)^2 + (-a + 24t)^2 = 144$

625t^2 – 720t + 81 = 0

t = 1.026h or t = 0.126h

Time for which they are in range

= 1.026 – 0.126 = 0.9h

Revision exercise 3

1. 1.At 8am ship A and ship Bare 11km apart with B due west of A. a and B move with a constant velocities (-4i +3j)kh/h and (2i + 4j)km/h respectively. Find the the

(i) least distance between the two ship in the subsequent motion[1.81km]

(ii) time to the nearest minute at which this situation occurs[9.47am]

2. 2. At 7.30am, two ship A and B are 8km apart with B due north of A. A and B move with constant velocities (12j)km/h and (-5i)km/h respectively. Find the

(i) least distance between the two ship in the subsequent motion[3.08km]

(ii) time to the nearest minute at which this situation occurs[8.04am]

3. A and B are two tankers at 13.00hrs, tanker B bas position vector of (4i+8j)km relative to A. A and B move with constant velocities (6i + 9j)km/h and (-3i + 6j)km/h respectively. Find the
   (i) least distance between the two ship in the subsequent motion[6.32km]
   (ii) time to the nearest minute at which this situation occurs[13.40hrs]
4. At 12 noon the position vectors r and velocity vectors v of two ship A and B are as follows
   r_A = (5i + j)km, V_A = (7i + 3j)km/h and r_B = (8i + 7j)km, V_B = (2i –j)km/h
   (i) Assuming velocities do not change , find the least distance between the ships in subsequent motion [2.81]
   (ii) Find the time when their distance of closest approach occur [12.57pm]

5. At a certain time, the position vectors r and velocity vectors V of two ship A and B are as follows

r_A = (3i + j)km, V_A = (2i + 3j)km/h at 11.00am

r_B = (2i - j)km, V_B = (3i + 7j)km/h at 12.00noon

Assuming velocities do not change; find the

(i) The position vector of A at noon [5i + 4j]
(ii) Distance between the ships at 12.00 noon [5.83km]
(iii) The least distance between A and B in the subsequent motion[1.7km]
(iv) Time at which the least separation occurs [1.21pm]

6. At 12 noon, the position vectors r and velocity vectors V of two battle ship A and battle B are as follows

r_A = (13i + 5j)km, V_A = (3i - 10j)km/h

r_B = (3i - 5j)km, V_B = (15i + 14j)km/h

(i) Assuming the velocities do not change, find the least distance between the ships in subsequent motion [4.47km]
(ii) The battle ship has guns with a range of up to 5km, find the length of time during which the cruiser is within range of the battle ships [10minutes]

7. At tine t = 0 the position vectors r and velocity vectors V of two battle ship A and battle B are as follows

$$r_A = \begin{pmatrix}1\\2\\3\end{pmatrix}m, V_A = \begin{pmatrix}-6\\0\\1\end{pmatrix}m/s$$
$$and$$
$$r_B = \begin{pmatrix}4\\-14\\1\end{pmatrix}m, V_A = \begin{pmatrix}-5\\1\\7\end{pmatrix}m/s$$

Assuming velocities do not change, find
(i) The potion vectors of B relative to A at time t seconds
$$\left[\left(\begin{pmatrix}7\\2\\2\end{pmatrix} + \begin{pmatrix}9\\13\\-7\end{pmatrix}t\right)km\right]$$
(ii) The least distance between the ships in the subsequent motion [15.9m]
(iii) The time taken to the closest distance $\left[\frac{25}{33}s\right]$

8. At time t = 0 the position vectors r and velocity vectors V of two battle ship A and battle B are as follows
r_A = (3i + j + 5k)m, V_A = (4i + j – 3k)m/s
r_B = (i - 3j+ 2k) m, V_B = (i + 2j + 2k)m/s
Assuming velocities do not change, find
(i) The position vector of B relative to A at time t second
$$\left[\begin{pmatrix}2+3t\\4-t\\3-5t\end{pmatrix}m\right]$$
(ii) The value of t when A and B are closed $\left(\frac{13}{35}\right)$
(iii) Least distance between A and B [4.917m]

9. At tine t = 0 the position vectors r and velocity vectors V of two battle ship A and battle B are as follows

$r_A = (\beta)m, V_A = \begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix} m/s\ and$

$r_B = (2\beta)m, V_A = \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix} m/s$

Where β is a constant, assuming velocities do not change show that the least distance between the ships in the subsequent motion is $\frac{\beta}{73}$ and their distance of closest approach is $\frac{6\beta\sqrt{2}}{\sqrt{73}}$.

10. A lizard on a wall at point A, has a position vector $r_A = \begin{pmatrix} 65 \\ 40 \\ 0 \end{pmatrix} cm$. At time t= 0 seconds a fly has a position vector $r_F = \begin{pmatrix} 37 \\ 16 \\ 22 \end{pmatrix} cm$ and velocity vector $V_F = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} cm/s$

    If the fly were to continue with this velocity, find the closest distance it would come to the lizard and the value of t when it occurs [$\sqrt{374}m$, 7s]

11. A particle P move with constant velocity (2i + 3j + 8k)m/s passes a point with position vector (6i – 11j + 4k)m. At the same instant particle Q passes through a point whose position vector is (i -2j + 5k)m moving at constant velocity of (3i + 4j- 7k)m/s. Find
    (a) Position of Q relative to P at that instant. [10.344m]
    (b) Shortest distance between the particles [10.32m]
    (c) Time that elapses before the particles are nearest to each other [0.0485s]

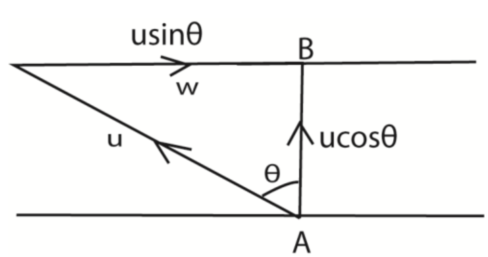
12. Two particles P and Q move with constant velocities (4i + j – 2k)m/s and (6i + 3k)m/s respectively. Initially P is at a point whose position vector is (i – 20j + 21k)m and Q is at a point whose position vector is (i + 3k)m. find
    (a) Time for which the distance between P and Q is least [2.2s]
    (b) Distance of P from the origin at the time when the distance between P and Q is least [28.8m]
    (c) Least distance between P and Q [24.14m]

Crossing a river

There are two cases to consider when crossing a river

Case I: Shortest route

If the water is not still and boatman wishes to cross **directly opposite** to the standing point. In order to cross from point A to point B directly opposite A (perpendicularly), then the course set by the boat must be upstream of the river.



u = speed of the boat in still water

w= speed of running water

At point B: w = usinθ

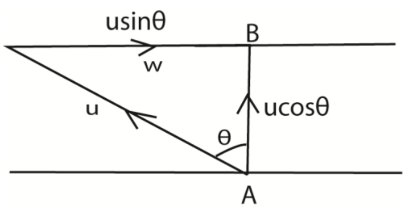
$\theta = \sin^{-1}\left(\frac{w}{u}\right)$

θ = is the direction to the vertical but the direction to the bank is $(90 - \theta)^0$

Time taken = $\frac{AB}{ucos\theta}$

Example 29

A man who can swim at 6km/h in still water would like to swim between two directly opposite points on the river banks of the river 300m flowing at 3km/h. Find the time taken to do this.



AB = 0.3km

$\theta = \sin^{-1}\left(\frac{w}{u}\right) = \sin^{-1}\left(\frac{3}{6}\right) = 30^0$

Time taken = $\frac{AB}{ucos\theta} = \frac{0.3}{6cos30}$

time = 0.058hrs = 3.46minute

He must swim at 300m to AB in order to cross directly and it takes him 3.46minutes.

Alternatively

Using Pythagoras theorem

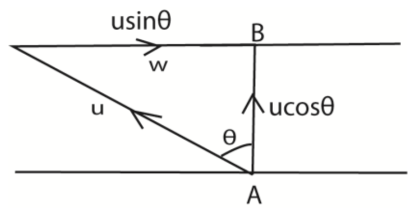
$6^2 = 3^2 + V_{AB}^2$

$V_{AB} = \sqrt{36 - 9} = 5.1962$km/h

Time = $\frac{AB}{V_{AB}} = \frac{0.3}{5.1963} = 0.058$hrs

Example 30

Two points A and B are on opposite banks of a river flowing at $\frac{5}{6}ms^{-1}$. A man who can swim at $\frac{25}{18}ms^{-1}$ in still water would like to swim directly from A to B. Find the width of the river if he takes 2minutes to cross the river.



$\theta = \sin^{-1}\left(\frac{w}{u}\right) = \sin^{-1}\left(\frac{5/6}{25/18}\right) = 36.87^0$

Time taken = $\frac{AB}{ucos\theta} = \frac{0.3}{\frac{25}{18}cos36.87}$

AB = 133.333m

Alternatively: using Pythagoras theorem

$\left(\frac{25}{18}\right)^2 = \left(\frac{5}{6}\right)^2 + V_{AB}^2$

$V_{AB} = 1.1111m$

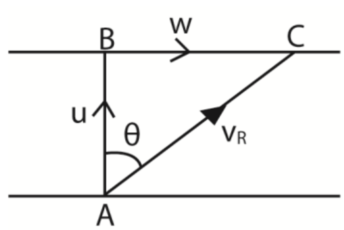
Time taken = $\frac{AB}{V_{AB}}$

$2 \times 60 = \frac{AB}{1.1111}$

AB = 133.333m

Case II: the shortest time taken/ as quickly as possible

If the boatman wishes to cross the river as quickly as possible, then he should steer his boat directly from A to B as shown. The river pushes the boat down stream



Time taken to cross the river, t = $\frac{AB}{u}$

Distance covered downstream = wt

Where w is the velocity at which the river flows

u is the velocity of the boat

Or

Distance downstream = $w\frac{AB}{u}$

tanθ = $\frac{w}{u}$ or θ = $\tan^{-1}\frac{w}{u}$

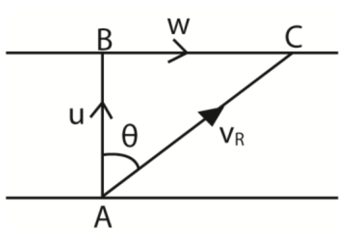
The resultant velocity downstream V_R

$$V_R = \sqrt{w^2 + u^2}$$

Example 31

A man who can swim at $2ms^{-1}$ in still water wishes to swim across a river 120m wide as quickly as possible. If the river flows at $0.5ms^{-1}$, find the time the man takes to cross far downstream he travels.

Solution



u = $2ms^{-1}$, w = $0.5ms^{-1}$, AB = 120m

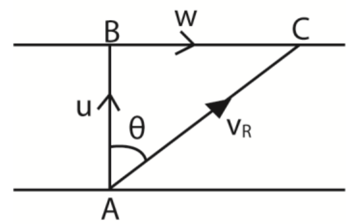
t = $\frac{AB}{u} = \frac{120}{2} = 60s$

Distance = wt = 0.5 x 60 = 30m

Example 32

A boat can travel at $3.5ms^{-1}$ in still water. A river is 80m wide and the current flows at $2ms^{-1}$, calculate

(a) the shortest time to cross the river and the distance downstream the oat is carried.

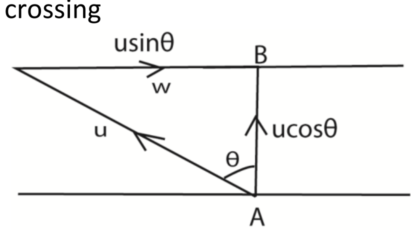


u = $3.5ms^{-1}$, w = $2ms^{-1}$, AB = 80m

t = $\frac{AB}{u} = \frac{80}{3.5} = 22.95s$

Distance, BC = wt = 2 x 22.95 = 45.8m

(b) the course that must be set to point exactly opposite the starting point and time taken for crossing



u = $3.5ms^{-1}$, w = $2ms^{-1}$, AB = 80m

course, θ = $\sin^{-1}\frac{w}{u} = \sin^{-1}\frac{2}{3.5} = 34.8^0$

Time for crossing = $\frac{80}{3.5cos34.8}$= 27.8s

Revision exercise 4

1. A man who can row at $0.9ms^{-1}$ in still water wishes to cross a river of width 1000m as quickly as possible. If the current flows at a rate of $0.3ms^{-1}$. Find the time taken for journey. Determine the direction in which he should point the boat and position of the boat where he lands. [111.11s, 71.57^0 to the bank, 333.33 downstream]
2. A man swims at $5kmh^{-1}$ in still water. Find the time it takes the man to swim across the river250m wide, flowing at $3kmh^{-1}$, if he swims so as to cross the river
   (a) the shortest route [225s]

(b) in the quickest time[180s]

3. A boy can swim in still water at $1ms^{-1}$, he swims across the river flowing at $0.6ms^{-1}$ which is 300m wide, find the time he takes
   (a) if he travels the shortest possible distance [375s]
   (b) if he travels as quickly as possible and the distance downstream, [300s, 180m]

Linear momentum

Linear momentum is a product of the body's mass and its velocity.

The S1 unit of momentum is $kgms^{-1}$

When a force F is applied to a body, it changes the body's velocity from u to v, the size of the force and the time for which it acts on a body.

From F = ma

$a = \frac{v-u}{t}$

$F = \frac{m(v-u)}{t}$

Ft = m(v-u)

Impulse of force is the product of force and the duration of its action or impulse is the change in momentum of the body which is acted on by the force

Example 33

A body of mass 3kg initially moving with a velocity of $5ms^{-1}$ is acted on by a horizontal force of 15N for 2s. Find the impulse and final speed.

Solution

Impulse = Ft

= 15 x 2

= 30N

Impulse = change in momentum

30 = m(v-u)

4. A boy wishes to swim across a river 100m wide as quickly as possible. The river flows at $3kmh^{-1}$ and the boy can swim at $4kmh^{-1}$ in still water. Find the time that the boy takes to cross the river and how far downstream he travels. [90s, 75m]

30 = 3(v-5)

$v = 15ms^{-1}$

Example 34

A tennis ball has a mass of 0.07kg. it approaches a racket with a speed of $5ms^{-1}$ and bounces off and returns to the way it come with a speed of $4ms^{-1}$. The ball is in contact with the racket for 0.2 seconds. Calculate

(i) The impulse given to the ball
(ii) The average force exerted on the ball by the racket.

Solution

(i) Impulse = Ft = m(v-u)

= 0.07(-4-5)

= -0.63Ns

(ii) $F = m\left[\frac{v-u}{t}\right] = \frac{0.63}{0.2} = 3.15N$

Collisions and principles of conservation of linear momentum

When two or more bodies collide, the total momentum of the system is conserved provided there is no external force on the sysytem.

Consider a body of mass m1 moving with a velocity u1 to the right. Suppose the body makes a head on collision with a nother body of mass m2 moving with velocity u2 in the same direction

Let v_1 and v_2 be the velocities of the 2 bodies respectively after collision

Let F1 be the force exerted on m_2 by m_1 and F_2 the force exerted on m_1 by m_2 using Newton's 2nd law.

$F_1 = m_1\left(\frac{v_1 - u_1}{t}\right)$, $F_2 = m_2\left(\frac{v_2 - u_2}{t}\right)$, where t is the time of collision

Using Newton's third law

$F_1 = -F_2$

$m_1\left(\frac{v_1 - u_1}{t}\right)$, = $-m_2\left(\frac{v_2 - u_2}{t}\right)$,

$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$

Hence, total momentum before collision = total momentum after collision, in other words momentum is conserved.

When two bodies collide, there is a short period of contact during which each exerts a force on each other at that instant, the force which each exert on each other is equal and opposite.

Types of collision

Collisions can be categorized as inelastic collision, perfectly inelastic, elastic or perfectly elastic collisions.

Elastic or perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
Kinetic energy is conserved	Kinetic energy is not conserved	Kinetic energy is not conserved
Linear momentum is conserved	Linear momentum is conserved	Linear momentum is conserved
Bodies separate after collision, e.g. collision of gas molecules	Bodies separate after collision e.g. a ball bouncing from a concrete floor	Bodies stick together and move with a common velocity. E.g. a trailer colliding with a saloon car.

Elastic collision

Momentum is conserved

$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ……………………(i)

Kinetic energy is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1(u_1^2 - v_1^2) = m_2(u_2^2 - v_2^2)$ ………….. (ii)

Equation (i) ÷ (ii)

$\frac{m_1(u_1 - v_1)}{m_1(u_1^2 - v_1^2)} = \frac{m_2(u_2 - v_2)}{m_2(u_2^2 - v_2^2)}$

$$\frac{(u_1-v_1)}{(u_1-v_1)(u_1+v_1)} = \frac{(u_2-v_2)}{(u_2-v_2)(u_2+v_2)}$$

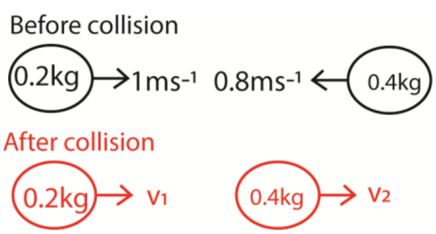
$$\frac{1}{(u_1+v_1)} = \frac{1}{(u_2+v_2)}$$

$$(u_1 - u_2) = (v_2 - v_1)$$

Example 35

A 200g block moves to the right at a speed of $100cms^{-1}$ and meets a 400g block moving to the left with a speed of $80cms^{-1}$. Find the final velocity of each block if the collision is elastic.

Solution



$(v_2 – v_1) = -(-0.8 – 1)$

$(v_2 – v_1) = -1.8$ …………………………………. (i)

Using conservation of momentum

$m_1v_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.2 \times 1) + (-0.4 \times 0.8) = 0.2v_1 + 0.4v_2$

$-0.12 = 0.2v_1 + 0.4v_2$

$-0.6 = v_1 + 2v_2$ …………(ii)

Eqn (i) and Eqn (ii)

$v_2 = -1.8 + v_1$

$-0.6 = v_1 + 2(-1.8 + v_1)$

$V_1 = 1ms^{-1}$

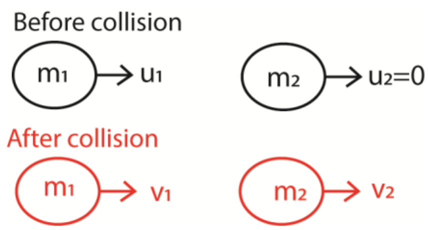
$V_2 = -0.8ms^{-1}$

Example 4

A particle of mass m_1, travelling with velocity u_1 makes a perfectly elastic collision with a stationary particle of mass m_2. After the collision, the first particle moves a velocity v_1 while the second particle moves in the same direction with velocity, v_2. Show that

$$v_2 = \frac{2m_1u_1}{m_1+m_2} \text{ and } v_1 = \frac{(m_1-m_2)u_1}{(m_1+m_2)}$$

Solution



From the principle of conservation of momentum

Momentum before collision = momentum after collision

$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$

$v_1 = \frac{m_1u_1 - m_2v_2}{m_1}$ ………………………………….(i)

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2 x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2 \text{ …… (ii)}$$

Substituting equation (i) into equation (ii)

$$m_1u_1^2 = m_1\left[\frac{m_1u_1 - m_2v_2}{m_1}\right]^2 + m_2v_2^2$$

$$m_1^2u_1^2 = m_1^2u_1^2 - 2m_1u_1m_2v_2 + m_2^2v_2^2 + m_1m_2v_2^2$$

$$2m_1u_1m_2v_2 = m_2^2v_2^2 + m_1m_2v_2^2$$

Dividing through by m_2v_2

$$2m_1u_1 = v_2(m_1 + m_2)$$

$$v_2 = \frac{2m_1u_1}{(m_1+m_2)}$$

From the principle of conservation of momentum

Momentum before collision = momentum after collision

$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$

$$m_2v_2 = m_1(u_1 - v_1)$$

$$v_2 = \frac{m_1((u_1-v_1)}{m_2}$$

$v_2^2 = \frac{m_1^2}{m_2^2}(u_1 - v_1)^2$ ....(i)

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2 x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$$

$$v_2^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2) \text{ ..... (ii)}$$

Equation (i) and (ii)

$\frac{m_1^2}{m_2^2}(u_1 - v_1)^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2)$

$\frac{m_1}{m_2}(u_1 - v_1) = (u_1 + v_1)$

$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1$

$u_1(m_1 - m_2) = v_1(m_2 + m_1)$

$$v_1 = \frac{u_1(m_1 - m_2)}{(m_2 + m_1)}$$

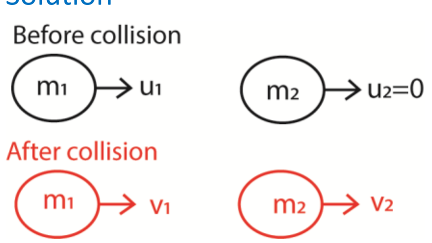
Example 36

A particle P of mass m_1, travelling with a speed u_1 makes a head on collision with a stationary particle Q of mass m_2. If the collision is elastic and speed of P and Q after impact are v_1 and v_2 respectively show that if b = $\frac{m_1}{m_2}$

(i) $\frac{u_1}{v_1} = \frac{b+1}{b-1}$

(ii) $\frac{v_2}{v_1} = \frac{2b}{b-1}$

Solution



From the principle of conservation of momentum

Momentum before collision = momentum after collision

$$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$$

$$v_2 = \frac{m_1(u_1 - v_2)}{m_2}$$

$$v_2 = b(u_1 - v_1) \text{ ....(i)}$$

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2 x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$$

$$v_2^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2)$$

$$v_2^2 = b(u_1^2 - v_1^2) \text{....... (ii)}$$

Squaring equation (i)

$v_2^2 = b^2(u_1 - v_1)^2$ ....(iii)

Equation (ii) and (iii)

$b(u_1^2 - v_1^2) = b^2(u_1 - v_1)^2$

$(u_1 + v_1) = b((u_1 - v_1)$

$u_1(b-1) = v_1(b+1)$

$$\frac{u_1}{v_1} = \frac{(b+1)}{(b-1)} \text{ .... (iv)}$$

(ii) Consider $\frac{u_1}{v_1} = \frac{(b+1)}{(b-1)}$

$$u_1 = v_1\frac{(b+1)}{(b-1)} \text{ ....(v)}$$

Substitution Eqn (v) into (i)

$$v_2 = b\left(v_1\frac{(b+1)}{(b-1)} - v_1\right)$$

Dividing through by v_1

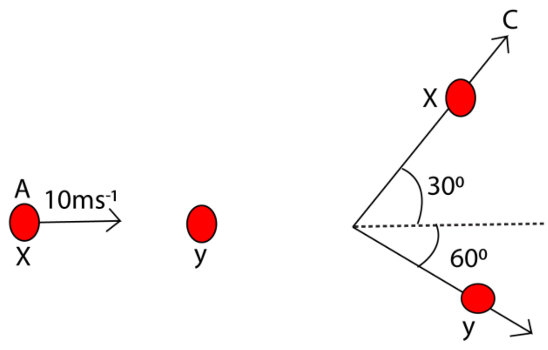
$$\frac{v_2}{v_1} = \frac{2b}{b-1}$$

Example 6

An object X of mass m moving with velocity $10ms^{-1}$ collides with a stationary object Y of equal mass. After collision, x, moves with speed u at an angle 30^0 to its initial direction, while Y moves with a speed of V at an angle 90^0 to the new direction of x.

(i) Calculate the speed u and v.

(ii) Determine whether the collision is inelastic or not.



Before collision After collision

Consider horizontal momentum

From the principle of conservation of conservation of momentum

m x 10 + m x 0 = m x ucos 30^0 + mvcos 60^0

$$10 = u\frac{\sqrt{3}}{2} + \frac{v}{2} \text{ ....(i)}$$

Consider vertical momentum

m x 0 + m x 0 = $\frac{u}{2} - v\frac{\sqrt{3}}{2}$

u = v$\sqrt{3}$ ....(ii)

Putting Eqn (ii) into Eqn (i)

20 = (v$\sqrt{3}$ x $\sqrt{3}$ + v

20 = 4v

v = 5ms^{-1}

u = 5$\sqrt{3}$ ms^{-1}

(ii) Kinetic energy before =

$\frac{1}{2}\ x\ m\ x\ (10)^2$ = 50m J

Kinetic energy after collision =

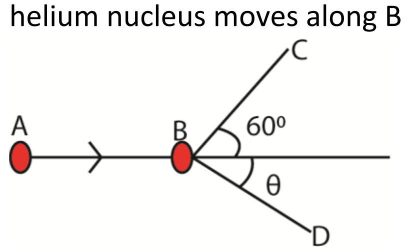
$\frac{1}{2}\ x\ m\ x\ (5)^2 + \frac{1}{2}\ x\ m\ x\ (5\sqrt{3}\ )^2$

=50m J

Hence the collision is elastic

Exercise 5

1. A bullet of mass 300g travelling at a speed of 8ms^{-1}hits a body of mass 450g moving in the same direction as the bullet at 1.5ms^{-1}. The bullet and the body move together after collision. Find the loss in kinetic energy. [Ans. 3.8025J]
2. A particle A of mass 4kg is incident with velocity V on a stationary helium nucleus B of mass 4kg. After collision, A moves in direction BC with velocity v/2 where BC makes an angle of 60^0 with the initial direction of AB and the helium nucleus moves along BD.



Calculate the angle made in direction AB and the velocity of the helium along BD. [θ = 30^0, velocity = $\frac{v\sqrt{3}}{2}$]

3. (a) State Newton's laws of motion
   (b) Use Newton's laws of motion to show that when two bodies collide, their momentum is conserved.
   (c) Two balls P and Q travelling in opposite direction with speeds 6ms^{-1} and 15ms^{-1} inelastic collision. If the masses of P and Q are 8kg and 5kg respectively, find the
      (i) final velocity [Ans. 2.08ms^{-1}]
      (ii) change in kinetic energy [Ans. 28.03J]
   (d) (i) What is an impulse of force?
      (ii) Explain why a long jumper should normally land on sand.
   [The force exerted on a long jumper on coming to rest is given by F = change in momentum over time taken. Since the change in momentum is constant, it Implies

that if the time taken when coming to rest is increased, then the force exerted on the knees of the jumper reduces. Sand increases the time taken for the jumper to stop reducing the damaging force to the knee.]

4. (a)(i) State the law of conservation of linear momentum
   (ii) Use Newton's laws to derive the above.
   (b) Distinguish between elastic and inelastic collisions
   (c) An object X of mass m moving with velocity 10ms-1 collides with a stationary object Y of equal mass. After collision, x, moves with speed u at an angle 30^0 to its initial direction, while Y moves with a speed of V at an angle 90^0 to the new direction.
   (i) Calculate the speed u and v. [Ans. u = $5\sqrt{3}$ ms^{-1}, v =$5ms^{-1}$]
   (ii) Determine whether the collision is inelastic or not. [Ans. kinetic energy on both sides = (50M)J, since kinetic energy is conserved, the collision is elastic]
5. (a) (i) Define linear momentum
   (ii) State the law of conservation of linear momentum.
   (iii) Show that in (ii) above follows Newton's laws of motion.
   (iv) Explain why, when catching a fast moving ball, the hands are drawn back while the ball is being brought to rest.
   (b) A car of mass 100kg travelling at uniform velocity of $20ms^{-1}$ collides perfectly inelastically with a stationary car of mass 1500kg. Calculate the loss in kinetic energy of the car as a result of collision. [Ans. 168000J]

Newton's laws of motion

First Newton's law of motion.

A body remains in its state of rest or uniform motion in straight line unless an external force acts on it.

This law, also called the principle of inertia. Inertia is reluctance of a body to start moving once at rest or stop moving once in motion.

Inertia depends on the mass of the body thus the mass is a measure of inertia.

Second Newton's Law of motion

The rate of change of momentum is directly proportional to the magnitude of the applied force producing it and takes place in the reaction of the applied force.

Momentum is the product of mass of the body and its velocity.

$F \propto \frac{mV - mU}{t}$

mV- mU = change in momentum

$F = \frac{m(V-U)}{t}$

But $\frac{V-U}{t} = a$

F = kMa, where k is a constant

A newton is a force which gives a mass of 1 kg an acceleration of $1ms^{-2}$

F = 1N

M = 1kg

$a = 1ms^{-1}$

1 = k x 1 x 1

k = 1

F = Ma

Third Newton's law of motion

For every action, there is an equal and opposite reaction.

For example 37

A block of mass 2kg is pushed along a table with constant velocity by force of 5N. When the push is increased to 9N, what is the resultant force and acceleration?

Resultant force F = 9 - 5 = 4N

But

F = ma

4 = 2a

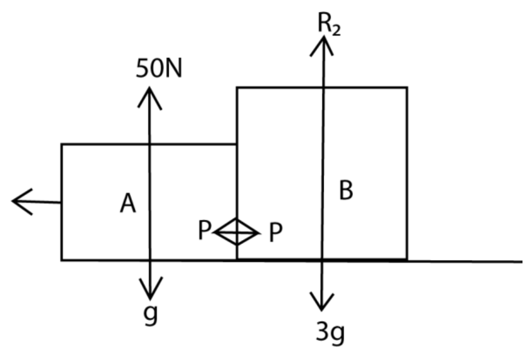
$a = 2ms^{-2}$

Example 38

Two blocks, A of mass 1kg and B of mass 3kg, are side by side and with contact with each other. They are pushed along the smooth flow under the action of a constant force 50N applied to A.

Find

i) The acceleration of the blocks
ii) The force exerted on B by A.



(a) F =ma

50 = (1+3)a

Acceleration, $a = 12.5ms^{-2}$

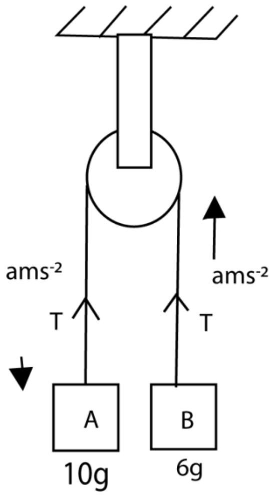
(b) Using A

50 – p = 1 x 12.5

p = 50 – 12 = 37.5N

Example 39

A light cord connects 2 objects of masses 10kg and 6kg respectively over a light frictionless pulley. Find the acceleration and tension in the cord.



Body A

10g –T = 10a ……………………………………(i)

Body B

T – 6g = 6a …………………………………… (ii)

Eqn (i) – eqn (ii)

4g = 16a

a = 2.45ms^{-2}

From equation (ii)

T = 6 x 2.45 + 6 x 9.81 = 73.6N

Example 40

Explain why the tension in a cable of a lift is different when the lift is accelerating upwards from when it is accelerating downwards.

Solution

The tension is greater when the lift is ascending because **it has to overcome both the weight of the lift and the force due to its acceleration upwards**.

Explanation

When the Lift is Ascending:

1. **Gravity's Force**: The weight of the lift and its contents (passengers, cargo) create a downward force due to gravity. This force is equal to the mass (m) of the lift multiplied by the acceleration due to gravity (g), resulting in a force F=mg
2. **Upward Acceleration**: When the lift ascends, it needs to overcome gravity and accelerate upwards. This requires an additional force. The total tension (T) in the cable must provide this extra force to achieve upward acceleration (a). Hence, the total tension is given by T=m(g+a)

When the Lift is Descending:

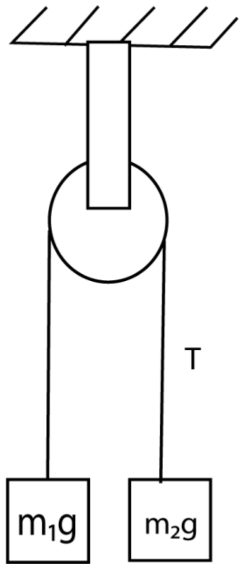
1. **Gravity's Force**: Similar to ascending, the lift and its contents exert a downward force due to gravity, F=mg
2. **Downward Acceleration**: When the lift descends, it accelerates downwards. However, the cable still needs to control this descent, effectively reducing the tension compared to when it is stationary or ascending. In this case, the tension (T) in the cable is calculated as T=m(g−a)

Example 41

(a) Two particles of mass m1 and m2 are connected as shown below by a light inextensible string passing over a smooth fixed pulley. If m1>m2 show that the acceleration of the system is given by

$$a = g\left[\frac{m_1 - m_2}{m_1 + m_2}\right]$$

(b) Find the expression for the tension in the string



Body m_1

$m_1g - T = m_1a$ ……………………………….(i)

Body m_2

$T - m_2g = m_2a$ ……………………… (ii)

Eqn (i) – eqn (ii)

$(m_1 - m_2)g = (m_1 + m_2)a$

$a = g\left[\frac{m_1 - m_2}{m_1 + m_2}\right]$

(b) From equation (ii)

$T = m_2 \times g\left[\frac{m_1 - m_2}{m_1 + m_2}\right] + m_2g$

$= m_2g\left(\left[\frac{m_1 - m_2}{m_1 + m_2}\right] + 1\right)$

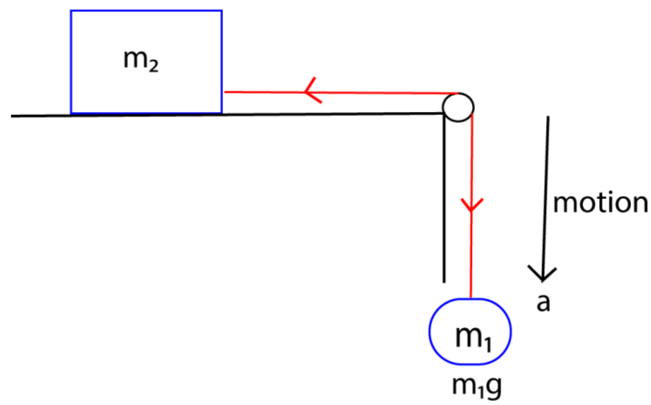
$= \left[\frac{2gm_1m_2}{m_1 + m_2}\right]$

Example 42

A mass m_2 lies on a smooth table connected by a light inextensible string passing over a small smooth pulley at the table to a mass, m_1, hanging freely.

(a) What is the acceleration?

Solution



Body m_1

$m_1g - T = m_1a$ ……………………………….(i)

Body m_2

$T - 0 = m_2a$ ………………………………………. (ii)

Substitution Eqn (ii) into eqn (i)

$m_1g - m_2a = m_1a$

$a = \left[\frac{m_1g}{m_1 + m_2}\right] ms^{-2}$

(b) Find the expression for the tension in the string

From equation (ii)

$T = m_2\left[\frac{m_1g}{m_1 + m_2}\right]$

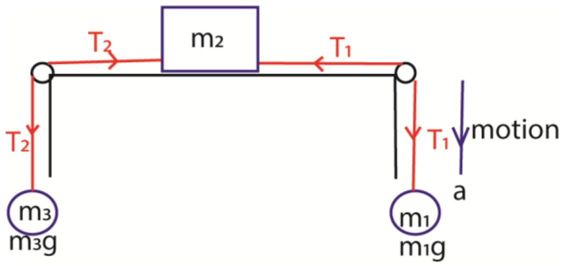
$= \left[\frac{m_1m_1g}{m_1 + m_2}\right]$

Example 43

The diagram below shows masses m_1, m_2 and m_3 connected by light inextensible string such that m_1 and m_3 hang vertically while m_2 lies on horizontal smooth surface with m_1 greater than m_3. Show that the acceleration due to gravity

$a = g\left[\frac{m_1 - m_3}{m_1 + m_2 + m_3}\right]$

Solution



Body m_1

$m_1g - T_1 = m_1a$ ....(i)

Body m_3

$T_2 - m_3g = m_3a$ .... (ii)

Body m_2

$T_1 - T_2 = m_2a$ .... (iii)

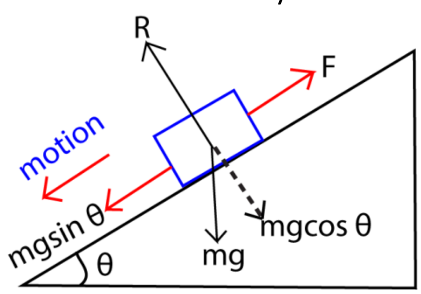
Eqn (i) + Eqn (ii) + Eqn (iii)

$m_1a + m_2a + m_3a = m_1g - m_2g$

$a = g\left[\frac{m_1 - m_3}{m_1 + m_2 + m_3}\right]$

Motion of a particle on an incline

Let the mass of the particle be m and R be the normal reaction. If the plane makes an angle θ to the horizontal, then the forces which act on the body are shown below;



Motion of a particle on inclined

From the diagram

$R = mg\cos\theta$

Result force = $mg\sin\theta - F$

$ma = mg\sin\theta - F$

if the system is at rest where a = 0

$m(0) = mg\sin\theta - F$

$F = mg\sin\theta$

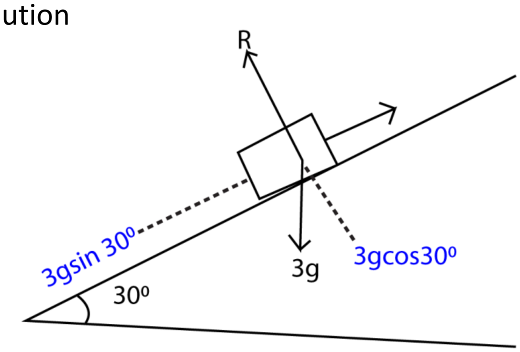
For example 44

A body of 3kg slides down a plane which is inclined at 30^0 to the horizontal. Find the acceleration of the body if

(a) the plane is smooth

(b) there is a frictional resistance of 9N.

Solution



R is the normal reaction

(a) F = ma

$3g\sin 30^0 = 3a$

$a = 4.9ms^{-2}$

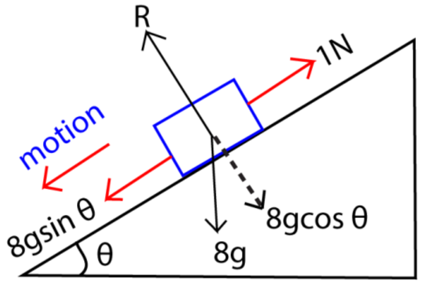
b) $F = 3g\sin 30^0 - 9 = 3a$

$a = 1.9ms^{-2}$

Note friction force acts in the opposite direction of motion.

Example 45

A body of mass 8kg is released from rest on a surface of a plane. If the resistance to the motion is 1N acting up the plane and the slope of the plane is 1 in 40. Calculate the acceleration of the body down the plane and the speed acquired in 6 seconds after the release if he resistance to the motion is N



From ma = mgsinθ – friction

But sinθ = $\frac{1}{40}$

8a = 8 x 9.81 x $\frac{1}{40}$ – 1

a = 0.12ms^{-2}

Using

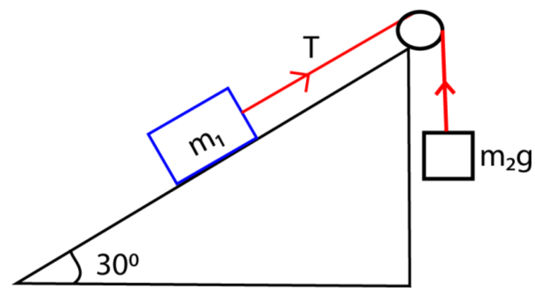
v = u + at

= 0 + 0.12 x 6 =0.72ms^{-1}

Example 16

A block m_1 is connected to another m_2 by a light inextensible string which passes over as smooth pulley. If the inclined plane is smooth and m_2>m_1, show that the acceleration of the system is given by

a = $\frac{(2m_2 - m_1)g}{2(m_1+m_2)}$



Since m_2>m_1, the acceleration a acts downwards

Consider m_2

m_2a = m_2g –T ……………………… (i)

Consider m_1

M_1a = T – $m_1$gsin30

= T- $\frac{m_1 g}{2}$ …………………… (ii)

Eqn (i) + Eqn (ii)

M_2a + m_1a = m_2g - $\frac{m_1 g}{2}$

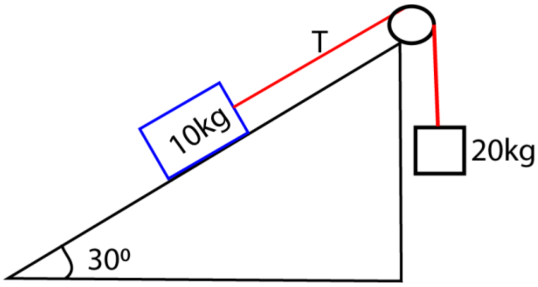
a(m_2+m_1) = $\frac{g(2m_2 - m_1)}{2}$

a = $\frac{g(2m_2 - m_1)}{2(m2+m1)}$

Exercise 6

1. A body A rests on a smooth horizontal table. Two bodies of mass 2kg and 10kg hanging freely are attached to opposite ends of A by light inextensible strings which pass over smooth pulleys at the edges of the table. The two strings are taut when the system is released from rest; it accelerates at 2ms^{-2}. Find the mass of A
[Answer 27.24kg]
2. A rectangular block of mass 10kg is pulled from rest a long a smooth inclined plane by a light inelastic string which passes over a frictionless pulley and carries a mass of 20kg as shown below with the plane inclined at angle

of 30^0 to the horizontal.



Find (i) the acceleration of the block

(ii) the tension, T, in the string

[Ans. A = 4.905ms^{-2}, T =98.1N

Thank you

Dr. Bbosa Science