SENIOR FIVE TERM 1

TOPIC 2/6: STATICS

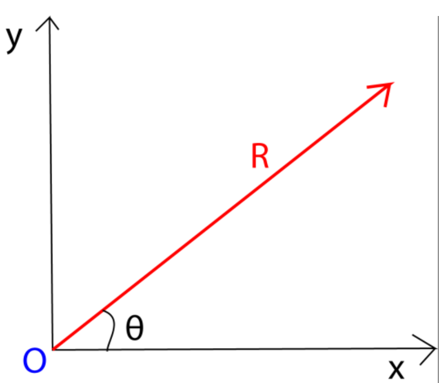
Scalar and vector quantities

Scalar quantities are physical quantities which have only magnitude; for, example speed, mass, volume, energy, time and temperature.

Vector quantities are physical quantities which have both magnitude and direction, for, example acceleration, momentum, weight, velocity, magnetic flux, force, pressure and displacement.

Resultant and composition of vectors or resolving vectors

Consider a force R acting of particle O at an angle θ to the horizontal shown the figure below



The horizontal component, x, of the vector = Rcos θ.

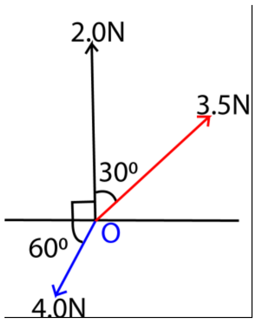
The vertical component, y, of the vector R = Rsin θ

It Implies that

R = $\sqrt{x^2 + y^2}$ and θ = $\tan^{-1}\left(\frac{y}{x}\right)$

Example 1

Three forces of 3.5N, 4.0N and 2N, act on point O as shown in the figure below. Find the resultant force.



Solution

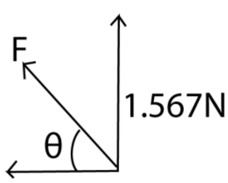
Magnitude of force

F_x = 2.0cos 90 + 3.5cos 60 – 4.0cos 60 = -0.25N

F_y = 2.0sin 90 + 3.5sin 30 – 4.0sin 60 = 1.567N

Resultant force , F =$\sqrt{-0.25^2 + 1.567^2}$ = 1.5868N

Direction of resultant force



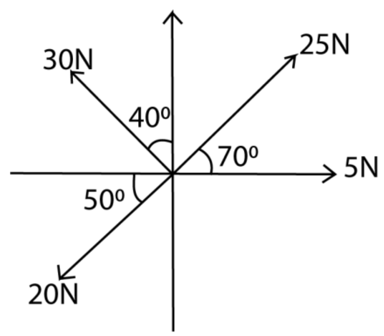
θ = $\tan^{-1}\left(\frac{1.567}{0.25}\right)$ = 80.9^0

The resultant force is 1.5868N acting in the direction of 80.94^0 to the horizontal.

Example 2

(i) Define vector and scalar quantities and give an example each.

(ii) A body M of mass 6k is acted on by the forces of 5N, 20N, 25N and 30N as shown in the figure below. Find the acceleration of M



Solution

F_x = -30cos 50 + 25cos 70 + 5cos 0 – 20 cos50 = - 18.59N

F_y =30sin 50 + 25sin 70 + 5 sin 0 – 20sin50 = 31.15N

Resultant F = $\sqrt{F_x^2 + F_y^2}$ = $\sqrt{(-18.59^2 + 31.15^2)}$ =36.28N

But F = ma

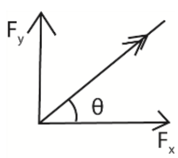
a = $\frac{F}{m} = \frac{36.28}{6} = 6.046ms^{-2}$

Example 3

Four forces of ai + (a-1)j, 3i + 2aj, 5i -6j, and –i -2j act on a particle. The resultant forces make an angle of 45^0 with horizontal. Find a. Hence determine the magnitude of the resultant force.

$R = \begin{pmatrix} a \\ a-1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2a \end{pmatrix} + \begin{pmatrix} 5 \\ -6 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

= $\begin{pmatrix} a+7 \\ 3a-9 \end{pmatrix}$



$\frac{a+7}{3a-9} = \tan^{-1}(45^0)$ = 1

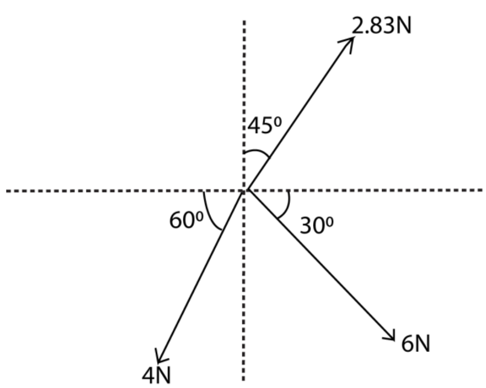
a + 7 = 3a – 9, a = 8

$R = \begin{pmatrix} a+7 \\ 3a-9 \end{pmatrix} = \begin{pmatrix} 8+7 \\ 3 \times 8 - 9 \end{pmatrix} = \begin{pmatrix} 15 \\ 15 \end{pmatrix}$

$|R| = \sqrt{15^2 + 15^2}$ = 21.21N

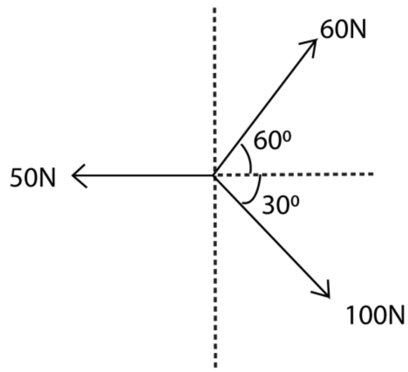
Exercise 1

1. Find the resultant force in the figure below



[Ans. 6.85N at angle θ =40.6^0 to the horizontal]

2. Three forces as shown below act on a body of 5.0kg. Find the acceleration of the body.



[Ans. 13.33ms^{-2}]

1. Find the resultant of each of the following forces
   (a) (6i + 2j)N, (-5i + j)N, (3i – 3j)N. [(4i)N]
   (b) (2i + 4j)N, (3i -5j), (6i +2j)N, (-7i – 7j)N. [(4i -6j)N]
   (c) (2i + 3j -7k)N, (2i + 5k)N, (3j + 4k)N. [(4i + 6j + 2k)N]
2. The resultant of forces (5i + 7j), (ai + bj) and (bi –aj)N is a force (11i + 5j)N. Find a and b. [a = 4, b = 2]
3. Find the magnitude and direction of the resultant of each of the following;
   (a) (-2i + 5j)N, (I + 2j)N. [7.07N at 98.1^0]
   (b) (6i + 2j)N, (4i – 3j)N. [10.05N at 354.3^0]
   (c) (3i + 2j), (-5i + j)N. [3.61N at 124^0]
4. A force of magnitude 50N acts on a body in the direction 24i + 7j. Find the force. [(48i + 14j)]
5. Two forces F_1 and F_2 have magnitude αN and βN and act in the direction i -2j and 4i + 3j respectively. Given that the resultant of F_1 and F_2 is (48i + 14j). Find the magnitude of αN and βN. [α = $8\sqrt{5}$N and β = 50N]
6. If a = 3i + 4j, b = 4i + 20j and c = 5i – 19j; find the
   (i) resultant of a and b [(7i + 24j)]
   (ii) resultant of a and c [(8i – 15j)]
   (iii) vector is parallel to a and has magnitude of 15 unit [(9i + 12j)]
   (iv) vector parallel to (a + b) and has a magnitude of 100 units [(28i + 96j]
7. If a = 2i + 5j, b = -7i + 7j and 14i. Find the;
   (i) resultant of a and b [(-5i + 12j)]
   (ii) resultant of a, b and c [(9i + 12j)]
   (iii) |b| [$7\sqrt{2}$]
   (iv) |a + b + c| [15units]
8. If a = i -3j + 2k, b = 5i + 4j and c = 3i + j + 4k. Find the
   (i) resultant of a and b [(6i + j + 2k)]
   (ii) resultant of a, b and c. [(9i + 2j + 6k)
   (iii) |a| [$\sqrt{14}$]
   (iv) |a + b + c|[11units]
9. If a = 2i + 7j + 7k, b = 6i -3j + 2k and c = -4j -3k. find the
   (i) resultant a and b [8i +4j + 9k]
   (ii) resultant a and c [2i + 3j + 4k

Moments

This is a product of the **force** multiplied by the perpendicular distance between its line of action and the axis of rotation.

The moment is also known at turning force or torque

Moment (Nm) = Force (N) x perpendicular distance (m)

A couple

Two forces that are equal in magnitude but opposite in direction (and acting along parallel lines), thus creating the turning effect of a torque or moment.

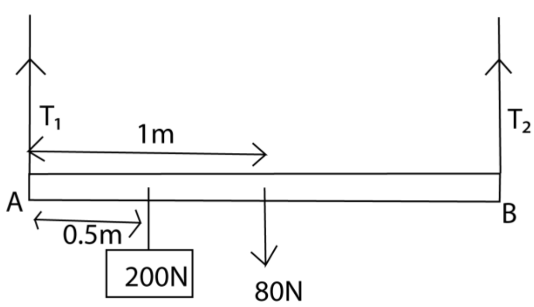
Principle of moments

For an object to be in equilibrium (with no movements about a turning point), the sum of anticlockwise moments is equal to the sum of clockwise moments.

Examples 4

A rod mass 8kg and 2m long is balance horizontally by two inextensible string tied to the end A and B of the rod when a mass of 20kg hangs 0.5m from A. Find the tensions in the strings at A and B. [take g = $10ms^{-2}$]

Solution



The sum of moments of initial at A = 0

$0 = 0.5 \times 200 + 1 \times 80 - T_2 \times 2$

$T_2 = 90$

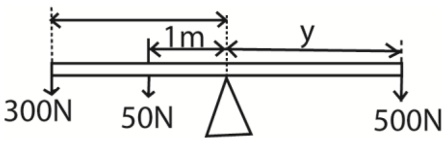
But $T_1 + T_2 = 200 + 80$

$T_1 + 90 = 280$

$T_1 = 190N$

Example 5

Given the diagram below. Find the value of y

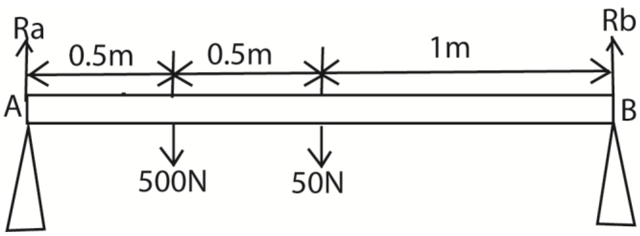


500 x y = (300 x 3) + (50 x 1)

y = $\frac{950}{500} = 1.9m$

Example 6

A uniform beam of weight 50N and length 2m rests horizontally on two supports pivoted at each end. A load of weight 500N is placed 0.5m from one end. Find the reaction on each support.



B: Ra x 2 =50 x 1 + 500 x 1.5

2Ra = 50 + 750

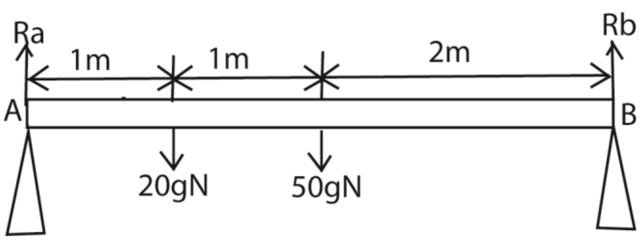
Ra = 400N

Also Ra + Rb = 500N + 50 N

Rb = 550N – 400N = 150N

Example 7

A uniform beam of mass 50kg and length 4m rests horizontally on two supports pivoted at each end. A load of 20kg is placed 1m from one end. Find the reaction on each support



B: Ra x 4 =50g x 2 + 20g x 3

4Ra = 100g + 60g = 160 x 9.8

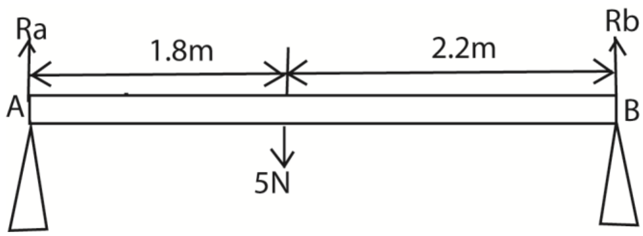
Ra = 392N

Also Ra + Rb = 500N + 50 N

Rb = 20gN + 50gN – 392N = 294N

Example 8

A non- uniform beam AB of length 4m has its weight 5N acting at a point 1.8m from end A. The beam rests horizontally on two supports pivoted at each end. Find the reaction on each support.



B: Ra x 4 =5 x 2.2 =11

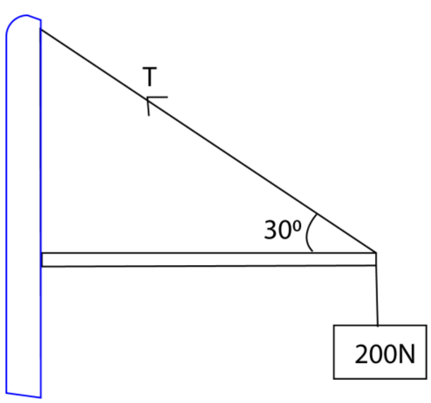
Ra = 2.75N

Also Ra + Rb = 5N

Rb = 5N – 2.75N = 2.25N

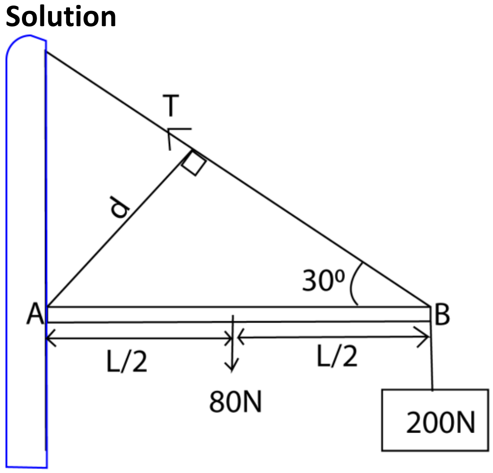
Example 9

A force of 200N hangs on a uniform rod of weight 80N and held at equilibrium by a string as shown in the figure below.



Calculate the tension in the string

Solution



Please find free notes, tests and marking guides of physics, chemistry, biology and mathematics from:

Taking moments at A

T x d = 80 x $\frac{L}{2}$ + 200L

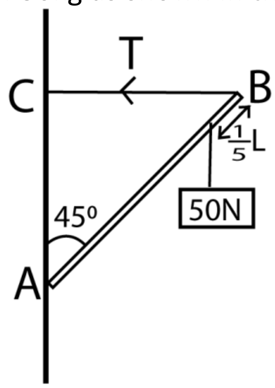
But d = L sin 30

TLsin 30 = 240L

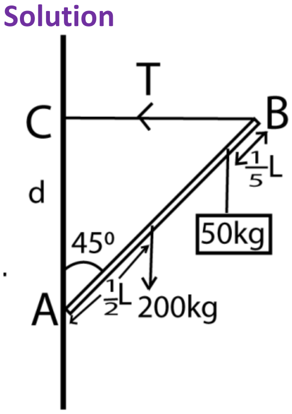
T = 480N

Example 10

A rod of length L weighing 200kg is held by a horizontal string BC in equilibrium with a mass of 50kg as shown in the diagram below. Find the tension T.



Solution



Moments about A

T x d = (200 x 9.8) x $\frac{1}{2}L\cos 45$ + (50 x 9.8) x $\frac{4}{5}L\cos 45$

But d = Lcos 45

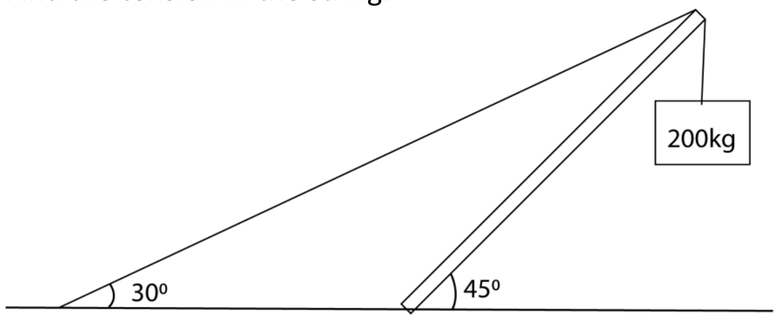
Therefore,

T x Lcos 45 = (200 x 9.8) x $\frac{1}{2}L\cos 45$ + (50 x 9.8) x $\frac{4}{5}L\cos 45$

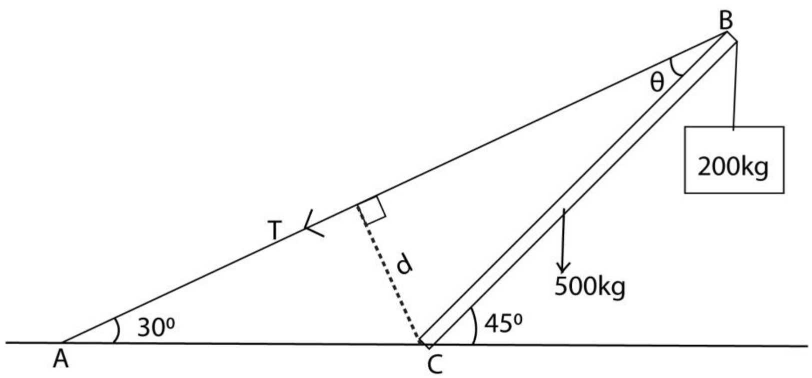
T = 1372N

Example 11

A rod of 500 N is balanced by a string AB and a mass of 200kg as shown in the diagram below. Find the tension in the string.



Solution



Taking moments at C

Let the length of the rod b L

$T \times d = (500 \times 9.8) \times \frac{L}{2}\cos 45 + (200 \times 9.8) \times L\cos 45$

But d = Lsinθ

$45^0 = 30^0 + \theta$

$\theta = 15^0$

It implies that

$T \times L\sin 15 = (500 \times 9.8) \times \frac{L}{2}\cos 45 + (200 \times 9.8) \times L\cos 45$

T =12047N

Exercise 2

1. A mass of 5kg is suspended from end A of a uniform bean of mass 1kg and length 1m. the end B of the beam is hinged to a wall. The beam is kept horizontal by a wire attached to point A and C on the wall at a height 0.75m above B.
   (i) Draw a sketch diagram to show the forces acting on the beam
   (ii) Calculate the tension in the wire. [T = 90N]

(iii) What is the force exerted by hinge on the beam [F = 72.1N]

2. A uniform ladder AB of length 12m is placed at an angle of 60^0 to the horizontal with one end B leaning against the wall and the other end A on the ground. Calculate the reaction force R of the wall at B and force F of the ground at A if the weight of the ladder AB is 200N. [R = 200N, F = 57.7N]

Center of gravity

This is a point where the resultant force of attraction of a body acts

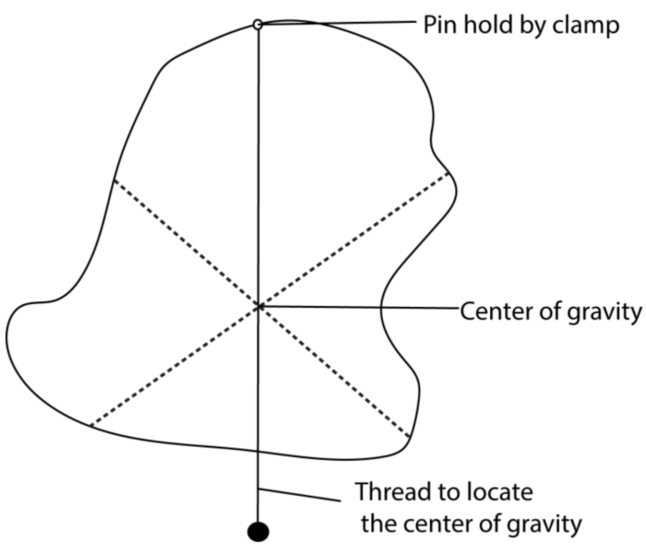
Center of gravity of regular object

For regular shape bodies, the center of gravity is at the geometric center of the body e.g. the center of gravity of a Uniform meter ruler is at 50cm mark, for circle, it is at the center. For a rectangular and square body it is at the point of intersection of the diagonals.

Center of gravity of Irregular body

The best way of finding the center of gravity of an irregular object is by use a plumb line. A plumb line is made from a thread of cotton with a loop at one end and a weight tied at other end.

For a irregular card board for instance, three small holes are made at well-shaped intervals around the edge of the card



A pin is then put through one of the holes and firmly by a clamp and stand so that the card board swings on it.

The card board will come to rest with its center of gravity below the point of support along the vertical line of plumb line.

The cardboard is hung through another hole, the point of interception of the two vertical lines is the center of gravity.

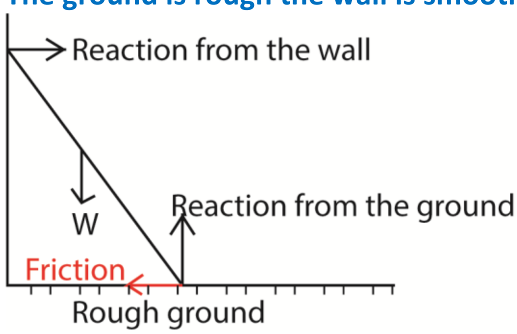
Factors that affect stability
1. The position of center of gravity, should be low.
2. Width of the base: the wider the width of the base, the more stable the body is.

Way of increasing stability
1. Increasing the base area
2. Lowering the center of gravity

Application of center of gravity
1. cars have very heavy framed to lower center of gravity
2. Racing cars have wide wheel base to lower center of gravity.

Dealing with Force on a ladder leaning against the wall

(a) The ground is rough the wall is smooth



At equilibrium

The resultant forces = 0

(→): Reaction from the wall = friction

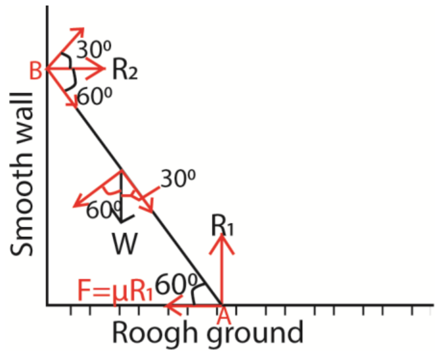
(↑): Reaction from the ground = weight

Example 13

A uniform ladder of length 2L and mass 20kg rests in equilibrium with its base on rough horizontal floor and its top against a smooth vertical wall. The ladder is inclined at 60^0 to the horizontal. If the ladder is on the point of slipping and the coefficient of friction is μ. Find the value of μ, the reactions at the wall and the floor.

Solution

Let the normal reaction at the floor = R1; the normal reaction at the wall= R_2; weight of the ladder be W and friction on the ground = F



(↑): $R_1 = W = 20 \times 9.8 = 196N$ ……………… (i)

(→): $R_2 = F$
$= \mu R_1$
$= 196\mu$ ……………………………………(ii)

Taking moments at A:

↶(A): $R_2\cos30(2L) = W\text{Cos}60\ (L)$

$$R_2 = \frac{196\cos60}{2\cos30}$$

$= 56.58N$

NB: cos 30 = sin60

From equation (ii)

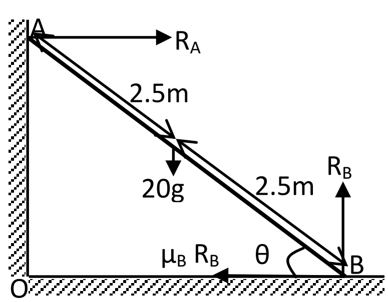
$$\mu = \frac{R_2}{196} = \frac{56.58}{196} = 0.29$$

Example 14

A uniform ladder which is 5m long and mass of 20kg leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The bottom of the ladder is 3m from the wall. Calculate

(i) The frictional force between the ladder and the ground

(ii) The coefficient of friction



$\theta = \cos^{-1}\frac{3}{5} = 53.13^0$

(i) (↑) $R_B = 20g = 20 \times 9.8 = 196N$

(→) $R_A = \mu_B R_B = 196\ \mu_B$

Taking moments about B

$R_A \times 5\sin\theta = 20g \times 2.5\cos\theta$

$R_A \times 5\sin53.13 = 20g \times 2.5\cos53.13$

$R_A = 73.5N$

Frictional force = 73.5N

(ii) $R_A = \mu_B R_B = 196\ \mu_B$

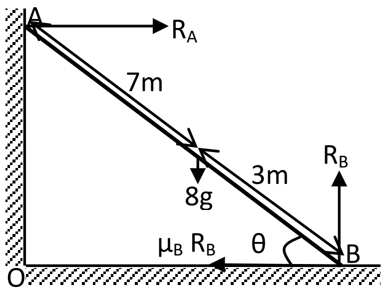
$73.5 = 196\ \mu_B$

$\mu_B = 0.375$

Example 15

A non-uniform ladder AB 10m long and 8kg lies in limiting equilibrium with its lower end resting on a rough horizontal ground and the upper end resting against a smooth vertical wall. If the centre of gravity of the ladder is 3m from the foot of the ladder and the ladder makes an angle of 30^0 with the horizontal, find the

(i) Coefficient of friction between the ladder and the ground
(ii) Reaction at the wall



(i) (↑) $R_B = 8g = 8 \times 9.8 = 78.4N$

(→) $R_A = \mu_B R_B = 78.4 \mu_B$

Taking moments about B

$R_A \times 10\sin\theta = 8g \times 3\cos\theta$

$R_A \times 10\sin 30 = 8g \times 3\cos 30$

$R_A = 40.738N$

Reaction at the wall = 40.738N

$R_A = \mu_B R_B = 78.4 \mu_B$

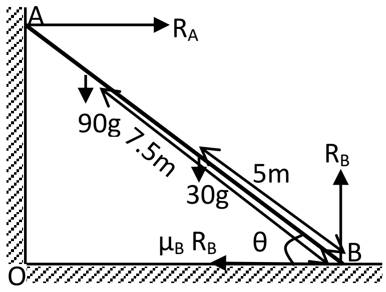
$78.4 \mu_B = 40.738$

$\mu_B = 0.5196$

Example 16

A uniform ladder AB 10m long and mass 30kg lies in a limiting equilibrium with its lower end resting on a rough horizontal ground and its upper end resting against a smooth vertical wall. If the ladder makes an angle of 60^0 with the horizontal, with a man of mass 90kg standing on the ladder at a point 7.5m from its base, find

(i) Magnitude of normal reaction and friction force at the ground
(ii) The minimum value of the coefficient of friction between the ladder and the ground that would enable the man to climb to the top of the ladder



(i) (↑) $R_B = 30g + 90g = 120 \times 9.8 = 1176N$

(→) $R_A = \mu_B R_B = 1176 \mu_B$

Taking moments about B

$R_A \times 10\sin 60 = 30g \times 5\cos 60 + 90g \times 7.5\cos 60$

$R_A = 466.788N$

$\mu_B R_B = R_A$

$1176 \mu_B = 466.788$

$\mu_B = 0.397$

(ii) Taking moments about B

R_A x 10sin60 =30g x 5cos60 + 90g x 10cos60

R_A = 594.093N

μ_B R_B = R_A

1176 μ_B = 594.093

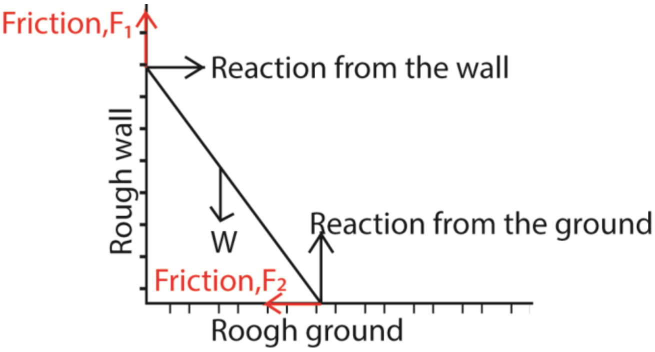
μ_B = 0.5052

Revision exercise 3

1. A non- uniform ladder AB 10m long and mass 8kg lies in limiting equilibrium with its lower end resting on a rough horizontal ground of angle of friction 17^0 and the upper end resting against a smooth vertical wall. if the centre of gravity is at point C and the ladder makes an angle of 63^0 with horizontal, find the length AC. [6m]

2. A uniform ladder of mass 30kg lies in limiting equilibrium with its lower end resting on a rough horizontal ground of coefficient of friction 0.4 and the upper end resting against a smooth vertical wall. If the ladder makes an angle of 60^0 with the horizontal, find the magnitude of the frictional force at the ground. [158.4N]
3. A ladder 12m long and weighing 200N is placed 60^0 to the horizontal with one end B leaning against a smooth vertical wall and the other end A on a rough horizontal ground. Find:
   (a) Reaction at the wall [57.7N]
   (b) Reaction at the ground [208.2N at 73.9^0 to the horizontal]
4. A uniform ladder of length 10m and weight w lies in limiting equilibrium with its lower end resting on a rough horizontal ground of coefficient of friction $\frac{1}{3}$ and the upper end resting against a smooth vertical wall. The ladder makes an angle of θ with the horizontal where tanθ = 1.7. A man of weight 2w starts to climb the ladder. Find
   (i) How far up the ladder the man can climb before slipping can occur. [6m]
   (ii) Find in terms of w, the magnitude of the frictional force at the ground to enable the man reach the top. $\left[\frac{8w}{17}\right]$
5. A uniform ladder of length 5m and weight 80N lies in limiting equilibrium with its lower end resting on a rough horizontal ground and the upper end resting against a smooth horizontal rail fixed 4m vertically above the ground. If the ladder makes an angle of θ with the vertical where tanθ ≤ 0.75 with the horizontal.
   (a) Find expressions in terms of θ for
      (i) Vertical reaction R of the ground [$80-50\sin^2\theta\cos\theta$]
      (ii) Friction F at the ground [$50\cos^2\theta\sin\theta$]
      (iii) Normal reaction N at the rail [50sinθcosθ]
   (b) Given that the ladder does not slip, show that F is maximum when tanθ = $\frac{1}{\sqrt{2}}$ and find its maximum value $\left[\frac{100}{3\sqrt{3}}N\right]$

(b) The ground is rough the wall is rough



At equilibrium

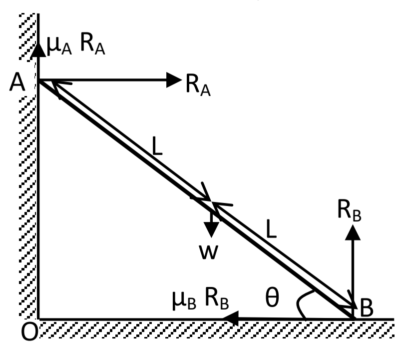
The resultant forces = 0

(→): Reaction from the wall = friction, F_2

(↑): Reaction from the ground + friction, F_1 = weight

Example 17

A uniform ladder rests with one end on a rough horizontal ground and the other against a rough vertical wall, the coefficient of friction being respectively $\frac{3}{5}$ and $\frac{1}{3}$. Find the inclination of the ladder to the vertical when it is about to slip



(↑) $R_B + \frac{1}{3}R_A = w$ …….. (i)

(→) $R_A = \frac{3}{5}R_B$

=> $R_B = \frac{5}{3}R_A$

Substituting for R_B in equation (i)

$\frac{5}{3}R_A + \frac{1}{3}R_A = w$

$R_A = \frac{w}{2}$

Taking moments about B

$R_A \times 2L\sin\theta + \frac{1}{3}R_A \times 2L\cos\theta = w \times L\cos\theta$ ...(ii)

Substituting for R_A in eqn. (ii)

$\frac{w}{2} \times 2L\sin\theta + \frac{1}{3} x \frac{w}{2} \times 2L\cos\theta = w \times L\cos\theta$

$\sin\theta = \frac{2}{3}\cos\theta$

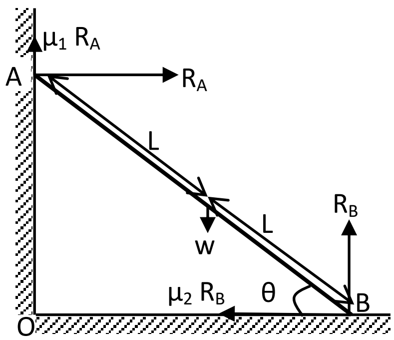
$\tan\theta = \frac{2}{3}$

$\theta = 33.7^0$

Angle to the vertical = 90 - 33.7 = 56.3^0

Example 18

A uniform ladder rests in limiting equilibrium with the top end against a rough vertical wall with coefficient of friction μ_1 and its base on a rough horizontal floor with coefficient of friction μ_2.If the ladder makes an angle of θ with the floor, prove that tanθ = $\frac{1-\mu_1\mu_2}{2\mu_2}$



(↑) $R_B + \mu_1 R_A = w$ .... (i)

(→) $R_A = \mu_2 R_B$ ....(ii)

$$R_B = \frac{1}{\mu_2}R_A$$

Substituting R_B in eqn. (i)

$$\frac{1}{\mu_2}R_A + \mu_1 R_A = w$$

$$R_A = \frac{\mu_2 w}{1+\mu_1\mu_2}$$

Taking moments about B

R_A x 2Lsinθ + $\mu_1 R_A$ x 2Lcosθ = w x Lcosθ ....(iii)

$\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lsinθ + μ_1x $\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lcosθ = w x Lcosθ

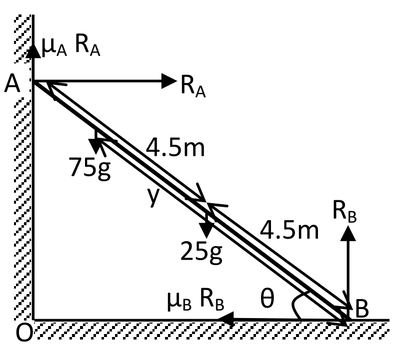
$\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lsinθ = w x Lcosθ - μ_1x $\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lcosθ

$\frac{2\mu_2}{1+\mu_1\mu_2}$ sinθ = $\frac{1+\mu_1\mu_2-2\mu_1\mu_2}{1+\mu_1\mu_2}$ cosθ

tanθ = $\frac{1-\mu_1\mu_2}{2\mu_2}$

Example 18

The foot of a ladder length of 9m and mass 25kg rests on a rough horizontal surface while the upper end rests in contact with a rough vertical wall. The ladder is in vertical plane perpendicular to the wall. If the first rug is 30cm from the foot and the rest at the interval of 30cm, find the highest rug to which a man of mass 75kg can climb without causing the ladder to slip, when the ladder is inclined at 60^0 to the horizontal and the coefficient of friction at each end is 0.25.



Let the man ascend a distance = ym

(↑) $R_B + \frac{1}{4}R_A = 25g + 75g$ ...... (i)

(→) $R_A = \frac{1}{4}R_B$

=> $R_B = 4R_A$ .... (ii)

Substituting for R_B in eqn. (i)

$$4R_A + \frac{1}{4}R_A = 25g + 75g$$

$$R_A = 230.6N$$

Taking moments about B

R_A x 9sinθ + $\frac{1}{4}R_A$ x 9cosθ = 25g x 4.5cosθ +75g x ycosθ

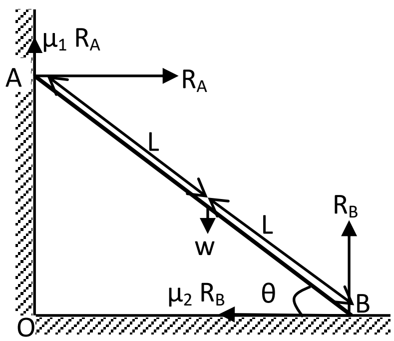
230.6 x 9sin60 + $\frac{1}{4}x$ 230.6 x 9cos60 = 25 x 4.5cos60 +75 x ycos60

y = 4

Example 19

A uniform ladder of length 2L and weight w rests in a vertical plane with one end on a rough horizontal ground and the other against a rough vertical wall, the angle of friction being respectively $\tan^{-1}\left(\frac{1}{3}\right)$ and $\tan^{-1}\left(\frac{1}{2}\right)$.

(a) Find the inclination of the ladder to the horizontal when it is in limiting equilibrium at either end



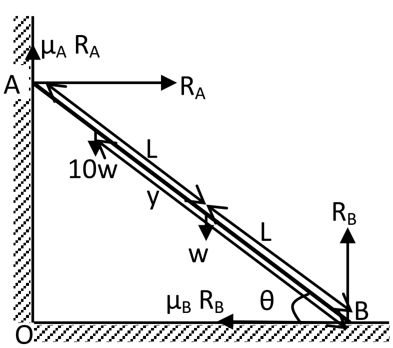
$\mu_A = \frac{1}{3}$ and $\mu_B = 2$

(↑) $R_B + \frac{1}{3} R_A = w$ …….. (i)

(→) $R_A = \frac{1}{2} R_B$

$R_B = 2R_A$ …………….(ii)

(b) A man of weight 10times that of the ladder begins to ascend it. How far will he climb before the ladder slips



Let the man ascend a distance = ym

(↑) $R_B + \frac{1}{3}R_A = 10w + w$ …… (i)

(→) $R_A = \frac{1}{2}R_B \Rightarrow R_B = 2R_A$ ….. (ii)

Substituting for R_B in eqn. (i)

$2R_A + \frac{1}{3}R_A = 11w$

$R_A = \frac{33}{7}w$

Taking moments about B

$R_A \times 2L\sin\theta + \frac{1}{3}R_A \times 2L\cos\theta = wL\cos\theta + 10wy\cos\theta$

$\frac{33}{7}w \times 2L\sin\theta + \frac{1}{3} \times \frac{33}{7}w \times 2L\cos\theta$

$= wL\cos\theta + 10wy\cos\theta$

$\frac{66}{7}L\sin\theta + \frac{15}{7}L\cos\theta = 10y\cos\theta$

$\frac{66}{7}L\tan\theta + \frac{15}{7}L = 10y$

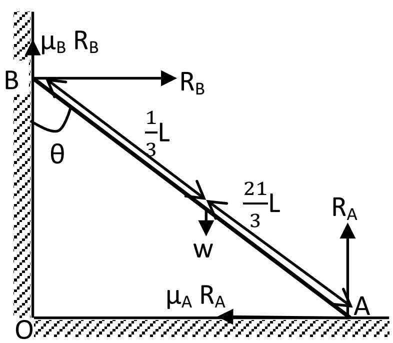
10L = 10y

=> y = Lm

Example 20

A non-uniform ladder AB is in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end B resting against a rough vertical wall. The ladder is in a vertical plane perpendicular to the wall. The centre of gravity of the ladder is t a G = $\frac{2}{3}AB$. The coefficient of friction between the ladder and the wall is twice that between the ladder and the floor. If the ladder makes an angle θ with the wall and the angle of friction between the ladder and the floor is λ;

(i) Show that 4tanθ = 3tan2λ
(ii) How far can a man of equal mass as the ladder ascend without the ladder slipping given that θ = 45^0 and coefficient of friction between the ladder and the floor is $\frac{1}{2}.\left[\frac{2}{3}AB\right]$



$\mu_A = \tan\lambda$ and $\mu_B = 2\tan\lambda$

(↑) $R_A + 2\tan\lambda = w$……………..(i)

(→) $R_B = \tan\lambda\ R_A$……………… (ii)

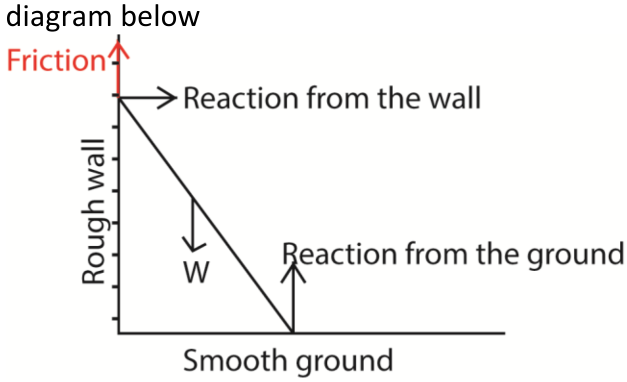
Substituting R_B in eqn. (i)

Revision exercise 4

1. A uniform ladder rests in limiting equilibrium with the top end against a rough vertical wall with coefficient of friction 0.25 and its base on a rough horizontal floor with coefficient of friction μ. If the ladder makes an angle of 300 with the vertical; find the value of μ [0.269]
2. A non-uniform ladder AB of length 6m is in limiting equilibrium with its lower end A resting on rough horizontal ground with coefficient of friction $\frac{1}{3}$ and the upper end B resting against a rough vertical wall with coefficient of friction $\frac{1}{4}$. The ladder is in a vertical plane perpendicular to the wall. The centre of gravity of the ladder is at C where AC = 4m. If the ladder makes an acute angle θ with the ground. Show that tanθ =$\frac{23}{12}$.
3. A uniform ladder AB is of weight 2w and length 10m rests in limiting equilibrium with the top end against a rough vertical wall with coefficient of friction $\frac{1}{3}$and its base on a rough horizontal floor with coefficient of friction $\frac{1}{3}$. If the ladder makes an angle an angle of θ with horizontal, such that tanθ = $\frac{16}{17}$. A man of weight 5w starts to climb the ladder.
   (a) How far up the ladder can a man climb before slipping [9m]
   (b) When a boy of weight Y stands on the bottom rung of the ladder at A, the man is just able to climb to the top safely. Find Y in terms of W $\left[\frac{7w}{11}\right]$
4. A non-uniform ladder AB of length 12m and mass 30 is in limiting equilibrium with its lower end A resting on a rough horizontal ground with coefficient of friction 0.25 and upper end B resting against a rough vertical wall with coefficient of friction 0.2. The ladder is in a vertical plane perpendicular to the wall. The centre of gravity of the ladder is at its trisection of the length nearer to A. The ladder makes an angle θ with the horizontal such that tanθ =$\frac{9}{4}$. A straight horizontal string connects A to a point at the base of the wall vertically below B. A man of mass 9okg begins to climb the ladder
   (i) How far up the ladder can the man climb without causing tension in the string [8m]
   (ii) What tension must the string be capable of withstanding if the man is to reach the tip of the ladder safely [126N]

(c) The ground is smooth and the wall is rough

In equilibrium the ladder experience a reaction from the wall, a reaction from the ground, weight (w) of the ladder acting from the center and a friction force as shown in the diagram below



Resolving forces
($\rightarrow$): Reaction from the wall
($\uparrow$): Reaction from the ground – weight + Friction
Note that equilibrium will never be attain because the reaction from the wall has no opposing force even if the resultant vertical force = 0 (vertical equilibrium)

For addition examples on forces acting on a ladder leaning against the wall. Refer the file 'A-level math paper 2 Coplanar forces (rigid bodies)' on the digitalteachers.co.ug website

Thank you
Dr. Bbosa Science