P 425/1
PURE MATHIMATICS
Paper 1
3hours.

POST MOCK EXAMINATIONS
Uganda Advanced Certificate of Education.
PURE MATHEMATICS
Paper 1
3 hours.

INSTRUCTIONS TO CANDIDATES:

Attempt ***all*** the eight questions in section A and any ***five*** questions from section B.
Begin each question on afresh sheet of paper.
Mathematical tables with lists of formulae and squared papers are provided.
Silent, non-programmable scientific calculators may be used.

SECTION A: (40 marks)

1. Solve: $\sqrt{(x + 2)} + \sqrt{(3x + 4)} = 2$.

2. The area enclosed by the curve $y^2 = 4x$ and the lines $y = 0$ and $y = 2x – 4$ in the first quadrant is rotated about the x-axis through one revolution. Calculate the volume swept out.

3. If $\sin(e^{xy}) = x$, show that $\dfrac{dy}{dx} = \dfrac{x - \sqrt{(1 - x^2)}(\ln\sin^{-1}x)\sin^{-1}x}{x^2\sqrt{(1 - x^2)}\sin^{-1}x}$

4. Solve : $2.5\cos x – 6\sin x = 3.25$; $0 \leq x \leq 360^0$

5. A car starts from rest and travels along a straight road. Its engine provides an acceleration of a m/s^2 . Air resistance and friction cause a deceleration of $\rho m/s^2$ for every metre per second of the car's velocity.

   (i) Show that after t seconds the car's velocity is $V = \dfrac{a}{\rho}(1 - e^{-\rho t})$

   (ii) If $a = 17.6\ m/s^2$, $\rho = 0.1$, find the limiting value of V as $t\ \rho \to \infty$.

6. Evaluate $\displaystyle\int_1^{\sqrt{2}} \frac{1 + \ln t}{t(3 + 2\ln t)}\,dt$

7. Use Maclaurin's theorem to express $(\sin x)\ln(1+ 2x)$ as a power series in ascending powers of x as far as the term in x^3 .State the range of values of x for which the series is valid.

8. Use Demoivre's theorem to evaluate $(2 + 2\sqrt{3}i)^6$

SECTION B : (60 marks)

9. P is the point $(ap^2,2ap)$ and Q is the point $(aq^2,2aq)$ on the parabola $y^2 = 4ax$.
   (i) Find the equation of the chord PQ.
   (ii) If P and Q vary on the parabola in such a manner that PQ remains a focal chord parallel to the y-axis, show that $pq = -1$ and $p + q = 0$.
   (iii) The tangents to the parabola at P and Q meet each other at R and that at the vertex at S and T respectively. Show that the locus of the centroid of triangle RST lies on a fixed line parallel to the y- axis.
   (iv) Find the area of triangle RST.

10. (a) Given that $Z_1= (1- 2i)(2 + 3i)$,$Z_2 = \dfrac{1}{1 + i}$, $Z_3 = \dfrac{(1 + i)^2}{2i}$

    Express Z_1, Z_2, Z_1 /Z_2 and Z_2 /Z_3 in polar form. Hence represent them on the argand diagram.

(b) O is the origin and A represents the point (1, 0) in the argand diagram.
If P represents a variable complex number Z, prove that PO is perpendicular to PA if the real part of $\frac{Z-1}{Z}$ is zero.
Deduce that if $Z = \frac{1}{1+pi}$, where P is a variable real number ,then the point representing Z describes a circle and find its centre and radius.

11. The lines L and M are given by the equations $2\mathbf{i} - 3\mathbf{j} + \mathbf{k} + s(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ and $8\mathbf{i} + 5\mathbf{j} + 13\mathbf{k} + t(3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$ respectively.
(i) Show that L and M intersect and find the vector **A** ,of their point of intersection.
(ii) Show that both L and M lie in the plane π given by $2x - z = 3$.
(iii) The point B is (12,5,6) and the point C is the foot of the perpendicular from B to π. Find the vector equation of BC.
(iv) Show that C lies in L.

12. Prove that : (i) $\sec^2\theta(\sin 2\theta - 1 + \cos 2\theta) = 2\tan\theta(1 - \tan\theta)$
(ii) $\sqrt{\left(\frac{(s-b)(s-c)}{s(s-a)}\right)} = \tan \frac{1}{2}A$
Where a, b ,c are sides of a triangle ABC and s is the semi-perimeter of the triangle.

13. Given the curve $y = \frac{x^2 + 4x - 5}{x + 7}$
(i) Find the turning points of the curve.
(ii) State the asymptotes of the curve
(iii) State the range of values of y within which the curve can not lie.
(iv) Sketch the curve.

14. (a) Show that $\int_{-1}^{1} \frac{dx}{x^{1/3}(1 + x^{1/3})} = 0$.

(b)Evaluate $\int_{0}^{1} \frac{4x \, dx}{(1 + x)(1 + x^2)}$

15.(a) Solve the simultaneous equations by row reducing the appropriate matrix to echelon form.
$$4x + y - z = -5$$
$$x - 3y + z = -2$$
$$2x + 5y + 2z = +7$$

(b) (i) Given that the equation $x^2 + (3k - 1)x + 2k + 10 = 0$ has real roots , find the values of k for which the equation has equal roots and determine those roots.
(ii) Show that the expression $2\tan x - \sin x \cos x - 4\ln\cos x + x$ increases as x increases for all real values of x.

16. (a) (i) Solve the equation $(x + 1)^2 \cos y \frac{dy}{dx} + 1 = 0$.

Given that $y = \frac{1}{2}\pi$ when $x = 0$.

(ii) If $(x^2 + xy)\frac{dy}{dx} = y^2$, show that $ye^{y/x} = C$, where C is a constant.

(b) A condenser has capacitance K faradays and leakage resistance R ohms. It is charged to a voltage V_0 and then allowed to discharge. The voltage V time t seconds from the beginning of the discharge conforms to the equation

$$\frac{dV}{dt} = -\frac{V}{KR}$$

Show that $V = V_0 e^{-t/kR}$ and find the ratio of the voltage after 10 seconds to the voltage when fully charged. If $k = 0.5\text{x}10^{-6}$ $R = 7\text{x}10^{7}$. Correct to 2d.p

END