

P425/1  
PURE  
MATHEMATICS  
Paper 1  
June, 2024  
3hrs

**SET 1**

**SMACON EXAMINATIONS 2024**  
**Uganda Advanced Certificate of Education**

**PURE MATHEMATICS**

**Paper 1**

3 hours

**INSTRUCTIONS TO CANDIDATES:**

- Attempt **all** the **eight** questions in **Section A** and **Not** more than **five** from
- **Section B.**
- Any additional question(s) will not be marked.
- All working must be shown clearly.
- Silent non-programmable calculators and mathematical tables with a list of formulae may be used.
- Graph papers are provided.

## SECTION A: (40MARKS)

Answer **all** the questions in this Section.

1. Find the sum of the numbers between 5 and 250 which are exactly divisible by 4. (5marks)
2. Given that the line;  $\frac{x-3}{4} = \frac{y-4}{-3} = \frac{z+3}{4}$  meets the plane  $4x - 3y - 4z = 3$  at  $M$ . Find the coordinates of  $M$ . (5marks)
3. Use the substitution  $x = \sin\theta$  to find the integral;  $\int \frac{2x^3}{\sqrt{1-x^2}} dx$ . (5marks)
4. Express  $\tan(45^\circ + x)$  in terms of  $\tan x$ . Hence prove that;  $\tan 75^\circ = 2 + \sqrt{3}$ . (5marks)
5. Given  $A(3, 4)$  and  $B(-2, 3)$ , find the equation of the locus of points  $P(x, y)$  which divide  $AB$  in the ratio 2:1. (5marks)
6. A women football team manager intends to take 18 players for a tournament. The manager has 2 goal keepers, 8 defenders, 4 mid fielders and 8 strikers. In how many ways can the team be chosen if it must contain both goal keepers, atleast 3 midfielders and 7 strikers. (5marks)
7. Solve the differential equation;  $\text{Cosec}x \frac{dy}{dx} = e^x \text{cosec}x + 3x$ . (5marks)
8. Solve for  $x$  in the equation;  $\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$ . (5marks)

## SECTION B (60MARKS)

Attempt any **five** questions from this Section.

9. Given that  $f(x) = \frac{x^3+2x^2+61}{(x+3)^2(x^2+4)}$ , express  $f(x)$  in partial fraction. Hence evaluate;  $\int_0^1 f(x)dx$ . (12marks)
10.  $P(ap^2, 2ap)$  and  $Q(aq^2, 2aq)$  are two variable points on the parabola  $y^2 = 4ax$ . If  $PQ$  subtends a right angle at the origin, prove that  $pq = -4$ .
  - a) Prove that  $PQ$  passes through a fixed point on the axis of the parabola.
  - b) The tangents at  $P$  and  $Q$  meet at  $R$ , find the equation of the locus of  $R$ . (6marks)

11. a) Differentiate  $\tan^{-1}\left(\frac{\sqrt{\ln X}}{e^{2x}}\right)$ . (6marks)
- b) Evaluate the integral;  $\int_0^{\frac{\pi}{6}} \frac{2\cos\theta + \sin\theta}{\cos\theta - \sin\theta} d\theta$ . (6marks)
12. a) P is the foot of the perpendicular from the point A(1, 1, 1) to the line  $\frac{X-1}{2} = \frac{y-1}{1} = \frac{Z-2}{1}$ . Determine the perpendicular distance of A from the line to 4 dp's. (5marks)
- b) Given the points A(-1, 2, 3) and P(2, 3, 4). If the point B(a, 2a, 3) lies on the plane  $2x - 3y + 4z + 8 = 0$ . Find the value of a and the angle between AP and AB. (7marks)
13. a) Solve the equation  $\tan\theta - \cot\theta = -1$  for  $0^\circ \leq \theta \leq 360^\circ$ . (5marks)
- b) Prove that  $\frac{\sin 3\theta}{1+2\cos 2\theta} = \sin\theta$ . Hence show that  $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$ . (7marks)
14. a) Prove that  $\log_a^b = \frac{1}{\log_{ba}}$ . hence solve the equation  $\log_2 x + \log_x 2 = 2.5$ . (5marks)
- b) A polynomial is given by  $P(x) = x^3 + Ax^2 + x - 6$ . The ratio of the remainder when  $P(x)$  is divided by  $(X + 1)$  to the remainder when divided by  $(x - 2)$  is  $-1:5$ . find the value of A. (7marks)
15. a) If  $Z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ , express Z in modulus argument form. (5marks)
- b) Use demoiver's theorem to prove that  $2\cos\theta = Z + \frac{1}{Z}$  then  $2\cos n\theta = Z^n + \frac{1}{Z^n}$ . Hence solve the equation  $5Z^4 - 11Z^3 + 6Z^2 - 11Z + 5 = 0$ . (7marks)
16. a) Determine the nature of the turning points of the curve  $y = x(1 - x)^2$ . (5marks)
- b) The acceleration of a particle is proportional to  $2t-3$ . If the velocity increases from  $4\text{ms}^{-1}$  to  $8\text{ms}^{-1}$  in the first 2 seconds of motion, find;
- i) its initial acceleration (5marks)
- ii) the velocity after 5 seconds. (2marks)

\*\*\*\* **END** \*\*\*\*