SECTION D: ELECTROSTATICS, CAPACITORS AND ELECTRICITY

Electrostatics (static electricity) is the study of electric charges at rest, the forces between them, and the electric fields associated with them.

Static electricity occurs when positive (+) or negative (–) electrical charges collect on an object's surface. There are several methods through which this condition can be caused.

One way is by rubbing certain materials together or pulling them apart. Another way is by bringing a charged material near to a neutral material, and also by sharing the charge on a body with another neutral insulated body when they come into contact with each other.

Electrification by friction / charging by rubbing or friction

- When two dissimilar bodies are rubbed together, heat is generated due to friction
- The heat is sufficient to make the material of lower work function to release some electron, which are taken up by other material.
- The one which lost electrons become positively charged while the one which gained electrons becomes negatively charged
- The number of electrons lost is equal to the number of electrons acquire therefore two insulating bodies rubbed together acquire equal and opposite charges.

Examples of charging by friction

- When a polythene rod (ebonite rod) is rubbed with fur (woolen duster), the ebonite rod becomes negatively charged while the duster becomes positively charged.
- If a glass rod (cellulose acetate) is rubbed with silk, a glass rod becomes positively charged while the silk becomes negatively charged.

Insulators, semiconductors and conductors

Conductor

This is a material with free electrons and it can allow electricity and heat to pass through it.
Examples: Copper, bronze

Insulator

This is a material without free electrons and it cannot allow electricity and heat to pass through it.
Examples: Dry wood, plastic

Semiconductors

These are materials which allow electric charges to pass through them with difficulty.
Examples: Moist air, paper

Law of electrostatics

Like charges repel each other and unlike charges attract each other.

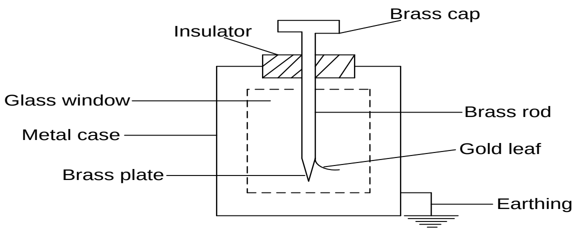
Precautions taken when carrying experiments in electrostatics

(i) Apparatus must be insulated
(ii) The surrounding must be free from dust and moisture

Attraction of neutral body by charged body

Consider the uncharged conductor being brought near a negatively charged ebonite rod. Negative charges on the ebonite rod repel the free electrons on the conductor to the remote end and positive charge is thus left near the end of the metal adjacent to the ebonite rod. So the conductor is now attracted by the ebonite rod.

GOLD LEAF ELETROSCOPE (GLE)



Uses of GLE

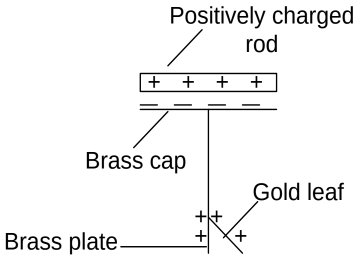
(i) Test for the presence of charge
(ii) Test the sign of the charge
(iii) To test the magnitude of charge
(iv) Measure $p.d$

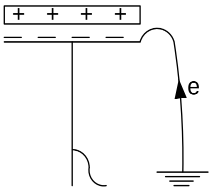
Electrostatic induction

It's a phenomenon that describes the formation of charges on a conductor when a charged body is brought near it.
The charge acquired is opposite to that of inducing body.

Charging a gold leaf electroscope by induction

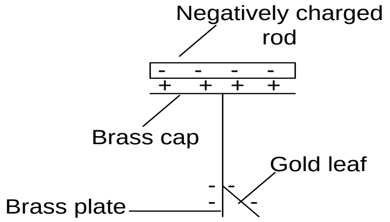
(a) Charging G.L.E negatively

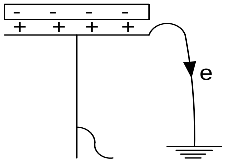




- ❖ A positively charged glass rod is brought near the cap of the G.L.E, negative charges are induced on the brass cap and positive charges on G.L and brass plate. The gold leaf diverges.
- ❖ With glass rod still in position, the G.L.E is earthed. Free electrons flow from the earth to the brass plate and gold leaf thus collapses.
- ❖ With the rod still in position, the earthing wire is removed.
- ❖ Glass rod is removed, the negative charges then redistribute themselves to the brass cap, plate and gold leaf thus causing he leaf to diverge. The electroscope is now negatively charged.

(b) Charging G.L.E negatively





- ❖ A negatively charged rod is brought near the cap of the G.L.E, positive charges are induced on the brass cap and negative charges on G.L and brass plate. The gold leaf diverges.
- ❖ With glass rod still in position, the G.L.E is earthed. Free electrons flow from the plate and leaf to the earth thus the leaf collapses.
- ❖ With the rod still in position, the earthing wire is removed.
- ❖ The rod is removed, the positive charges then redistribute themselves to the brass cap, plate and gold leaf thus causing he leaf to diverge. The electroscope is now positively charged

Testing for the sign of charge on a body

- ❖ Charge an electroscope negatively and the divergence <u>noted</u>. Bring the body under test near the cap of *GLE*. If the leaf divergence <u>increases</u> then that body is <u>negatively</u> charged, but if the divergence of the leaf <u>decreases</u>, then that body has either **positive** charge or it is **neutral body**
- ❖ To differentiate between the two alternatives, discharge the *GLE* and now charge it positively
- ❖ Bring the same body under test near the cap of appositively charged *GLE*. If the leaf divergence increases again, then that body has positive charges but if the leaf divergence decreases then that body is neutral.

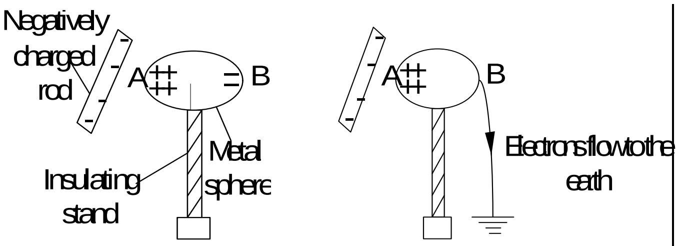
Note: Repulsion is the only confirmatory test for the sign of the charge

Summary

Charge on GLE	Charge brought near cap	Effect on leaf divergence
+	+	Increase(repulsion)
-	-	Increase(repulsion)
+	-	Decrease (attraction)
-	+	Decrease (attraction)
+ or -	Uncharged body	Decrease (attraction)

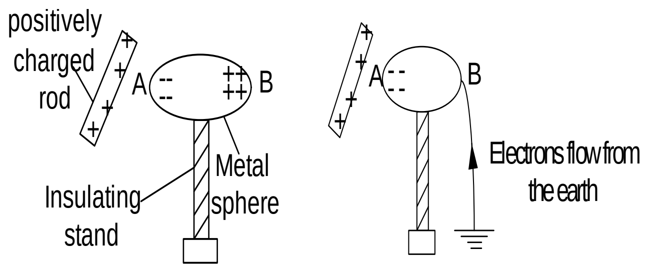
Charging a conductor by induction

a) Positively



- Metal sphere on an insulating stand is placed near the negatively charged body. Free electrons in the metal sphere are repelled to the far end of the sphere.
- The sphere is earthed while the charged body is still in position. Free electrons move from the sphere to the earth.
- The earthing wire is removed while the charged rod is still in position
- The charged body is removed and charges distributes themselves all over the sphere. Hence the metal sphere is now positively charged.

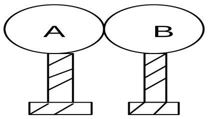
b) Negatively



- Metal sphere on an insulating stand is placed near the positively charged body. Free electrons in the metal sphere are attracted to the near end of the sphere.
- The sphere is earthed while the charged body is still in position. Free electrons move from the earth to the sphere.
- The earthing wire is removed while the charged rod is still in position
- The charged body is removed and charges distributes themselves all over the sphere. Hence the metal sphere is now negatively charged.

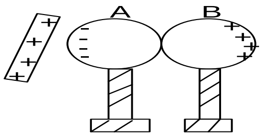
Separation of conductors

i)



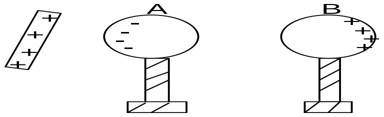
Two identical brass spheres A and B are placed together so that they touch one another.

ii)



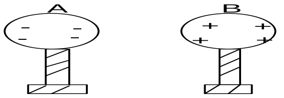
A positively charged rod is now brought near end A and as a result negative charge is induced at A and positive charges repelled to B.

iii)



Keeping the positive rod in position, sphere B is moved a short distance away from B

iv)



The charged rod is now removed and charges redistribute

Explain how two spherical conductors made of brass can be changed oppositely and simultaneously by induction.

How to distinguish a conductor and an insulator using an electroscope

- An electroscope is given charge and the divergence noted. The material is brought near the cap of the electroscope
- If there is no change in divergence, the material is an insulator. If the leaf divergences material is a conductor

Charging a body negatively at zero potential

- A positively charged glass rod is brought near end A of the conductor. Negative charges are induced at the near end and positive charges at the far end of the neutral body.
- With the glass rod still in position, body is earthed. Body is now negatively charged at zero potential

Electrophorus

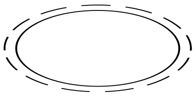
This an instrument for produce unlimited supply of charge but it is not source of energy though converts mechanical energy to electrical energy

Distribution of charge over the surface of a conductor.

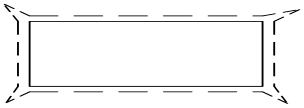
Surface charge density:

This is the quantity of charge per unit area over the surface of the conductor. Charge is mostly concentrated at sharp points.

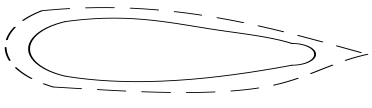
Spherical conductor



Rectangular conductor



Triangular conductor



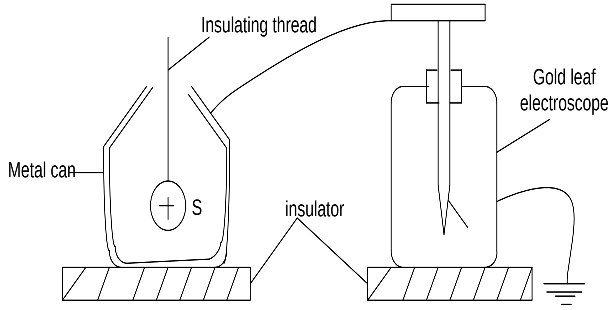
Note:

Charge only resides on the outside of a hollow conductor

Investigating charge distribution on a pear shaped conductor

- A proof plane is placed on the surface of the conductor. A sample of charge acquired by the proof plane is then transferred to a hollow metal can placed on the cap neutral electroscope and the deflection of the electroscope is noted
- The proof plane is then used to pick samples charges from different parts of the conductor and each time the deflection of the electroscope is noted
- The greatest deflection is obtained when the sample of charge are picked from the pointed end of the conductor. Therefore the surface charge density of charges is greatest where the curvature is greatest

Experiment to show distribution of charge in a hollow conductor. (Faraday's ice pail experiment)



- A positively charged metal sphere, S is lowered into a metal can (without touching it) connected to a gold leaf electroscope. The leaf of the electroscope diverges
- S is withdrawn, the leaf of the electroscope collapses
- S is again lowered inside the metal (without touching it), the leaf of the electroscope diverges to the same extent as before.
- S is then allowed to touch the can. The divergence of the leaf remains unchanged
- S is withdrawn and on testing , it is found to have no charge
- There must have been charge inside the can equal and opposite to the charge on S. since the leaf remains diverged, the charge on the can must be residing on the out side of it. This charge is equal to that which was originally on S

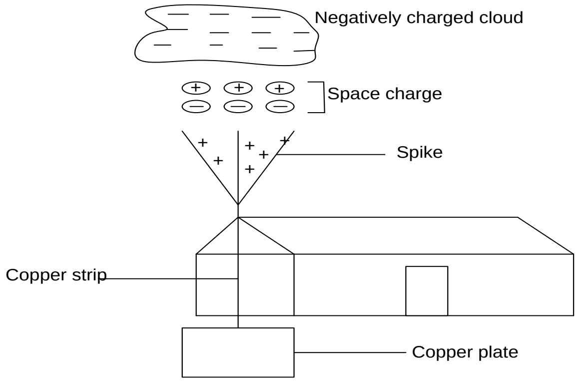
Action at sharp points [Corona discharge]

The high electric field intensity at the sharp points of a charged conductor, ionizes the air molecules around the sharp points. The ions of opposite charge are attracted to the sharp point and neutralize the charge there. This way the conductor loses charge and the process is called **corona discharge.**

Applications of action at sharp points

(a) Lightening Conductor

Action of a ligtening conductor

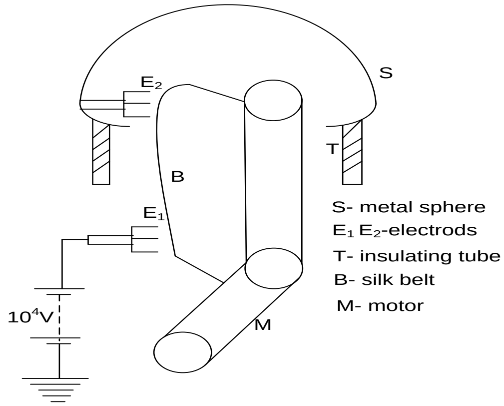


- When a charged cloud passes over a lightening conductor, it induces opposite charge on the spikes of the conductor which results to high electric field intensity
- The high electric field intensity on the spikes ionizes the air around it. Charge similar to those on the spikes is repelled to the clouds and neutralize charge on the cloud, while those opposite are attracted and discharged at the spikes
- This way charge from the cloud is safely conducted to the ground

Effect of Lightening

Clouds in relative motion become charged due to friction. The resulting charge builds up leading to a high p.d between the clouds and the earth. Large discharge currents through the building can cause them to burn

(b) Vander graaf generator



- The electrode E_1 is made 10^4V positive relative to the earth. The high electric field intensity at the sharp points of E_1 ionizes air around it repelling positive charges on to the belt.
- The belt driven by a motor carries this charge into the sphere. As it approaches E_2, it induces negative charge at the spikes of E_2 and positive charge on the sphere
- The high electric field intensity around E_2 ionizes air there, repelling negative charge onto the belt. The negative charge neutralizes positive charge on the belt before it goes over the upper pulley.
- The process is repeated many times until the potential of the sphere is about 10^6V relative to the earth

COULOMBS LAW OF ELECTROSTATICS

It states that the force between any two point charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance of separation of the charges.

$$\boxed{F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}}$$

For vacuum or air

$$\boxed{F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}}$$

$$\varepsilon_o = 8.85x10^{-12}Fm^{-1}$$

$$\frac{1}{4\pi\varepsilon_o} = 9x10^9$$

$$\boxed{F = \frac{9x10^9Q_1Q_2}{r^2}}$$

This law holds for all sign of charges. If Q_1 and Q_2 are unlike then the force is attractive but if they are like then the force is repulsive

Illustration

Example

1. Find the force between two point charges $+4\mu C$ and $-3\mu C$ placed 10cm apart

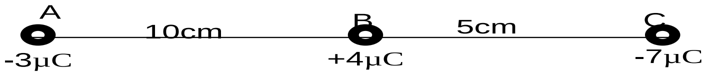
Solution

$$F = \frac{9x10^9Q_1Q_2}{r^2}$$

$$F = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.1^2}$$

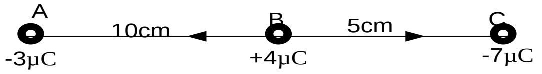
$F = 10.8N$ attractive

2. Three points charges are placed at point A, B and C as shown below



Find the resultant force on the charge at B

Solution



$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$

$F_A = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.1^2}$

$F_A = 10.8N$ attracted towards A

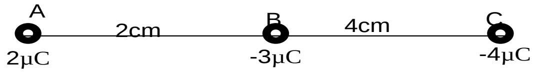
$F_c = \frac{9x10^9x4x10^{-6}x7x10^{-6}}{0.05^2}$

$F_c = 100.8N$ attracted towards C

$F_B = 100.8 - 10.8$

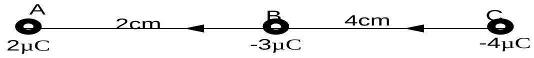
$F_B = 90N$ attracted towards C

3.



Calculate the force on $-3\mu C$

Solution



$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$

$F_A = \frac{9x10^9x2x10^{-6}x3x10^{-6}}{0.02^2}$

$F_A = 135N$ towards left

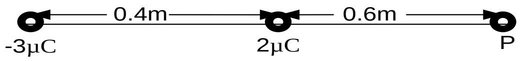
$F_c = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.04^2}$

$F_c = 67.5N$ towards left

$F_B = 135 + 67.5$

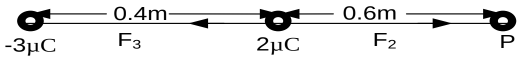
$F_B = 202.5N$ towards left

4.



Find the resultant force acting at point P if a charge of $1\mu C$ is placed at point P.

Solution



$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$

$F_3 = \frac{9x10^9x1x10^{-6}x3x10^{-6}}{1^2}$

$F_3 = 0.027N$ towards left

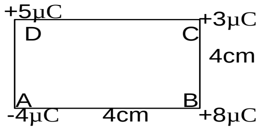
$F_2 = \frac{9x10^9x2x10^{-6}x1x10^{-6}}{0.6^2}$

$F_2 = 0.05N$ towards right

$F_P = 0.05 - 0.027$

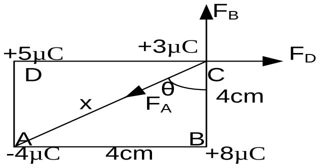
$F_P = 0.023$ towards right

5.



Find the resultant force at C

Solution



$$x^2 = 4^2 + 4^2$$

$$x = \sqrt{32} = 5.66cm$$

$$tan\theta = \frac{4}{4}$$

$$\theta = tan^{-1}(1) = 45°$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$

$$F_A = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.0566^2}$$

$$F_A = 33.75N$$

$$F_B = \frac{9x10^9x8x10^{-6}x3x10^{-6}}{0.04^2}$$

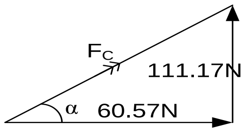
$F_B = 135N$upwards

$$F_D = \frac{9x10^9x5x10^{-6}x3x10^{-6}}{0.04^2}$$

$F_D = 84.375N$towards right

$$F = \begin{pmatrix}-33.7cos45\\-33.7sin45\end{pmatrix} + \begin{pmatrix}0\\135\end{pmatrix} + \begin{pmatrix}84.375\\0\end{pmatrix}$$

$$F = \begin{pmatrix}60.57\\111.17\end{pmatrix}$$

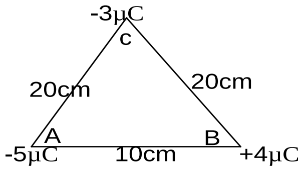


$$F^2 = 60.57^2 + 111.17^2$$
$$F = 126.59N$$

$$tan\alpha = \frac{111.17}{60.57}$$
$$\alpha = tan^{-1}\left(\frac{111.17}{60.57}\right) = 61.42°$$

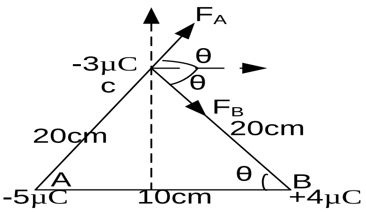
Resultant forces at C is 126.59N at $61.42°$ **to the horizontal**

6.



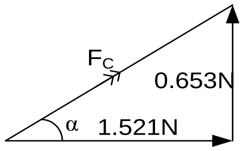
Find the resultant force at c

Solution



$$cos\theta = \frac{5}{20}$$
$$\theta = cos^{-1}\left(\frac{1}{4}\right) = 75.5°$$
$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$
$$F_A = \frac{9x10^9x5x10^{-6}x3x10^{-6}}{0.2^2}$$
$$F_A = 3.375N$$
$$F_B = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.2^2}$$

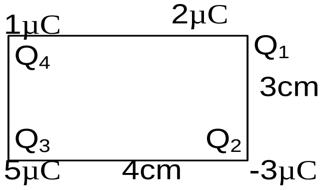
$$F_B = 2.7N$$
$$F = \begin{pmatrix} 3.375cos75.5 \\ 3.375sin75.5 \end{pmatrix} + \begin{pmatrix} 2.7cos75.5 \\ -2.7sin75.5 \end{pmatrix}$$
$$F = \begin{pmatrix} 1.521 \\ 0.653 \end{pmatrix}$$



$$F^2 = 1.521^2 + 0.653^2$$
$$F = 1.655N$$
$$tan\alpha = \frac{0.653}{1.521}$$
$$\alpha = 23.23°$$

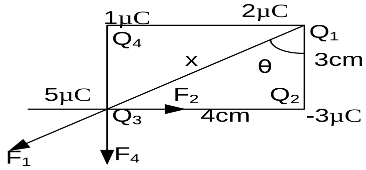
Resultant forces at C is 1.655N at $23.23°$ **to the horizontal**

7. Four point charges Q_1, Q_2, Q_3 and Q_4 are placed at different corners of rectangle



Find the resultant force at Q₃

Solution



$$x^2 = 4^2 + 3^2$$
$$x = \sqrt{25} = 5cm$$
$$tan\theta = \frac{4}{3}$$

$$\theta = 53.13°$$
$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$
$$F_4 = \frac{9x10^9x1x10^{-6}x5x10^{-6}}{0.03^2}$$

$F_4 = 50N$ downwards

$$F_2 = \frac{9x10^9x5x10^{-6}x3x10^{-6}}{0.04^2}$$

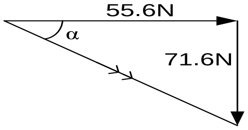
$F_2 = 84.4N$ towards the right

$$F_1 = \frac{9x10^9x5x10^{-6}x2x10^{-6}}{0.05^2}$$

$$F_1 = 36N$$

$$F = \begin{pmatrix} 0 \\ -50 \end{pmatrix} + \begin{pmatrix} 84.4 \\ 0 \end{pmatrix} + \begin{pmatrix} -36\sin 53.13 \\ -36\cos 53.13 \end{pmatrix}$$

$$F = \begin{pmatrix} 55.6 \\ -71.6 \end{pmatrix}$$



$$F^2 = 55.6^2 + 71.6^2$$

$$F = 90.66N$$

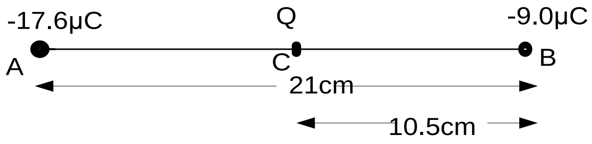
$$\tan\alpha = \frac{71.6}{55.6}$$

$$\alpha = 52.2°$$

Resultant forces at Q₃ is 90.66N at 52.2° to the horizontal

8. Two points charges A nad B of $-17.6\mu C$ and $-9.0\mu C$ respectively are placed in vacuum at a distance of 21cm apart. When a third charge C is placed mid way between A and B, the net force on B is zero
   - (i) Determine the charge on C
   - (ii) Sketch the electric field lines for the above charge distribution

Solution



$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$

$$F_A = \frac{9x10^9x17.6x10^{-6}x9x10^{-6}}{0.21^2}(\leftarrow)$$

$$F_C = \frac{9x10^9x9x10^{-6}xQ}{0.105^2}(\rightarrow)$$

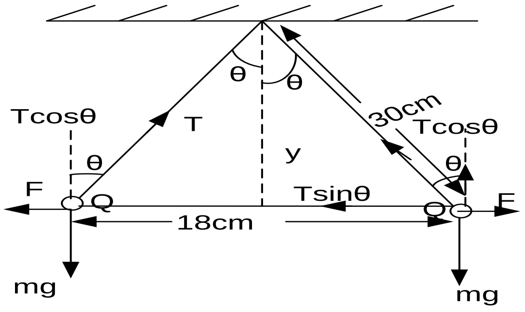
$$F_A = F_C$$

$$\frac{9x10^9x17.6x10^{-6}x9x10^{-6}}{0.21^2} = \frac{9x10^9x9x10^{-6}xQ}{0.105^2}$$

$$Q = 4.4x10^{-6}\mathbf{C}$$

9. Two pith balls Pand Q each of mass 0.1g are separately suspended form the same point by threads 30cm long. When the balls are given equal charges, they repel each other and come to rest 18cm apart. Find the magnitude of each charge.

Solution



$$\sin\theta = \frac{9}{30}$$

$$\theta = 17.5°$$

$$(\uparrow)\ T\cos\theta = mg$$

$$T\cos 17.5° = 0.1x10^{-3}x9.81$$

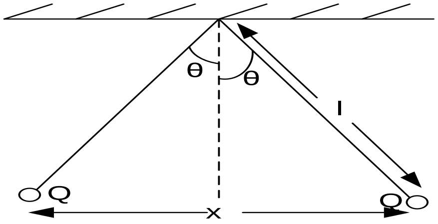
$$T = 1.029x10^{-3}N$$

$$(\rightarrow)T\sin\theta = \frac{Q^2}{4\pi\varepsilon_o x(0.18)^2}$$

$$1.029x10^{-3}x\frac{9}{30} = \frac{9x10^9xQ^2}{(0.18)^2}$$

$$\boldsymbol{Q = 3.33x10^{-8}C}$$

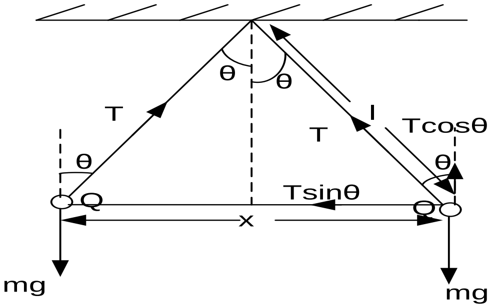
10. Two identical conducting balls of mass m are each suspended in air from a thick of length l, When the two balls are each given identical charge Q, they move apart as shown below



If at equilibrium each thread makes an angle θ with vertical and separated x is given by

$$x = \left(\frac{Q^2 l}{2\pi\varepsilon_o mg}\right)^{1/3}$$

Solution



$$Tsin\theta = \frac{Q^2}{4\pi\varepsilon_o x^2}\text{.............}(1)$$

$$Tcos\theta = mg \dots\dots\dots\dots..(2)$$

$$eqn1 \div eqn2$$

$$\frac{Tsin\theta}{Tcos\theta} = \frac{\left(\frac{Q^2}{4\pi\varepsilon_o x^2}\right)}{mg}$$

$$x^2 = \frac{Q^2}{4\pi\varepsilon_o mgtan\theta}$$

But for small angles in radians $tan\theta \approx sin\theta$

$$sin\theta = \frac{x}{2}\Big/_{l} = \frac{x}{2l}$$

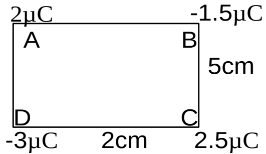
$$x^2 = \left(\frac{Q^2}{4\pi\varepsilon_o mg\frac{x}{2l}}\right)$$

$$x^3 = \left(\frac{Q^2 l}{2\pi\varepsilon_o mg}\right)$$

$$x = \left(\frac{Q^2 l}{2\pi\varepsilon_o mg}\right)^{1/3}$$

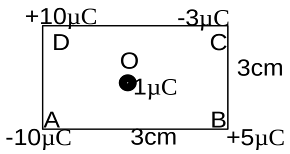
Exercise

1. Two points charges A nad B of $47.0\mu C$ and $24.0\mu C$ respectively are placed in vacuum at a distance of 30cm apart. When a third charge C of $-35.0\mu C$ is placed between A and B at a distance of 20cm from A. find the net force on C **An(-643.2N)**
2. Two point charges of 5μC and 2μC are placed in liquid of relative permittivity 9 at distance 5cm apart. Calculate the force between them. **An(3.998N)**
3. Two insulating metal spheres each of charge $5x10^{-8}C$ are separated by distance of 6cm. What is the force of repulsion if;
   (a) The spheres are in air **An(0.00625N)**
   (b) The spheres are in air with the charge in each sphere doubled and their distance apart is halved **An(0.1N)**
   (c) The two sphere are placed in water whose dielectric constant is 81 **An(**$7.7x10^{-5}N$**)**
4.



Find the resultant force at charge B. **An(**$\mathbf{12.58}N\ \boldsymbol{at}\ \mathbf{88.5}\ ^0\boldsymbol{to\ the\ horizontal}$**)**

5.



Calculate the resultant force at charge O, where O is the mid- point of the square **An(**$523.166N\ at\ 46.8\ ^0 to\ the\ horizontal$**)**

ELECTRIC FIELD

An electric field is a region within which an electric force is experienced.
Electric fields can be represented by electric field line.

Definition

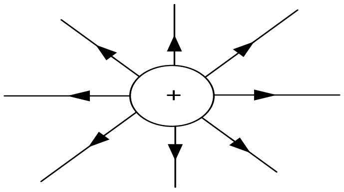
An electric field line is the path taken by a small positive charge placed in the field

Properties of electric field lines

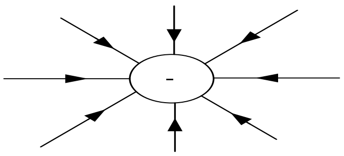
- They originate from positive and end on negative.
- they are in a state of tension which causes them to shorten
- they repel one another side ways
- they travel in straight lines and never cross each other
- the number of filed lines originating or terminating on a charge is proportional to the magnitude of the charge

Field patterns

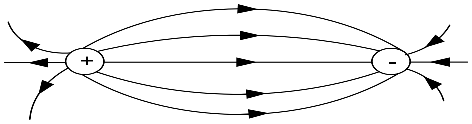
i) Isolated positive charge



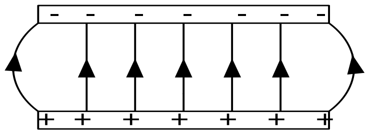
ii) Isolated negative charge



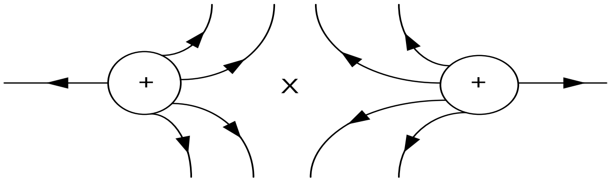
iii) Two equal opposite charges



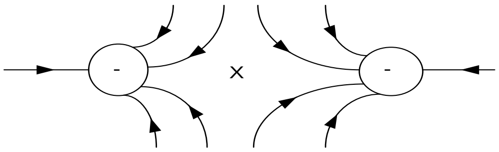
iv) Two parallel plates



v) Two positive charges near each other



vi) Two negative charges near each other



X is Neutral point;

Neutral point is the point in the electric field where the resultant electric force is zero.

Explain what happens to the potential energy as two point charges of the same sign are brought together

- Like charges repel. Work has to be done against the repulsive forces between them to bring them closer
- This work is stored as electric potential energy of the system
- The potential energy of the two like charges therefore increases when the charges are brought closer together

ELECTRIC FIELD INTENSITY/ ELECTRI C FIELD STRENGTH

Electric field intensity at a point is the force experienced by a positive one coulomb charge placed in an electric field.

$E = \frac{F}{Q}$

Generally

$$\boxed{E = \frac{Q}{4\pi\varepsilon r^2}}$$

But in air

$$\boxed{E = \frac{Q}{4\pi\varepsilon_o r^2}}$$

$$\varepsilon_o = 8.85x10^{-12}Fm^{-1}$$

$$\frac{1}{4\pi\varepsilon_o} = 9x10^9$$

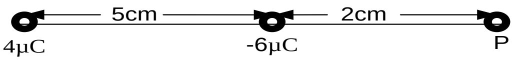
$$\boxed{E = \frac{9x10^9 Q}{r^2}}$$

S.I unit of electric field intensity is NC^{-1} or Vm^{-1}

Electric field intensity is a vector quantity and therefore direction is important. The direction of E is radially outwards if the point charge is positive and radially inwards if the pint charge is negative

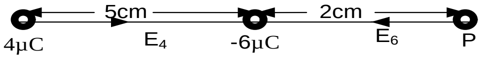
Examples

1.



Find Electric field intensity at P

Solution



$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_4 = \frac{9x10^9x4x10^{-6}}{0.07^2}$$

$E_4 = 7.347x10^6NC^{-1}$ towards the right

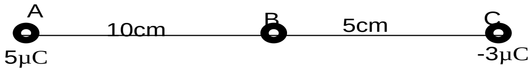
$$E_6 = \frac{9x10^9x6x10^{-6}}{0.02^2}$$

$E_6 = 1.35x10^8NC^{-1}$ towards the left

$$E = 1.35x10^8 - 7.347x10^6$$

$E = 1.2765x10^6NC^{-1}$ Towards left

2.



Find electric field intensity at B

Solution

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_A = \frac{9x10^9x5x10^{-6}}{0.1^2}$$

$E_A = 4.5x10^6NC^{-1}$ towards the right

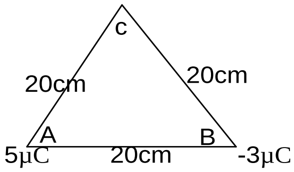
$$E_C = \frac{9x10^9x3x10^{-6}}{0.05^2}$$

$E_C = 10.8x10^6NC^{-1}$ towards the right

$$E = 10.8x10^6 + 4.5x10^6$$

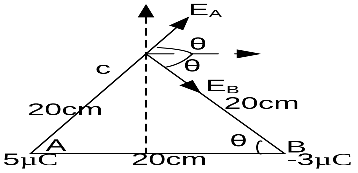
$E = 15.3x10^6NC^{-1}$ Towards Right

3.



Find electric field intensity at B

Solution



$$cos\theta = \frac{10}{20}$$

$$\theta = cos^{-1}\left(\frac{1}{2}\right) = 60°$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_A = \frac{9x10^9x5x10^{-6}}{0.2^2}$$

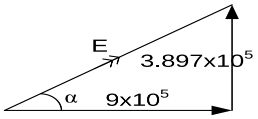
$$E_A = 1.125x10^6NC^{-1}$$

$$E_B = \frac{9x10^9x3x10^{-6}}{0.2^2}$$

$$E_B = 6.75x10^5NC^{-1}$$

$$E = \begin{pmatrix} 1.125x10^6cos60 \\ 1.125x10^6sin60 \end{pmatrix} + \begin{pmatrix} 6.75x10^5cos60 \\ -6.75x10^5sin60 \end{pmatrix}$$

$$E = \begin{pmatrix} 9.0x10^5 \\ 3.8971x10^5 \end{pmatrix}$$



$$E^2 = (9.0x10^5)^2 + (3.8971x10^5)^2$$

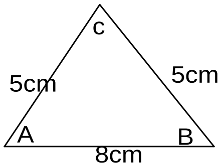
$$E = 9.81x10^5NC^{-1}$$

$$tan\alpha = \frac{3.8971x10^5}{9.0x10^5}$$

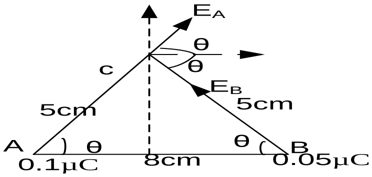
$$\alpha = 23.4°$$

Resultant electric field is $9.81x10^5NC^{-1}$**at** $23.4°$ **to the horizontal**

4. **Two point charges A and B of charges** $0.10\mu C$**and** $0.05\mu C$ **respectively placed 8cm apart as shown below**



Solution



$$cos\theta = \frac{4}{5}$$

$$\theta = cos^{-1}\left(\frac{4}{5}\right) = 36.9°$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_A = \frac{9x10^9x0.1x10^{-6}}{0.05^2}$$

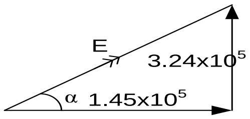
$$E_A = 3.6x10^5NC^{-1}$$

$$E_B = \frac{9x10^9x0.05x10^{-6}}{0.05^2}$$

$$E_B = 1.8x10^5NC^{-1}$$

$$E = \begin{pmatrix} 3.6x10^5cos36.9 \\ 3.6x10^5sin36.9 \end{pmatrix} + \begin{pmatrix} -1.8x10^5cos36.9 \\ 1.8x10^5sin36.9 \end{pmatrix}$$

$$E = \begin{pmatrix} 1.45x10^5 \\ 3.24x10^5 \end{pmatrix}$$



$$E^2 = (1.45x10^5)^2 + (3.24x10^5)^2$$

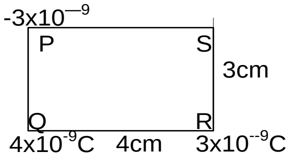
$$E = 3.55x10^5NC^{-1}$$

$$tan\alpha = \frac{3.24x10^5}{1.45x10^5}$$

$$\alpha = 65.89°$$

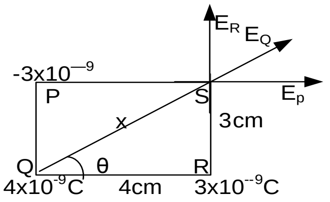
Resultant electric field is $3.55x10^5NC^{-1}$ **at** $65.89°$ **to the horizontal**

5. **Three charges**$-3x10^{-9}C$ **,** $3x10^{-9}C$ **and** $4x10^{-9}C$ **are placed in a vacuum at the vertices PRQ respectively at rectangle PQRS of sides 3cm by 4cm as shown below**



Calculate the resultant electric field strength at S

Solution

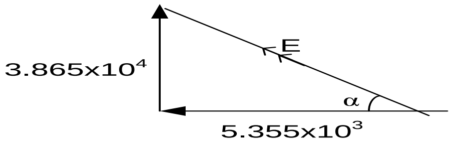


$$x^2 = 4^2 + 3^2$$
$$x = \sqrt{25} = 5cm$$

$tan\theta = \frac{3}{4}$

$\theta = 36.9°$

$$E = \begin{pmatrix} -5.355x10^3 \\ 3.865x10^4 \end{pmatrix}$$



$$E^2 = (-5.355x10^3)^2 + (3.865x10^4)^2$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$
$$E_P = \frac{9x10^9x3x10^{-9}}{0.04^2}$$
$E_P = 1.687x10^4NC^{-1}$ towards left
$$E_R = \frac{9x10^9x3x10^{-9}}{0.03^2}$$
$E_R = 3x10^4NC^{-1}$ upwards
$$E_Q = \frac{9x10^9x4x10^{-9}}{0.05^2}$$
$$E_Q = 1.44x10^4NC^{-1}$$

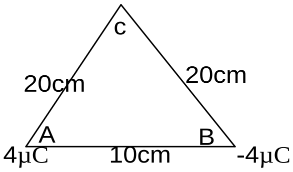
$$E = \begin{pmatrix} -1.687x10^4 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3x10^4 \end{pmatrix} + \begin{pmatrix} 1.44x10^4cos36.9 \\ 1.44x10^4sin36.9 \end{pmatrix}$$

$$E = 3.9x10^4NC^{-1}$$
$$tan\alpha = \frac{3.865x10^4}{5.355x10^3}$$
$$\alpha = 82.11°$$

Resultant electric field is $3.9x10^4NC^{-1}$ at $82.11°$ to the horizontal

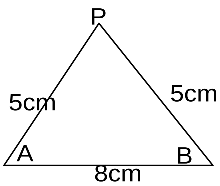
Exercise

1. The electric field intensity at the surface of the earth is about $1.2x10^2Vm^{-1}$at points towards the centre of the earth. Assuming that the earth is sphere of radius $6.4x10^6m$. Find the charge held by the earth surface **An($5.46x10^5C$).**
2. Two point charges $+4\mu C$ and $-4\mu C$ are placed 10cm apart in air.



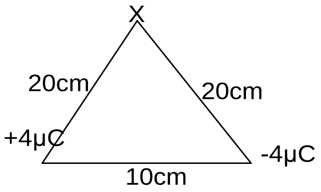
Find the electric field intensity at point C which is a distance of 20cm form each charge.
An($4.5x10^5NC^{-1}$).

3.



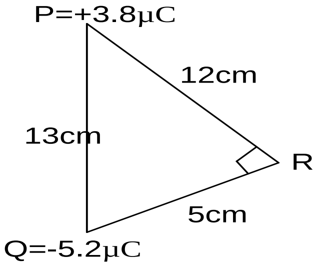
The point charges A and B of charges + 0.10 μc and +0.05μ c are separated by a distance of 8.0 cm along the horizontal as shown above. Find the electric field intensity at P.
An($3.55x10^5NC^{-1}$ at $66°$ to $horizontal$).

4. Two point charges +4.0μC and – 4 .0μC are separated by 10.0 cm in air as shown below



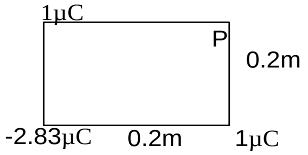
Find the electric field intensity at point x a distance of 20.0 cm from each charge . **An($4.5x10^5 NC^{-1}$ at $75.52°$ to the horizontal)**

5. Two point charges $+3.8\mu C$ and $-5.2\mu C$ are in air at points P and Q as shown below**.**



Find the electric field intensity at R. **An($1.89x10^7 NC^{-1}$ at $7.2°$ to the horizontal)**

6. Find the electric field strength at P

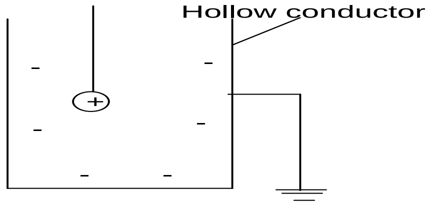


An($142.8 NC^{-1}$ at $45°$ to the horizontal)

7. The electric intensity at the surface of the earth is about 1.2 x 10^2 V m $^{-1}$ and points towards the centre of the earth. Assuming that the earth is a sphere of radius 6.4 x 10^6 m, find the charge held by the earth's surface.

ELECTROSTATIC SHEILDING OR SCREEENING

It is the creation of an electrically neutral space in the neighborhood of an electric field however strong it is by enclosing it in a hollow conductor (faraday cage)



- The charged body is enclosed in a hollow conductor which is earthed.
- Equal but opposite charge is induced on the inner walls of the hollow conductor
- Electric field outside will not affect the charged body inside the conductor

ELECTRIC FLUX $\emptyset$

This is the product of electric field strength at any point and area normal to the field

$$\emptyset = AE$$

TOTAL ELECTRIC FLUX

Consider a spherical surface of radius concentric with point charge

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

But $\emptyset = AE$

$$\emptyset = A\frac{Q}{4\pi\varepsilon r^2}$$

But $A = 4\pi r^2$

$$\emptyset = 4\pi r^2\frac{Q}{4\pi\varepsilon\, r^2}$$

$\boxed{\emptyset = \frac{Q}{\varepsilon}}$ This is called Guass's law of electrostatics

Guass's theorem of electrostatic states that the total electric flux passing normally through a closed surface, whatever its shape is always constant

Electric field intensity due to hollow charged sphere

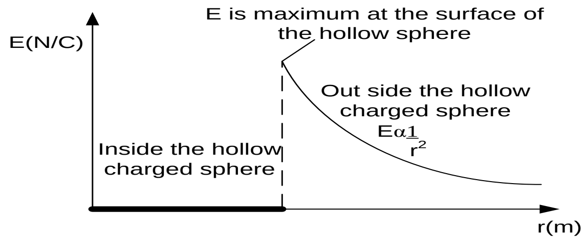
(i) Outside the sphere

Since $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ there $E \propto \frac{1}{r^2}$

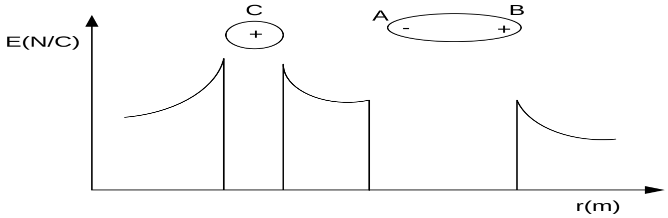
(ii) Inside the sphere

No charge resides on the inside of a hollow conductor therefore $E = 0$

A graph of E against the distance of a charge from a hollow charged sphere



A graph of E against the distance due to charges

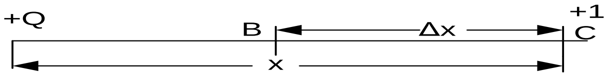


ELECTRIC POTENTIAL

This is the work done in moving a positive one coulomb charge from infinity to a point against an electric field.

Expression for electric potential

Consider $+1C$ charge xm away from $+Q$ being moved from C to B through a small displacement Δx without affecting the electric field due to $+Q$



Force on 1C of charge, $F = \frac{Q}{4\pi\varepsilon x^2}$

Work done to move the charge through Δx against the field is $\Delta w = -F\Delta x$

Total work done to bring the charge form infinity to a point a distance r from the charge of

$$w = \int_{\infty}^{r} -F dx$$

$$= \int_{\infty}^{r} -\frac{Q}{4\pi\varepsilon x^2}dx$$

$$= -\frac{Q}{4\pi\varepsilon}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= -\frac{Q}{4\pi\varepsilon}\left(-\frac{1}{r} - -\frac{1}{\infty}\right)$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon r}}$$

Note: Electric potential is a scalar quantity

Examples

1. A hollow spherical conductor of diameter $21.4\ cm$ carrying a charge of $6.9\ x\ 10^{-10}$ C is raised to a potential of $50V$. Find the permitivity of the surrounding medium

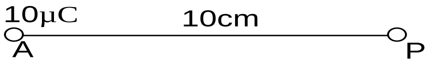
Solution

$$V = \frac{Q}{4\pi\varepsilon r}$$

$$\varepsilon = \frac{6.9x10^{-10}}{4\pi x 10.7x10^{-2}x50}$$

$$\boldsymbol{\varepsilon} = 1.026x10^{-11}Fm^{-1}$$

2. Find the potential at P



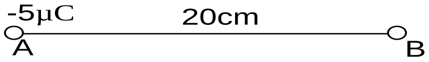
Solution

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \frac{10x10^{-6}x9x10^{9}}{0.1}$$

$$\boldsymbol{V} = 9x10^{5}V$$

3. Find the potential at B



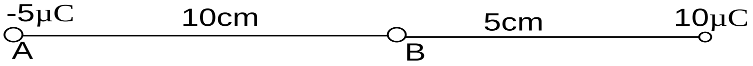
Solution

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \frac{-5x10^{-6}x9x10^{9}}{0.2}$$

$$\boldsymbol{V} = -2.25x10^{5}V$$

4. Find the potential at B

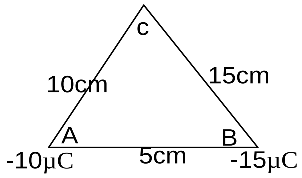


$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = 9x10^{9}x10^{-6}x\left(\frac{-5}{0.1} + \frac{10}{0.05}\right)$$

$$\boldsymbol{V} = 1.35x10^{6}V$$

5.



Find;

(a) Electric potential at C

(b) The potential energy at C if a charge of 10μC is placed at C

(c) Work done to move $-10\mu C$ charge from A to C

Solution

(a) $V = \frac{Q}{4\pi\varepsilon_0 r}$

$$V_c = 9x10^{9}x10^{-6}x\left(\frac{-15}{0.1} - \frac{15}{0.05}\right)$$

$$\boldsymbol{V_c} = -1.8x10^{6}V$$

(b) $\boldsymbol{W = QV}$

$$\boldsymbol{P.e} = 10x10^{-6}\boldsymbol{x} - 1.8x10^{6}$$

$$\boldsymbol{P.e} = -18J$$

(c)

$$V_A = \frac{9x10^9x - 15x10^{-6}}{0.05}$$

$$V_A = -2.7x10^6V$$

$$\boldsymbol{W = QV_{AB}}$$

$$\boldsymbol{w} = -10x10^{-6}\boldsymbol{x}(-2.7x10^6 - -1.8x10^6)$$

$$\boldsymbol{W = 9J}$$

Potential due to hollow charged sphere

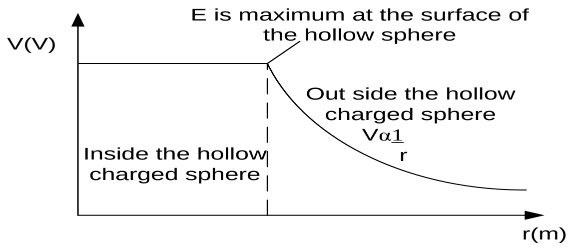
(i) Outside the sphere

Since $V = \frac{Q}{4\pi\varepsilon_0 r}$ there $V \propto \frac{1}{r}$

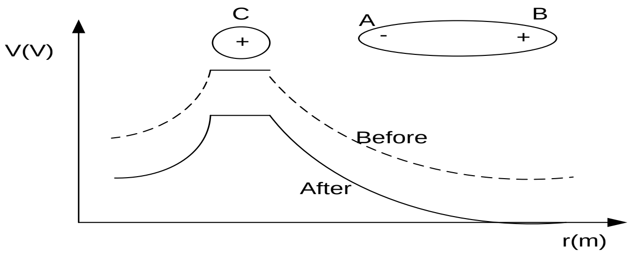
(ii) Inside the sphere

No charge resides on the inside of a hollow conductor$E = 0$, therefore there no work done is done to transfer a charge from the suffice of the sphere to inside hence potential remains constant

A graph of V against the distance of a charge from a hollow charged sphere



A graph of V against the distance due to charges

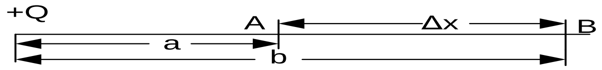


ELECTRIC POTENTIAL DIFFERENCE

Electric potential difference between two points is the work done to transfer +1C of charge form one point to the other against an electric field

Expression for electric potential

Consider two points A and B in an electric field which are Δxm apart



Force on 1C of charge,$F = \frac{Q}{4\pi\varepsilon x^2}$

Work done to move the charge through Δx against the field is $\Delta w = -F\Delta x$

Total work done to move the charge from point A to B

$$w = \int_a^b -Fdx$$

$$= \int_a^b -\frac{Q}{4\pi\varepsilon x^2}dx$$

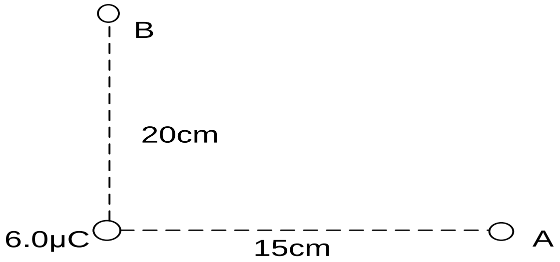
$$= -\frac{Q}{4\pi\varepsilon}\left[-\frac{1}{x}\right]_{\boldsymbol{a}}^{\boldsymbol{b}}$$

$$=-\frac{Q}{4\pi\varepsilon}\left(-\frac{1}{b}--\frac{1}{a}\right)$$

$$\boxed{V_{AB}=\frac{Q}{4\pi\varepsilon}\left(\frac{1}{b}-\frac{1}{a}\right)}$$

Examples

1. Consider two points A and B at distances of 15.0 cm and 20.0 cm respectively, from a point charge of 6.0 μC as shown below



(i) Find the electric potential difference between A and B

(ii) Calculate the energy required to bring a charge of + 1.0 μC from infinity to point A

Solution

(i) $V=\frac{Q}{4\pi\varepsilon_o r}$

$$V_A=\frac{6.0x10^{-6}x9x10^{9}}{0.15}=3.6x10^{5}V$$

$$V_B=\frac{6.0x10^{-6}x9x10^{9}}{0.2}=2.70x10^{5}V$$

$$\boldsymbol{V_{AB}=V_A-V_B}$$

$$\boldsymbol{V_{AB}}=3.6x10^{5}-2.7x10^{5}$$

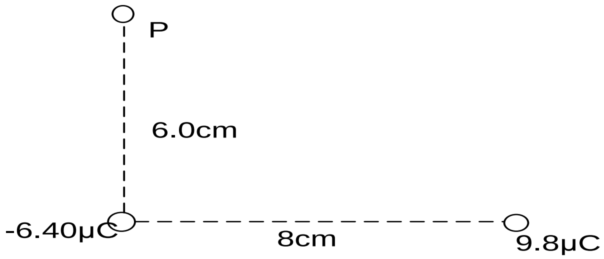
$$\boldsymbol{V_{AB}}=9.0x10^{4}V$$

(ii) $\boldsymbol{W=QV_A}$

$$\boldsymbol{W}=1.0x10^{-6}x3.6x10^{5}$$

$$\boldsymbol{W=0.36J}$$

2. Consider two point charges 9.8 μC and -6.4 μC, are placed as in figure below in airow



Find the potential energy of a charge of 2.5 μC placed at P

Solution

$$x^2=6^2+8^2$$

$$x=10cm$$

$$V=\frac{Q}{4\pi\varepsilon_o r}$$

$$V_{9.8}=\frac{9.8x10^{-6}x9x10^{9}}{0.1}=8.82x10^{5}V$$

$$V_{6.4}=\frac{-6.4x10^{-6}x9x10^{9}}{0.06}=-9.6x10^{5}V$$

$$\boldsymbol{V_P}=V_{6.4}+V_{9.8}$$

$$\boldsymbol{V_P}=-9.6x10^{5}+8.82x10^{5}$$

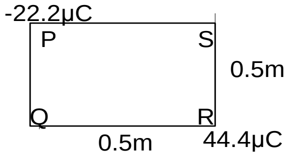
$$\boldsymbol{V_P}=-7.8x10^{4}V$$

$$\boldsymbol{P.e=QV_P}$$

$$\boldsymbol{P.e=2.5}x10^{-6}x-7.8x10^{4}$$

$$\boldsymbol{P.e=-0.195J}$$

3. The figure below shows point charges $44.4\ \mu C$ and $-22.2\mu C$ placed at the corners of a square of side 0.5m as shown below



Calculate;;

(i) Electric potential at S

(ii) Potential energy of 10μC charge placed at S

Solution

(i) $V = \frac{Q}{4\pi\varepsilon_0 r}$

$$V_R = \frac{44.4x10^{-6}x9x10^9}{0.5} = 7.99x10^5V$$

$$V_P = \frac{-22.2x10^{-6}x9x10^9}{0.5} = -3.99x10^5V$$

$$\boldsymbol{V_S = V_R + V_P}$$

$$V_S = 7.99x10^5 + -3.99x10^5$$

$$V_S = 4.0x10^5V$$

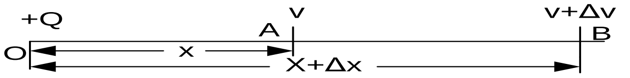
(iii) $\boldsymbol{P.e = QV_S}$

$$\boldsymbol{P.e} = 10x10^{-6}x4.0x10^5$$

$$\boldsymbol{P.e = 4J}$$

ELECTRIC POTENTIAL GRADIENT (relation between E and V)

Consider two points A and B in an electric field which are Δxm apart



If the potential at A is v and that at B is $v + \Delta v$. Then potential difference between A and B is

$$V_{AB} = V_A - V_B$$

$$V_{AB} = v - (v + \Delta v)$$

$$V_{AB} = -\Delta v \ldots\ldots\ldots\ldots\ldots..(1)$$

Work done to move 1C of change from A to B is equal to p.d and is given by

$$V_{AB} = E\Delta x \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

$$E\Delta x = -\Delta v$$

$$E = \frac{-\Delta v}{\Delta x}$$

Limit as $\Delta x \to 0$

$$\boxed{E = -\frac{dv}{dx}}$$

EQUIPOTENTIAL SURFACES

An equipotential surface is any two dimensional surface over which the electric potential is constant and work done moving charge from one point on surface to another is zero

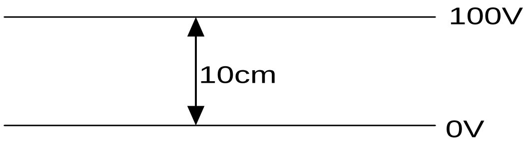
The direction of force is always at right angles to equipotential surfaces. This implies that the is no component of electric field inside the surface

Properties of equipotential surface

- Work done along an equipotential surface is zero
- Electric field intensity along surfaces is zero
- The surfaces are at right angles to the line of force

Examples

1. Calculate the electric field intensity between plates

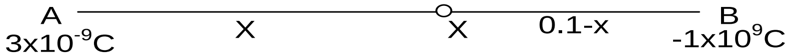


Solution

$$E = \frac{dv}{dx} \qquad \Bigg| \qquad E = \frac{100-0}{0.1} \qquad \Bigg| \qquad E = 1000Vm^{-1}$$

2. Points A and B are 0.1m apart, a point charge of $3x10^{-9}C$ is placed at A and another point charge $-1x10^{-9}C$ is placed at B. X is a point on straight line through A and B but between A and B where electric potential is zero. Calculate the distance AX

Solution



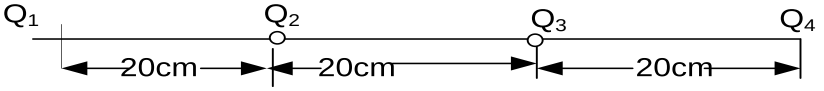
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$0 = 9x10^9x10^{-9}x\left(\frac{3}{x} - \frac{1}{0.1-x}\right)$$

$$\boldsymbol{x = 0.075m}$$

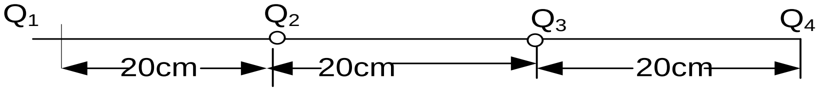
Exercise

1. Two point charge $3x10^{-9}C$ and $-1x10^{-9}C$ are placed at points A and B respectively. A and B are 0.2m apart and x is a point on a straight line through A and B but between A and B. Calculate distance BX for which electric potential at x is zero. **An($\boldsymbol{0.15m}$)**
2. The figure shows charges Q_1, Q_2, Q_3, and Q_4, of $-1\mu C, 2\mu C, -3\mu C$ and $4\mu C$ are arranged on a straight line in vacuum



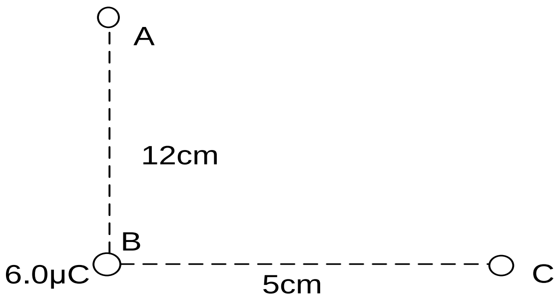
   (a) Calculate potential energy at Q2 **An($\boldsymbol{-1.8x10^{11}J}$)**
   (b) what is the significance of the sign of the potential energy above

3. Alpha particles of charge 2e each having kinetic energy $1.0x10^{-12}J$ are incident head on, on a gold nuclide of charge 79e in a gold foil. Calculate the distance of closest approach of an alpha particle and gold foil.($e = 1.6x10^{-19}C$) **An($\boldsymbol{3.64x10^{-14}m}$)**
4. The figure shows charges Q_1, Q_2, and Q_3, of $5\mu C, 6\mu C$,and $-20\mu C$ are arranged on a straight line in vacuum



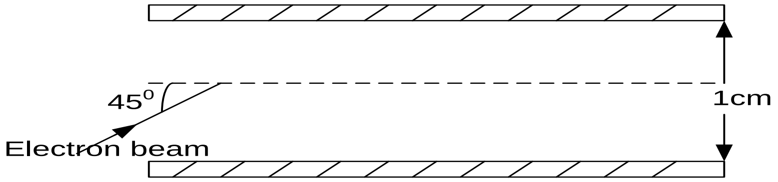
   (a) Calculate electric field intensity midway between Q_1 and Q_2 **An(4.44$\boldsymbol{x10^6Vm^{-1}}$)**
   (b) Calculate electric potential midway between Q_1 and Q_2 **An($\boldsymbol{7.85x10^5V}$)**

5. Consider two points A and C at distances of 12.0 cm and 5.0 cm respectively, from a point charge of 6.0 μC situated at B as shown below



Calculate the energy required to bring a charge of + 2.0 μC from A to point C. **An (1.26J)**

6. Two large oppositely charged plates are fixed 1.0 cm apart as shown below. The p.d between the plates is 50V.



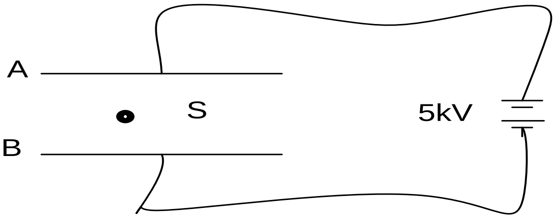
An electron beam enters the region between the plates at an angle of 45° as shown. Find the maximum speed the electrons must have in order for them not to strike the upper plate
[Mass of an electron = $9.11\ x\ 10^{-31}$ kg.]

7. A conducting sphere of radius 9.0 cm is maintained at an electric potential of 10kV. Calculate tahe charge on the sphere. **An**$(\mathbf{1x10^{-7}C})$

Uneb 2016

(a) (i) Explain an equipotential surface. (04marks)
(ii) Give an example of an equipotential surface. (01mark)

(b) (i) State **coulomb's law.** (01mark)
(ii) With the aid of a sketch diagram, explain the variation of electric potential with distance from the centre of a charged metal sphere. (03marks)
(iii) Two metal plates A and B, 30cm apart are connected to a 5kV d.c supply as shown below



When a small charged sphere, S, of mass $9.0x10^{-3}kg$ is placed between the plates, it remains stationary. Indicate the forces acting on the sphere and determine the magnitude of the charge on the sphere. (04marks)

(c) (i) Define **electric field intensity** (01mark)
(ii) With the aid of a diagram, explain electrostating shielding. (04marks)

(d) Explain briefly a neutral metal body is attracted to a charged body when broguth near it. (02marks)

Uneb 2015

(a) (i) Define **electric potential** (01mark)
(ii) Derive an expression for the electric potential at a point of a distance r, from a fixed charge. (04marks)

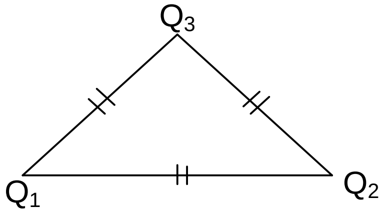
(b) With reference to a charged pear-charged conductor.

(i) Describe an experiment to show the distribution of charge on it. (03marks)

(ii) Show that the surface of the conductor is an equipotential surface. (03marks)

(c) Explain how a lightning conductor protects a house from lightning. (04marks)

(d) Three charges Q_1, Q_2 and Q_3 of magnitude $2\mu C$, $-3\mu C$, and $5\mu C$ respectively are situated at corners of an equilateral triangle of sides 15cm as shown below.



Calculate the net force on Q_3. (05marks)

CAPACITORS

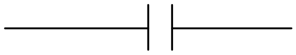
A capacitor is a device which stores charge

A capacitor consists of a pair of oppositely charged plates separated by an insulator called a dielectric.

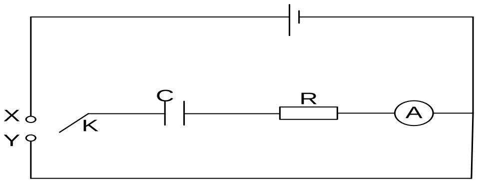
A dielectric is an insulator which breaks down when the potential difference is very high

The dielectric is can be air, oil, glass or a paper

The symbol of a capacitor is

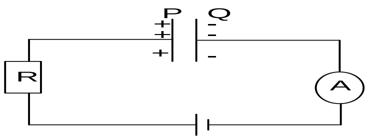


Charging and Discharging process



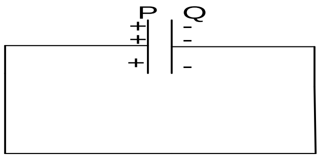
- When switch k is brought to contact x, the capacitor, c charges. Current flowing through the ammeter is initially high but slowly comes to zero with time when the capacitor is fully charged
- If switch k is brought in contact with y, capacitor c is discharged. The current is initially high but eventually comes to zero and in opposite direction to that when the capacitor is being charged.

Explanation of charging process

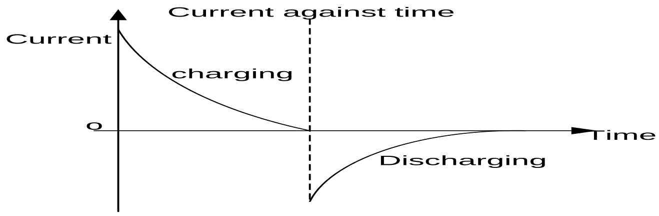


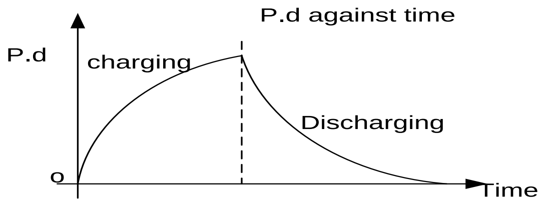
- When the capacitor is connected to a battery, electrons flow from the negative terminal of the battery to the adjacent plate of the capacitor and at the same rate electrons flow from plate P of the capacitor towards the positive terminal of the battery leaving positive charges at P.
- Positive and negative charges therefore appear on the plate and oppose the flow of electrons that cause them
- As charge accumulates the p.d between the plates increase and charge current falls to zero when the p.d between the plates of the capacitor is equal to battery voltage

Explanation of discharging process



Connect a wire from the positive plate to the negative plate. Electrons flow form the negative plate to positive plate through wire until the p.d is zero. The capacitor is fully discharged





Note

Energy changes in charging a capacitor include Chemical energy is changed to heat and electrical energy which is stored in the plates of the capacitor.

Capacitance of capacitor

This is the ratio of the magnitude of charge on either of the plates of a capacitor to the p.d between the plates of the capacitor

$$C = \frac{Q}{V}$$

The S.I unit of capacitance is farad, F

Definition

The farad is the capacitance of the capacitor when one coulomb of charge changes its potential difference by one volt.

Examples

Given the capacitance of capacitor of 4μF and charge on the plate is 5μC. Find the p.d across the plate.

Solution

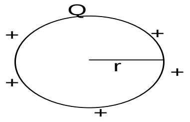
$$C = \frac{Q}{V}$$

$$V = \frac{5x10^{-6}}{4x10^{-6}}$$

$$V = 1.25V$$

Capacitance of an isolated sphere

Consider an isolated sphere of radius r. If the conductor is given charge Q, then its p.d is



$$V = \frac{Q}{4\pi\varepsilon_O r}$$

Where ε_O – is permittivity of free space

$$4\pi\varepsilon_O r = \frac{Q}{V}$$

$$\boxed{C = 4\pi\varepsilon_O r}$$

Example

Calculate the capacitance of the earth given that the radius of the earth is 6.4x10^{6}m

Solution

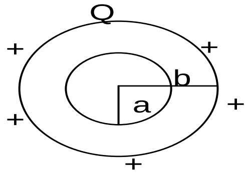
$C = 4\pi\varepsilon_O r$

$C = 4x3.14x8.85x10^{-12}x6.4x10^{6}$

$C = 7.12x10^{-4}F$

Capacitance of concentric spheres

Consider two concentric sphere A and B each of radius a and b respectively.



$$V = \frac{Q}{4\pi\varepsilon_O}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\frac{V}{Q} = \frac{1}{4\pi\varepsilon_O}\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{V} = 4\pi\varepsilon_O\left(\frac{ab}{b-a}\right)$$

$$C = 4\pi\varepsilon_O\left(\frac{ab}{b-a}\right)$$

Example

1. Find the capacitance of concentric spheres of radius 9cm and 10cm. Given that $\varepsilon_O = 8.85x10^{-12}Fm^{-1}$

   Solution

   $$C = 4\pi\varepsilon_O\left(\frac{ab}{b-a}\right)$$

   $$C = 4x3.14x8.85x10^{-12}\left(\frac{0.1x0.09}{0.1-0.09}\right)$$

   $$C = 1.0x10^{-10}F$$

2. Given two concentric sphere of radius 5cm and 2cm separated by material of permittivity $8.0x10^{-11}Fm^{-1}$. Calculate it capacitance

Solution

$$C = 4\pi\varepsilon_O\left(\frac{ab}{b-a}\right)$$

$$C = 4x3.14x8.0x10^{-11}\left(\frac{0.02x0.05}{0.05-0.01}\right)$$

$$C = 3.352x10^{-11}F$$

Capacitance of a parallel plate capacitor

Consider two parallel plate of capacitors each having charge Q and an area A separated by a distance d by a dielectric of permittivityε . Total electric flux $\emptyset$ through the surface is given by: $\emptyset = AE$......................(1)

Where E is electric field intensity

From Gauss law $\emptyset = \frac{Q}{\varepsilon}$...................................(2)

Equating (1) and (2) $\frac{Q}{\varepsilon} = AE$

But $E = \frac{V}{d}$

$$\frac{Q}{\varepsilon} = A\frac{V}{d}$$

$$\frac{Q}{V} = \frac{\varepsilon A}{d}$$

$$\boxed{C = \frac{\varepsilon A}{d}}$$

For parallel plate capacitor placed in vacuum or air $\boxed{C = \frac{\varepsilon_o A}{d}}$

Example

4. Calculate the capacitance of a parallel capacitor whose plates are 10 cm by 10 cm separated b an air gap of 5 mm

Solution

$$C = \frac{\varepsilon_o A}{d}$$

$$C = \frac{8.85x10^{-12}x0.1x0.1}{0.005}$$

$$C = 1.77x10^{-11}F$$

5. A parallel plate capacitor consists of two separate plates each of size 25cm and 3.0mm apart. If a p.d of 200V is applied to the capacitor. Calculate the charge in the plates

Solution

$$C = \frac{\varepsilon_o A}{d}$$

$$C = \frac{8.85x10^{-12}x0.25x0.25}{0.003}$$

$$C = 1.854x10^{-10}F$$

$$C = \frac{Q}{V}$$

$$Q = 1.854x10^{-10}x200$$

$$Q = 3.708x10^{-8}C$$

6. The plates of a parallel plate capacitor each of area 2.0 cm^2 are 5 mm apart. The plates are in vacuum and a potential difference of 10,000V is applied across the capacitor. Find the magnitude of the charge on the capacitor.

RELATIVE PERMITIVITY / DIELECRTIC CONSTANT

It is defined as the ratio of capacitance of a capacitor when the insulating material (dielectric) between its plates to the capacitance of the same capacitor with a vacuum between it plates

$\boldsymbol{\varepsilon_r = \frac{C}{C_O}}$.................(1)

$$\boxed{\boldsymbol{C = \varepsilon_r C_O}}$$

But

$C = \frac{\varepsilon A}{d}$ and $C = \frac{\varepsilon_o A}{d}$ put into (1)

$$\boldsymbol{\varepsilon_r} = \frac{\left(\frac{\varepsilon A}{d}\right)}{\left(\frac{\varepsilon_o A}{d}\right)}$$

$\boldsymbol{\varepsilon_r} = \frac{\varepsilon}{\varepsilon_O}$............(2)

$$\boxed{\boldsymbol{\varepsilon = \varepsilon_r \varepsilon_O}}$$

Relative permittivity can also be defined as the ratio of the permittivity of a material to permittivity of free space

Examples

1. A parallel plate capacitor was charged to 100V and then isolated. When a sheet of a dielectric is inserted between its plates, the p.d decreases to 30V. Calculate the dielectric constant of the dielectric.

Solution

By law of conservation of charge

$$Q_0 = Q$$
$$C_0V_0 = CV$$
$$\frac{V_0}{V} = \frac{C}{C_0}$$
$$\varepsilon_r = \frac{100}{30}$$
$$\varepsilon_r = 3.33$$

2. A $2\mu F$ capacitor that can just withstand a p.d of 5000V uses a dielectric with a dielectric constant 6 which breaks down if the electric field strength in it exceeds $4x10^7 Vm^{-1}$. Find the;
   (i) Thickness of the dielectric
   (ii) Effective area of each plate
   (iii) Energy stored per unit volume of dielectric

Solution

(i) $E = \frac{V}{d}$

$$4x10^7 = \frac{5000}{d}$$
$$d = 1.25x10^{-4}m$$

(ii) $C = \frac{\varepsilon A}{d}$

$$\varepsilon = \varepsilon_r\varepsilon_o$$
$$2x10^{-6} = \frac{6x\,8.85x10^{-12}xA}{1.25x10^{-4}}$$
$$A = 4.71m^2$$

(iii) $\frac{Enegry}{volume} = \frac{\frac{1}{2}CV^2}{Ad}$

$$= \frac{\frac{1}{2}x2x10^{-6}x5000^2}{4.71x1.25x10^{-4}}$$
$$= 4.246x10^4Jm^{-3}$$

3. A parallel plate capacitor has an area of $100cm^2$, plate separation of 1cm and charged initially with the p.d of 100V supply, it is disconnected and a slab of dielectric 0.5cm thick and relative permittivity 7 is then placed between plates.
   (a) Before the slab was inserted calculate;
      (i) Capacitance
      (ii) Charge on the plates
      (iii) Electric field strength in the gap between plates
   (b) After the dielectric was inserted, find;
      (i) Electric field strength
      (ii) P.d between the plates
      (iii) capacitance

Solution

(a) (i) $C = \frac{\varepsilon_o A}{d}$

$$C = \frac{8.85x10^{-12}x100x10^{-4}}{1x10^{-2}}$$
$$C = 8.85x10^{-12}F$$

(ii) $Q = CV$

$$Q = 8.85x10^{-12}x100$$
$$Q = 8.85x10^{-10}C$$

(iii) $E = \frac{V}{d}$

$$E = \frac{100}{1x10^{-2}}$$
$$E = 1x10^4Vm^{-1}$$

(b) (i) $E = \frac{Q}{\varepsilon A}$

$$\varepsilon = \varepsilon_r\varepsilon_o$$
$$E = \frac{8.85x10^{-10}}{7x8.85x10^{-12}x100x10^{-4}}$$
$$E = 1.43x10^3Vm^{-1}$$

(ii) $E = \frac{V}{d}$

$$1.43x10^3 = \frac{V}{0.5x10^{-2}}$$
$$V = 7.15V$$

(iii) $C = \frac{\varepsilon A}{d}$

$$\varepsilon = \varepsilon_r\varepsilon_o$$
$$C = \frac{7x\,8.85x10^{-12}x100x10^{-4}}{0.5x10^{-2}}$$
$$C = 1.24x10^{-10}F$$

DIELECRTIC STRENGHT

It is the maximum electric field intensity an insulator can with stand without conducting

Or Is the maximum potential gradient an insulator can with stand without conducting

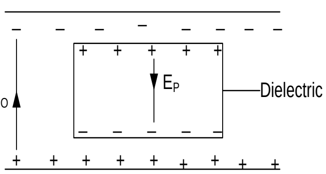
USES OF DIELECTRIC

- It should increase the capacitance of a capacitor
- It is used to separate the plates of a capacitor
- It reduces the chance of dielectric breakdown

QUALITIES OF GOOD DIELECTRIC

- It should have a large dielectric constant
- It should have high dielectric strength

ACTION OF DIELETRIC



- The molecules of the insulator get <u>polarized</u>. Charge inside the material cancel each other's influence but the surfaces adjacent to the plates develop charge <u>opposite</u> to that on the <u>near plate.</u>
- Since charges are bound, <u>electric field intensity,</u>E_P develops between the <u>opposite faces</u> of the insulator in <u>opposition to the applied field,</u> E_O
- The <u>resultant</u> electric field intensity between the plates is thus <u>reduced</u>. But electric field intensity, $E = \frac{V}{d}$ thus p.d between the plates <u>reduces</u>, since capacitance, $C = \frac{Q}{V}$ hence capacitance <u>increases</u>

Note:

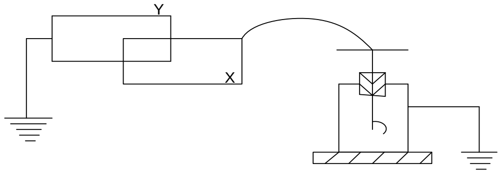
If a conductor instead a dilectric is palced between the plates of a charged capacitor, charge reduces to zero on the plates. This is because electrons move form the negative palet to the positive plate to neutralize the positive charge

FACTORS THAT AFFECT CAPAITANCE OF A CAPACITOR

Capacitance of a capacitor is affected by;

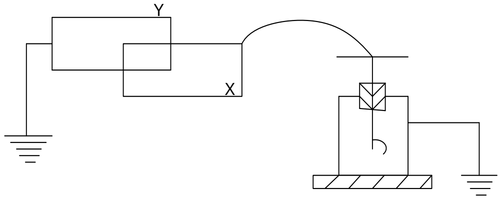
(i) Area of overlap of the plates
(ii) Distance of separation of the plates
(iii) Dielectric

Experiment to show the effect of area of overlap on capacitance



- Plate x is <u>charged</u> and <u>divergence</u> of the leaf of the electroscope <u>noted</u>
- Plate y is then displaced <u>upwards</u> relative to x and the <u>divergence</u> of the leaf of the electroscope is seen to <u>increase</u>
- The p.d between the plates has increased. Since $C = \frac{Q}{V}$, capacitance has <u>decreased</u> with <u>decrease</u> in area and $C \propto A$

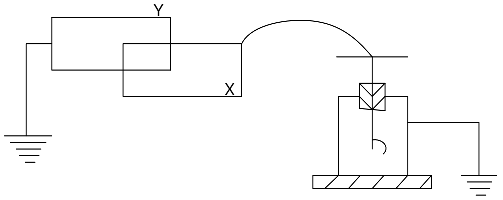
Experiment to show the effect of plate separation on capacitance



- XY are metal plates near each other but not touching.

- Plate x is charged and divergence of the leaf of the electroscope noted
- Plate y is then moved closer to x and the divergence of the leaf of the electroscope is seen to decrease
- The p.d between the plates has decreased. Since $C = \frac{Q}{V}$, capacitance has increased with decrease plate separation and $C \propto \frac{1}{d}$

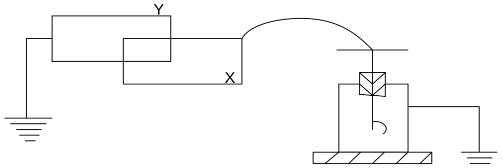
Experiment to show the effect of dielectric on capacitance



- XY are metal plates near each other but not touching.

- Plate x is charged and divergence of the leaf of the electroscope noted
- Insert a dielectric between the plates and the divergence of the leaf of the electroscope is seen to decrease
- The p.d between the plates has decreased. Since $C = \frac{Q}{V}$, capacitance has increased and $C \propto \varepsilon$

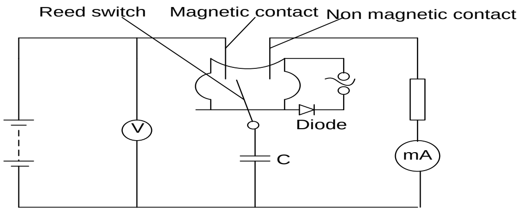
Investigation of all factors that affect capacitance of a parallel plate capacitor



- Plate x is charged and divergence of the leaf of the electroscope noted
- Plate y is then displaced upwards relative to x and the divergence of the leaf of the electroscope is seen to increase. The p.d between the plates has increased. Since $C = \frac{Q}{V}$, capacitance has decreased with decrease in area and $C \propto A$
- Plate y is now restored to its intial position. Plate y is then moved closer to x and the divergence of the leaf of the electroscope is seen to decrease. The p.d between the plates has decreased. Since $C = \frac{Q}{V}$, capacitance has increased with decrease plate separation and $C \propto \frac{1}{d}$
- The plates are restored, an insulator Inserted between the plates. Divergence of the leaf decreases. Since $C = \frac{Q}{V}$, capacitance has increased and $C \propto \varepsilon$

Measurement of capacitance

(a) Using a reed switch



- The circuit is connected as above

- The switch is closed, the microameter reading I is taken together with the voltmeter reading
- Knowing the frequency f of the A.C in the read switch circuit, the capacitance of the capacitor is calculated form

$$C = \frac{I}{fV}$$

Example

1. A capacitor filled with a dielectric is charged and then discharged through a milliameter. The dielectric is then withdrawn half way and the capacitor charged to the same voltage, and discharged through the milliammeter again, show the relative permitivity, ε_r of the dielectric is given by

$$\varepsilon_r = \frac{I}{2I^1 - I}$$

Where I, and I^1 are the readings of the milliammeter respectively

Solution

$$I = cfV$$

$$I = \frac{\varepsilon A}{d} fV \ldots\ldots\ldots\ldots (i)$$

When the dielectric is withdrawn half way, the area is halved and both portions one with a dielectric and the other with out a dielectric cont1ribute to current.

$$I^1 = \frac{\varepsilon AfV}{2d} + \frac{\varepsilon_0 A fV}{2d}$$

$$2\,I^1 = \frac{\varepsilon AfV}{d} + \frac{\varepsilon_0 A fV}{d}$$

$$2\,I^1 = I + \frac{\varepsilon_0 A fV}{d}$$

$$2\,I^1 - I = \frac{\varepsilon_0 A fV}{d} \ldots\ldots\ldots\ldots\ldots (ii)$$

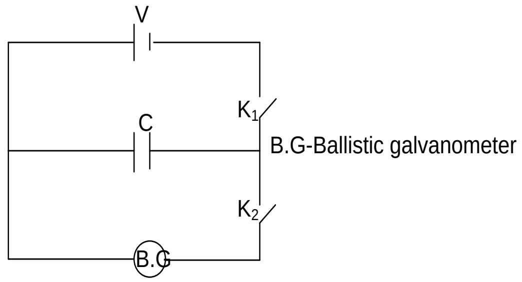
$$eq\ (i)\ \div (ii)$$

$$\frac{I}{2\,I^1 - I} = \frac{\varepsilon A f V}{d} \Big/ \frac{\varepsilon_0 A fV}{d}$$

$$\frac{I}{2\,I^1 - I} = {}^{\varepsilon}/_{\varepsilon_0} = \varepsilon_r$$

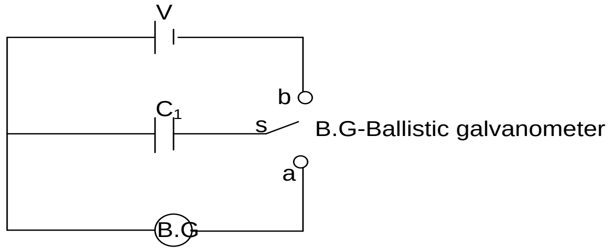
$$\varepsilon_r = \frac{I}{2I^1 - I}$$

(b) Using a Ballistic galvanometer



- The circuit is connected as shown above
- First with a standard capacitor of capacitance, C_s switch K_1 is closed and after a short time it is opened
- Switch K_2 is now closed and the deflection of B.G, θ_s is noted
- The capacitor is replaced with the test capacitor of capacitance C and the procedure repeated. The deflection, θ of B.G is noted
- Capacitance, C is calculated from $C = \frac{\theta}{\theta_s} C_s$

Comparing capacitance using B.G

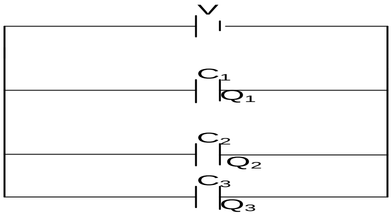


- The capacitor of capacitance C_1 is charged by connecting s to b. After sufficiently charging, s is now connected to a. The deflection θ_1 of the B.G is noted
- The capacitor of capacitance C_1 is then replaced by one of capacitance C_2
- The capacitor is charged by connecting s to b. It is then discharged through a B.G by connecting s to a. The deflection θ_2 of the B.G is noted

$$Then\ \frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

CAPACITOR NETWORKS

(a) Capacitors in parallel



For capacitors connected in parallel p.d across the plate of capacitors is the same

$$Q = Q_1 + Q_2 + Q_3$$

But $Q_1 = C_1V, Q_2 = C_2V$ and $Q_3 = C_3V$

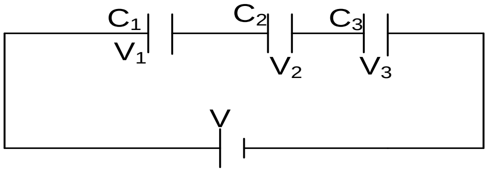
$$Q = C_1V + C_2V + C_3V$$

$$\frac{Q}{V} = C_1 + C_2 + C_3$$

$$but\ \frac{Q}{V} = C$$

$$\boxed{C = C_1 + C_2 + C_3}$$

(b) Capacitors in series



For capacitors connected in series charge stored on the plates of capacitor is the same

$$\boldsymbol{V = V_1 + V_2 + V_3}$$

But $V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}$ and $V_3 = \frac{Q}{C_3}$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

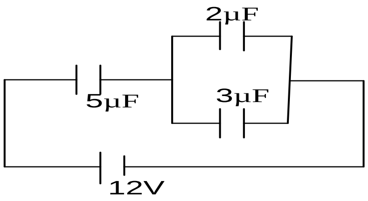
$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$but\ \frac{V}{Q} = \frac{1}{C}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Examples

1.



A battery of emf 12V is connected across a system of capacitor. Calculate the total energy stored in capacitor network.

Solution

$$C_P = 2\mu F + 3\mu F$$

$$C_P = 5\mu F$$

$$\frac{1}{C} = \frac{1}{C_5} + \frac{1}{C_P}$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{5}$$

$$\frac{1}{C} = \frac{5+2}{2x5}$$

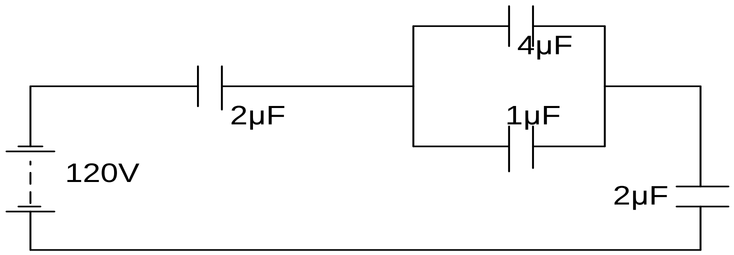
$$C = 2.5\mu F$$

$$Energy\ stored = \frac{1}{2}CV^2$$

$$Energy\ stored = \frac{1}{2}x2.5x10^{-6}x12^2$$

$$Energy\ stored = 1.8x10^{-4}J$$

2.



The diagram above shows a network of capacitors connected to a 120V supply. Calculate the;

(i) Charge on the $4\mu F$ capacitor

(ii) Energy stored in 1 μF capacitor

Solution

$$C_P = 4\mu F + 1\mu F$$
$$C_P = 5\mu F$$
$$\frac{1}{C} = \frac{1}{C_2} + \frac{1}{C_P} + \frac{1}{C_2}$$
$$\frac{1}{C} = \frac{1}{2} + \frac{1}{5} + \frac{1}{2}$$
$$\frac{1}{C} = \frac{6}{5}$$
$$C = \frac{5}{6}\mu F$$

Total charge flowing in the circuit, $Q = CV$

$$Q = \frac{5}{6}x10^{-6}x120$$
$$Q = 1.0x10^{-4}C$$

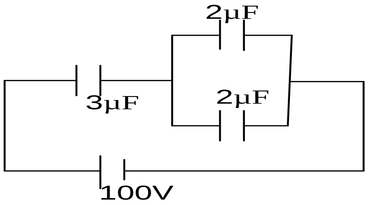
p.d across the paralle combination; $V_p = \frac{Q}{C}$

$$V = \frac{1.0x10^{-4}}{5x10^{-6}}$$
$$V = 20V$$

Charge on $4\mu F$ Capacitor $Q_4 = CV$

$$Q_4 = 4x10^{-6}x20$$
$$Q_4 = 8.0x10^{-5}C$$
$$Energy\ stored\ 1\mu F\ \text{Capacitor} = \frac{1}{2}CV^2$$
$$Energy\ stored = \frac{1}{2}x1x10^{-6}x20^2$$
$$Energy\ stored = 2x10^{-4}J$$

3.



Calculate energy stored in a system of capacitors, if the space between the $3\mu F$ is filled with an insulator of dielectric constant 3 and capacitors are fully charged

Solution

$$C_P = 2\mu F + 2\mu F$$
$$C_P = 4\mu F$$
$$C_3{}' = \boldsymbol{\varepsilon_r} C_0$$
$$C_3{}' = 3x3\mu F$$
$$C_3{}' = 9\mu F$$
$$\frac{1}{C} = \frac{1}{C_3{}'} + \frac{1}{C_P}$$
$$\frac{1}{C} = \frac{1}{9} + \frac{1}{4}$$
$$\frac{1}{C} = \frac{4+9}{9x4}$$
$$C = \frac{36}{13}\mu F$$
$$Energy\ stored = \frac{1}{2}CV^2$$
$$Energy\ stored = \frac{1}{2}x\frac{36}{13}x10^{-6}x12^2$$
$$Energy\ stored = 13.8x10^{-3}J$$

4. **A** $47\mu F$ capacitor is used to power the flash gun of a camera. The average power output of the gun is 4.0kW for a duration of the flash which is 2.0ms. Calculate the;

(i) Potential difference between the terminals of the capacitor immediately before a flash

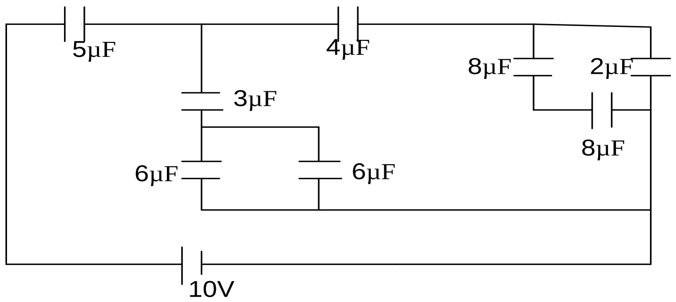
(ii) Maximum charge stored by the capacitor

(iii) Average current provided by the capacitor during a flash

Solution

$$\frac{1}{2}CV^2 = pt$$
$$\frac{1}{2}x47x10^{-6}x(V)^2$$
$$= 4x10^3x2x10^{-3}$$
$$\boldsymbol{V = 583.5V}$$
$$\boldsymbol{Q = CV}$$
$$\boldsymbol{Q} = 47x10^{-6}x583.5$$
$$\boldsymbol{Q} = 2.74x10^{-2}C$$
$$\boldsymbol{I = \frac{Q}{t}}$$
$$\boldsymbol{I} = \frac{2.74x10^{-2}}{2x10^{-3}}$$
$$\boldsymbol{I = 13.7A}$$

5.



The figure above shows a network of capacitors connected to a 10V battry. Calculate the total energy stored in the network.

Solution

$8\mu F$ and $8\mu F$ are in series

$$\frac{1}{C} = \frac{1}{8} + \frac{1}{8}$$

$$\frac{1}{C} = \frac{8+8}{8x8}$$

$$C = 4\mu F$$

$2\mu F$ is in parallel with $4\mu F$

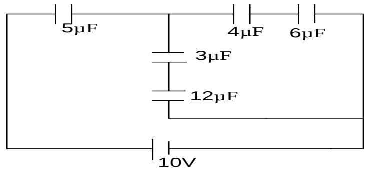
$$C = 4 + 2$$

$$C = 6\mu F$$

$6\mu F$ is in parallel with $6\mu F$

$$C = 6 + 6$$

$$C = 12\mu F$$



$4\mu F$ and $6\mu F$ are in series

$$\frac{1}{C} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{C} = \frac{4+6}{4x6}$$

$$C = \frac{12}{5}\mu F$$

$3\mu F$ and $12\mu F$ are in series

$$\frac{1}{C} = \frac{1}{3} + \frac{1}{12}$$

$$\frac{1}{C} = \frac{3+12}{3x12}$$

$$C = \frac{12}{5}\mu F$$

$$C_P = \frac{12}{5}\mu F + \frac{12}{5}\mu F$$

$$C_P = \frac{24}{5}\mu F$$

$$\frac{1}{C} = \frac{1}{C_5} + \frac{1}{C_P}$$

$$\frac{1}{C} = \frac{1}{5} + \frac{1}{\left(\frac{24}{5}\right)}$$

$$\frac{1}{C} = \frac{\frac{24}{5} + 5}{\frac{24}{5}x5}$$

$$C = \frac{120}{49}\mu F$$

$$Energy\ stored = \frac{1}{2}CV^2$$

$$E = \frac{1}{2}x\frac{120}{49}x10^{-6}x12^2$$

$$E = 1.224x10^{-4}J$$

ENERGY STORED IN A CAPACITOR

Suppose the p.d between the plates at some instant was V. When a small charge of $+\delta q$ is transferred from the negative plate to the positive plate, the p.d increases by δv.

Work done to transfer charge,

$$\delta w = (V + \delta v)\delta q$$

$$\delta w \approx V\delta q$$

But $V = \frac{q}{c}$

$$\delta w = \frac{q}{C}\delta q$$

Total work done to charge the capacitor to Q is

$$W = \int_0^Q \frac{q}{C}dq$$

$$= \frac{1}{2}\frac{Q^2}{C}$$

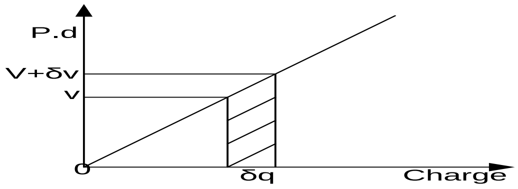
$$but\ Q = CV$$

$$\boxed{W = \frac{1}{2}CV^2}$$

ALTERNATIVELY

From $q = CV$

V is proportional to q, giving a graph of v against q



Area of the shaded part $= \frac{1}{2}(V + V + \delta v)\delta q$

=work done to increase charge on the capacitor from $q = 0$ to $q = Q$

Work done w$=$ $average\ voltage\ x\ charge$

$$= \frac{1}{2}(0 + V)Q$$

$$= \frac{1}{2}QV$$

$$but\ Q = CV$$

$$\boxed{W = \frac{1}{2}CV^2}$$

JOINING TWO CAPACITORS

When two capacitors are joined together;

- Charge flows until p.d across the capacitors is the same
- Total charge on the circuit is conserved
- Capacitors are in parallel ie $\boldsymbol{C = C_1 + C_2}$

NOTE

There is loss of energy when capacitors are joined together. This is because charge flows until the p.d across the capacitor is the same. The flow of charge results in heating of the wire and hence loss in energy

Examples

b. A 5μF capacitor is charged by a 40V supply and then connected to an un charged 20μF capacitor. Calculate;

(i) Final p.d across each capacitor
(ii) Final charge on each
(iii) Energy lost

Solution

(i) $C = C_1 + C_2$

$C = 5x10^{-6} + 20x10^{-6}$

$C = 2.5x10^{-5}F$

Charge before= charge after connection

$\boldsymbol{Q_1 + Q_2 = Q}$

$\boldsymbol{C_1V_1 + C_2V_2 = CV}$

$5x10^{-6}\boldsymbol{x40} + 20x10^{-6}x0 = 2.5x10^{-5}V$

$V = 8V$

(ii) $\boldsymbol{Q = Q_1 + Q_2}$

$\boldsymbol{Q = C_1V_1 + C_2V_2}$

$Q = 5x10^{-6}\boldsymbol{x40} + 20x10^{-6}x0$

$\boldsymbol{Q = 2x10^{-4}C}$

OR

$\boldsymbol{Q = CV}$

$Q = 2.5x10^{-5}x8$

$Q = \boldsymbol{2x10^{-4}C}$

(iii) Energy lost = energy before—energy after

$$= \left(\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{C_1V_1}^2 + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{C_2V_2}^2\right) - \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{CV^2}$$

$$= \left(\frac{\mathbf{1}}{\mathbf{2}}x5x10^{-6}\boldsymbol{x40^2} + \frac{\mathbf{1}}{\mathbf{2}}x20x10^{-6}\boldsymbol{x0^2}\right)$$

$$- \left(\frac{\mathbf{1}}{\mathbf{2}}x2.5x10^{-5}x\boldsymbol{8^2}\right)$$

$$= 0.0032J$$

c. A capacitor of 20μF is connected across 50V battery supply. When it has fully charged it is then disconnected and joined to capacitor of 40 μF having a p.d of 100V. Calculate;

(i) Effective capacitance after joining
(ii) The p.d on each capacitor
(iii) Energy lost

Solution

(i) $C = C_1 + C_2$

$C = 20x10^{-6} + 40x10^{-6}$

$C = 6.0x10^{-5}F$

(ii) Charge before= charge after connection

$\boldsymbol{Q_1 + Q_2 = Q}$

$\boldsymbol{C_1V_1 + C_2V_2 = CV}$

$20x10^{-6}\boldsymbol{x50} + 40x10^{-6}x100 = 6.0x10^{-5}V$

$V = 83.33V$

(iii) Energy lost = energy before−energy after

$$= \left(\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{C_1V_1}^2 + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{C_2V_2}^2\right) - \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{CV^2}$$

$$= \left(\frac{\mathbf{1}}{\mathbf{2}}x20x10^{-6}x\mathbf{50^2} + \frac{\mathbf{1}}{\mathbf{2}}x40x10^{-6}\boldsymbol{x100^2}\right)$$

$$- \left(\frac{\mathbf{1}}{\mathbf{2}}x6.0x10^{-5}x\mathbf{83.33^2}\right)$$

$$= 0.017J$$

d. A paralle plate air-capacitor is charged to a potential difference of 20V. It is then connected in parallel with an uncharged capacitor of similar dimensions but having ebonite as its dieelectric medium. The potential difference of the combination falls to 15V. Calculate the dielectric constant of the ebonite

Solution

Charge before= charge after connection

$\boldsymbol{Q_0 = Q}$

$\boldsymbol{C_0V_0 = CV}$

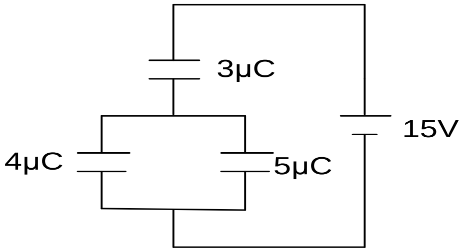
$\boldsymbol{C_0V_0 = (C_0 + \varepsilon_r C_0)V}$

$\mathbf{29 = (1 + \varepsilon_r)15}$

$\boldsymbol{\varepsilon_r = 0.33}$

Exercise

1. A capacitor is charged by a 30V d.c supply. When the capacitor is fully charged, it is found to carry a charge of 6.0μC. find the;
   (i) Capacitance of the capacitor
   (ii) Energy stored in the capacitor
   An((i) $\mathbf{2.0x10^{-7}F}$, **(ii)** $\mathbf{9.0x10^{-5}J, 1.5x10^{-2}J}$**)**
2. Two capacitors of 2μF and 3μF are charged to a p.d of 50V and 1000V respectively. Calculate;
   (i) Charge stored in each
   (ii) Energy stored in each
   (iii) Suppose capacitors are now joined with plate of the same charge connected together. Find the energy lost in the circuit
   An((i) $\mathbf{1x10^{-4}C, 3x10^{-4}C}$ **(ii)** $\mathbf{2.5x10^{-3}J, 1.5x10^{-2}J}$**(iii)** $\mathbf{1.5x10^{-3}J}$**)**
3. A 100 μF is charged from a supply of 1000V. it is then disconnected and then connected to an uncharged 50 μFcapacitor. Calculate;
   (i) Total energy stored initially and finally in the two capacitors
   (ii) Energy lost **An(50J, 0.333J, 49.667J)**
4. A 20 μF capacitor was charged to 1000V and then connected across an uncharged 60 μF capacitor. Calculate the p.d across a 60 μF capacitor **An(10V)**
5. A 10 μF capacitor was charged to 300V and then connected across an uncharged 60 μF capacitor. Calculate the total energy stored in both capacitors before and after connection. **An(0.45J, 0.064J)**
6. A 60 μF capacitor was charged from a 100V supply and then connected across an uncharged 15 μF capacitor. Calculate the final p.d across the combination and the energy lost **An(80V, 23.7J)**
7. A 20μF capacitor is charged to 40V and then connected across an uncharged 60μF capacitor. Calculate the potential difference across the 60μF capacitor. **An(10V)**
8. An air capacitor of capacitance 400μF. is charged to 180V and the connected across un charged capacitor of capacitance 500μF.
   (i) Find the energy stored in the 500μF capacitor
   (ii) With the two capacitors still connected, a dielectric of dielectric constant 1.5 is inserted between the plates of the 400 μF. capacitor. If the separation between the plates remains the same, find the new p.d across the two capacitors. **An(116J, 65.5J)**
9. A battery of e.m.f 15V is connected across a system of capacitors as shown below .

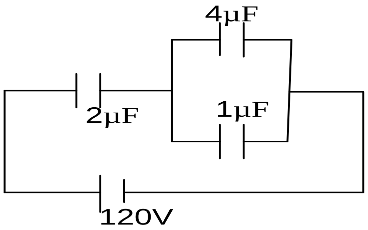


Find the

(i) charge on the 4μF capacitor

(ii) energy stored in the 3μF capacitor

10. Figure 9 shows a network of capacitors connected to a d.c. supply of 120 V.



Calculate the

(i) charge on 4 μF capacitor

(ii) energy stored in 1μF capacitor

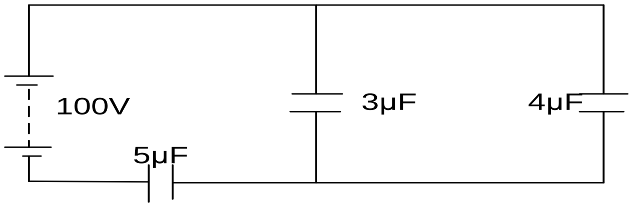
11. A 20 μF capacitor is charged to 40V and then connected across an uncharged 60 μF capacitor. Calculate the potential difference across the 60 μF capacitor

12. A 60μF capacitor is charged from a 100V supply. It is then connected across the terminals of a 15μF uncharged capactiro. Calculate

(i) the final p.d across the combination **An(80V)**

(ii) the difference in the initial and final energies stored in the capacitors and comment on the difference **An(0.06J)**

13.

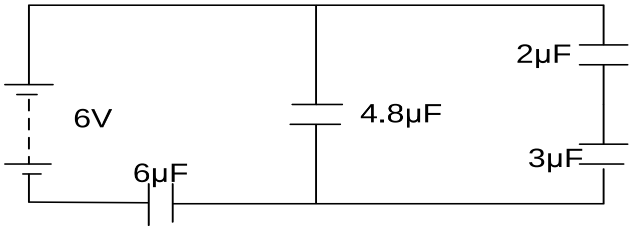


(i) Find the resultant capacitance in the circuit

(ii) Calculate the charge stored in wach capacitor

An($\frac{35}{12}\mu F$**,** $\frac{35}{12}x10^{-4}C, 1.25x10^{-4}C, 1.67x10^{-4}C$**)**

14.

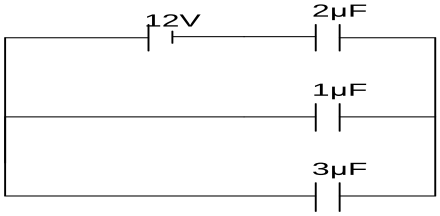


Find;

(i) Effective capacitance

(ii) Energy stored in the 2 capacitor **An(** $3\mu F$**,** $3.24x10^{-6}J$**)**

15.



Find the energy stored in the capacitor of capacitance 3μF shown in figure above, when the capacitor is fully charged

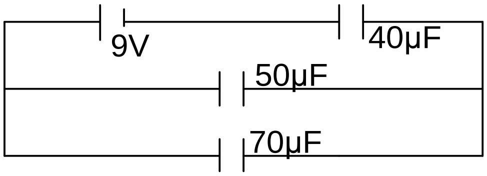
Uneb 2016

(a) (i) What is meant by capacitance of a capacitor. (01mark)

(ii) A parallel plate capacitor is connected across a battery and charged fully. When a dielectric material is now inserted between its plate, the amount of charge stored in the capacitor changes. Explain the change. (04marks)

(iii) Describe an experiment to determine the relative permittivity of a dielectric. (04marks)

(b) A network of capacitors of capacitance 40μF, 50μF, and 70μF is connected to a battery of 9V as shown below.



Calculate;

(i) Charge stored in the $50\mu F$ capacitor (05marks)

(ii) Energy stored in the $40\mu F$ capacitor (03marks)

(c) Explain **corona discharge**. (03marks)

Uneb 2015

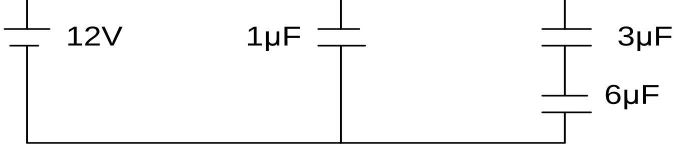
(a) (i) Define **capacitance** of a capacitor. (01mark)

(ii) Describe briefly an experiment to show the effect of placing a sheet of glasss or mica between the plates of a capacitor on capacitance. (05marks)

(b) Describe how capacitance eof a capacitor can be measured using a ballistic galvanometer. (04marks)

(c) Explain briefly how a charged capacitor can be fully discharged. (02marks)

(d) A network of capacitors of capacitance 3μF, 6μF, and 1μF is connected to a battery of 12V as shown below.



Calculate;

(i) Charge stored by each capacitor (05marks)

(ii) Energy stored in the $6\mu F$ capacitor when fully charged (03marks)

CURRENT ELECTRICITY

Current is the rate of flow of electric charge.

If charge Q, coulombs flows through a circuit in a time t seconds, then the current I, amperes is given by

$$I = \frac{Q}{t}$$

$$\boxed{Q = I\,t}$$

The S.I unit of current is Amperes(A) and current is measured using an instrument called an Ammeter.

Submultiples of A:

(i) $1\,mA = 1\,x\,10^{-3}\,A$

(ii) $1\,\mu A = 1\,x\,10^{-6} A$

(iii) $1\,nA = 1\,x\,10^{-9} A$

(iv) $1\,pA = 1\,x\,10^{-12} A$

Ampere;

Ampere is the current which, if flowing in two straight parallel wires of infinite length placed one meter apart in a vacuum, will produce on each of the wires a force of $2\,x\,10^{-7}$Nm^{-1}.

The S.I unit of charge is coulomb.

Coulomb;

Is the quantity of electricity which passes any point in a circuit in 1 second when a steady current of 1 ampere is flowing.

Examples

1. A charge of 180C flows through a lamp every 2 minutes. What is the electric current in the lamp.

Solution

$$I = \frac{Q}{t} \qquad I = \frac{180}{2\,x\,60} \qquad I = 1.5A$$

2. A charge of $20\,kC$ crosses two sections of a conductor in 1minute. Find the current through the conductor.

Solution

$$I = \frac{Q}{t} \qquad I = \frac{20\,x\,1000}{1\,x\,60} \qquad I = 333.33A$$

POTENTIAL DIFFERENCE ($P.d$)

$P.d$ is defined as the work done in moving one coulomb charge from one point to another a cross a conductor.

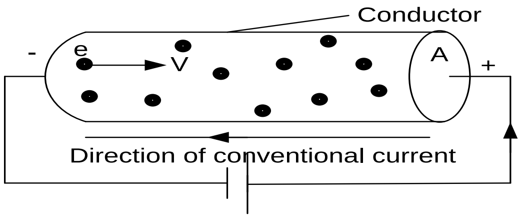
Current will flow through a conductor if there **a potential difference** between the ends of a conductor.

The $S.I$ unit of $P.d$ is volt and $P.d$ is measured using an instrument called a voltmeter

A volt;

A volt is the potential difference between two points when one joule of work is done in transferring one coulomb of charge from one point to another.

Mechanism of electrical conduction in metals



- ❖ In metals there are free electrons in random motion. When a battery (cell) is connected across the ends of a metal, an electric field is set up between its ends.
- ❖ The conduction electrons are accelerated by the field and gain velocity and energy. On collision with atoms vibrating about fixed

mean positions, they give up some of their energy to the atoms.

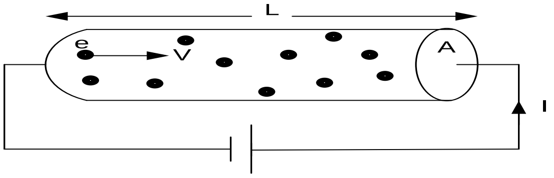
- ❖ The amplitude of vibration of the atoms increases and the temperature of the metal rises. The electric field continuously accelerates the free electrons and on average the electrons drift in the direction of the field with a mean speed.
- ❖ The continuous drift of the electrons in the same direction constitutes an electric current. The current produced is direct current (d.c) because the direction of flow is constant.

Heating Effect of Current

- ❖ When a p.d is applied across the wire, the conduction electrons gain kinectic enrgy form the applied field.
- ❖ As the electrons drift along the wire in a direction opposite to the applied field, they collide with the ions and lose their kinectic energy to the ions.
- ❖ The inos vibrate with increased amplitudes as a result temperature of the wire rises due to collision

Drift velocity of electrons

Consider a conductor of length ***l*** and cross –sectional area ***A*** having ***n*** free electrons per unit volume each carrying a charge, ***e***.



Volume of the conductor $= \boldsymbol{Al}$

Number of free electrons in the conductor $= \boldsymbol{nAl}$.

Total charge, ***Q*** of free electron $= \boldsymbol{nAle}$.

Suppose a battery connected across the ends of the conductor causes a total charge *Q* to pass through the conductor in time *t* with average drift velocity, *v*.

The resulting steady current I, is given by

$$I = \frac{Q}{t}$$

$$I = \frac{nAle}{t}$$

But the mean speed, $v = \frac{l}{t}$

$$\boxed{I = nAve}$$

From $I = nAve$

we can write $\frac{I}{A} = nve$. The quantity $\frac{I}{A}$ is called the current density and is denoted by J.

Thus $J = \frac{I}{A}$

Definition: *Current density is the current flowing through a conductor of cross sectional area 1 m²*.

Example

(e) A current of 10A flows through a copper wire of area 1mm^2. The number of free electrons per m^3 is 10^{29}. Find the drift velocity of the electron.

Solution

$$v = \frac{I}{nAe}$$

$$v = \frac{10}{10^{29}x1x10^{-6}x1x10^{-6}}$$

$$v = 6.25x10^{-4}m/s$$

(f) A metal wire contains 5x10^{22} electrons per cm^3 and has cross sectional area off 1mm^2. If the electrons move along the wire with a mean drift velocity of 1mms^{-1}, Calculate:

(i) current density
(ii) current in the wire.

Solution

(i) current density $= \frac{I}{A} = nev = 5x10^{22}x10^{6}x10^{-3}x1.6x10^{-19} = 8x10^{6}Am^{-2}$

(ii) current = current density x Area, $= 8x10^{6}x10^{-6}$

Conductivity, σ

The conductivity of a material is the reciprocal of its resistivity. It is denoted by σ.

$$\sigma = \frac{1}{\rho}$$

The S.I unit of conductivity is $\Omega^{-1}m^{-1}$

Example:

(a) Calculate the drift velocity of the free electrons in a copper wire of cross-sectional area 1.0 mm^2 when the current flowing through the wire is $2.0\ A$. (Number of free electrons in copper is $1\ x\ 10^{29}\ m^{-3}$)

(b) Explain how the drift velocity of free electrons in a metal conductor carrying a steady current changes when
  - (i) The p.d between the ends of the conductor is increased
  - (ii) The temperature of the conductor increases but the current remains unchanged.

Solution:

(a) Using the equation, $I = nAve$

Drift velocity, $v = \frac{I}{nAe}$

$$= \frac{2.0}{(1.0 \times 10^{29})(1.0\ \times 10^{-6})(1.6 \times 10^{-19})}$$

$v = 1.25 \times 10^{-4}$ m s^{-1}

(b) (i) The drift velocity increases as the higher p.d would accelerate the free electrons to a higher velocity.

(ii) The drift velocity decreases as the temperature increases. At a higher temperature, the ions in the metal lattice vibrate with greater amplitude, the mean free path decreases and the free electrons are unable to accelerate to a higher velocity. This results in increase of resistance to current flow.

OHM'S LAW

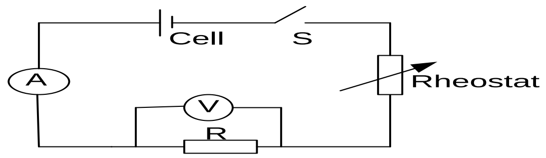
It states that the current flowing through a conductor is proportional to the potential difference across it's ends provided temperature and other physical conditions remain constant.

Ie $V \propto I$ at constant temperature

$$\boxed{V = I\,R}$$

R is resistance , V is p.d, I is current

Verification of ohm's law



A-ammeter, V- voltmeter, R-resistor, S- switch

- ❖ Arrange the apparatus as shown above.
- ❖ Switch is closed and rheostat adjusted so that A and V read suitable values of I and V respectively
- ❖ Ammeter reading I and voltmeter reading V are note and recorded
- ❖ The procedures above are repeated to obtain several values of I and V
- ❖ Tabulate the results in a suitable table
- ❖ A graph of V against I is plotted.
- ❖ A straight line through the origin verifies Ohms law

Limitations of ohm's law

- ➢ It does not apply to semiconductors and gases.
- ➢ It is only obeyed if physical conditions like temperature are constant.

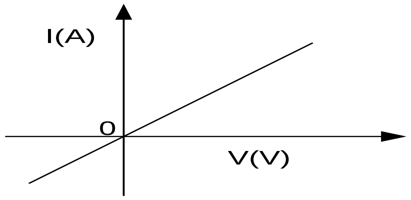
Ohmic and non ohmic conductors

An ohmic conductor is one which obeys ohm's law.

Non ohmic conductor s one which doesnot obey ohm's law.

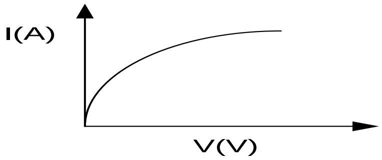
When we plot I against V between ends of a conductor, the shape of the curve is known as the characteristic of the conductor.

a) Ohmic conductors eg a metal

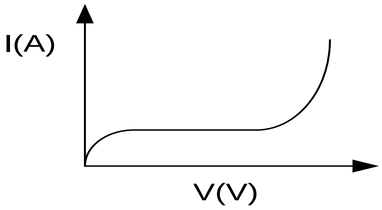


b) Non ohmic conductor

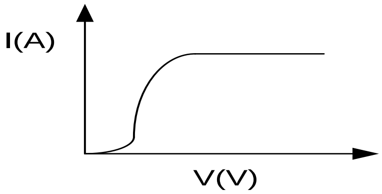
i) Filament lamp



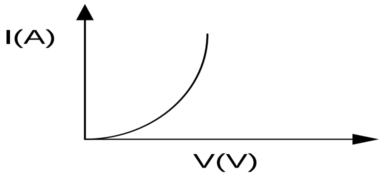
ii) Gas discharge tube



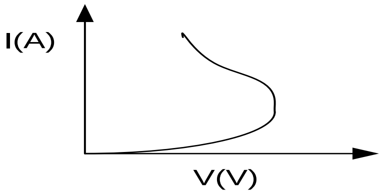
iii) Thermionic diode



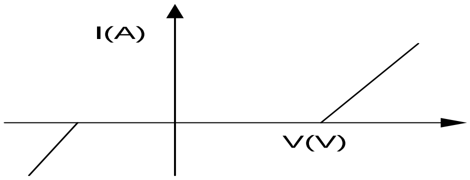
iv) Junction diode



v) Thermistor



vi) Electrolyte eg dilute sulphuric acid

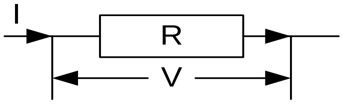


Resistance

This is the opposition of a conductor to the flow of current.
It is measured in ohms (Ω)

OR : Resistance of a conductor is the ratio of the potential difference across its ends to the current flowing through it.

A good conductor has low resistance while a good insulator has high resistance



$R = \frac{V}{I}$ 1

The S.I unit of electrical resistance is the ohm, symbol Ω.

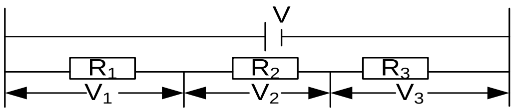
Definition

An ohm (Ω)

is defined as the resistance of a conductor if a current of 1A flows through when the p.d across it is 1V. (1 Ω = 1 V A^{-1})

ARRANGEMENT OF RESISTORS

a) Resistors in series



When resistors are in series, <u>current</u> flowing through them is the <u>same</u>.

Total $P.d, V = V_1 + V_2 + V_3$

$V_1 = I\,R_1, V_2 = I\,R_2 \; and \; V_3 = I\,R_3$

$V = I\,R_1 + I\,R_2 + I\,R_3$

$V = I\,(R_1 + R_2 + R_3)$

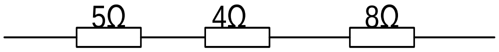
$\frac{V}{I} = R_1 + R_2 + R_3$ but $\frac{V}{I} = R$

$\boxed{R = R_1 + R_2 + R_3}$

Where R is equivalent reisisitance

Example

Find the total resistance in the circuits below

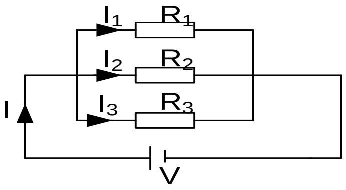


Solution

R = $(5 + 4 + 8)\,\Omega$

$R = 17\Omega$

b) Resistors in parallel



When resistors are in parallel, <u>p.d</u> across the ends is the <u>same</u>.

Total current, I = $I_1 + I_2 + I_3$

Where $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$ and $I_3 = \frac{V}{R_3}$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$ but $\frac{I}{V} = \frac{I}{R}$

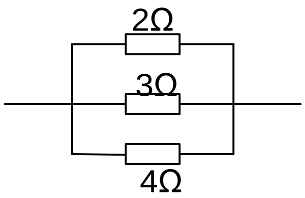
$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Where R is equivalent reisistance

Example

a) Find the effective resistance of the following circuit

1.



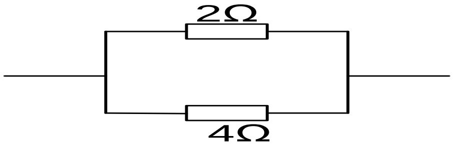
Solution

$$\frac{1}{R}=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

$$\frac{1}{R}=\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)$$
$$\frac{1}{R}=\frac{13}{12}$$
$$R=\frac{12}{13}$$

R = 0.92Ω

2.

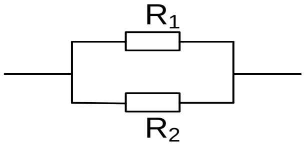


Solution

$$\frac{1}{R}=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$\frac{1}{R}=\left(\frac{1}{2}+\frac{1}{4}\right)$$
$$\frac{1}{R}=\frac{3}{4}$$
$$R=\tfrac{4}{3}$$

R = 1.33Ω

Note for two resistors in parallel

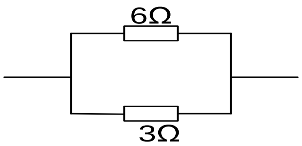


$$\frac{1}{R}=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$\frac{1}{R}=\frac{R_1+R_2}{R_1\,R_2}$$

$$R=\frac{R_1\,R_2}{R_1+R_2}$$
$$R=\frac{product\ of\ resisstance}{sum\ of\ resistance}$$

b) Calculate the effective resistance in each of the following circuits

(i)



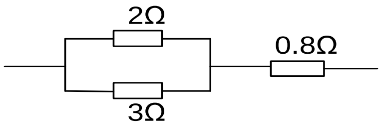
Solution

$$R=\frac{product\ of\ resisstance}{sum\ of\ resistance}$$
$$R=\frac{6x3}{6+3}$$

$$R=\frac{18}{9}$$

R = 2Ω

(ii)

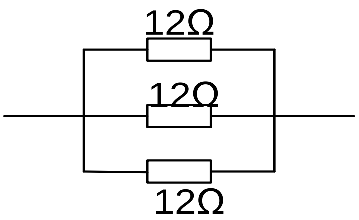


Solution

$$R=0.8+\frac{2x3}{2+3}$$
$$R=0.8+\frac{6}{5}$$

R = 2Ω

(iii)

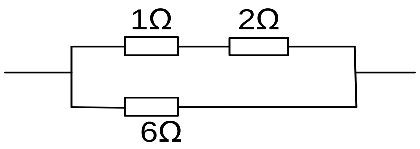


Solution

$$\frac{1}{R}=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$
$$\frac{1}{R}=\left(\frac{1}{12}+\frac{1}{12}+\frac{1}{12}\right)$$
$$\frac{1}{R}=\frac{3}{12}$$
$$R=\tfrac{12}{3}$$

R = 4Ω

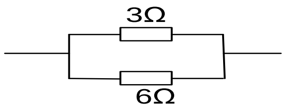
(iv)



Solution

For series

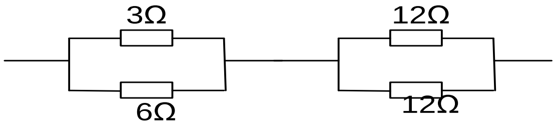
R = $(1+2)\Omega$ =**3**Ω



$$R = \frac{6x3}{6+3}$$

$$R = \frac{18}{9} = 2\Omega$$

(v)



Solutions

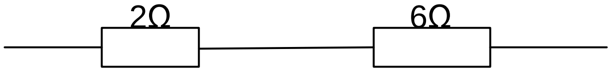
For the first set of parallel resistors

$$R = \frac{6x3}{6+3}$$

R = 2Ω

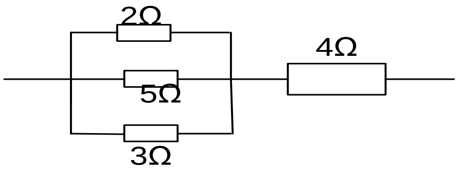
For the second set of parallel resistors

$$R = \frac{12x12}{12+12} = 6\Omega$$



Total resistance =$2+6=8\Omega$

(vi)



Solution

For parallel combination

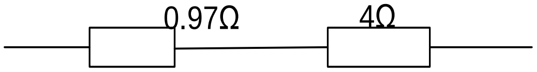
$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R} = \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{3}\right)$$

$$\frac{1}{R} = \frac{31}{30}$$

$$R = \frac{30}{31}$$

R = 0.97Ω

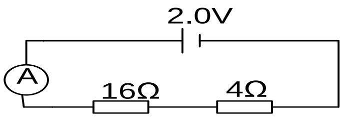


Total resistance =$0.97+4=4.97\Omega$

Further examples

1. Find the ammeter readings in each of the circuits below

i)



Solution

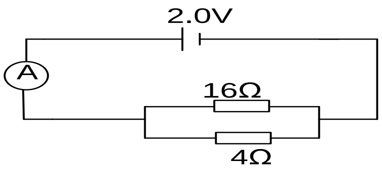
$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{2}{16+4}$$

$$I = 0.1\ A$$

ii)



Solution

$$V = IR$$

$$I = \frac{V}{R}$$

$$R = \frac{product\ of\ resisstance}{sum\ of\ resistance}$$

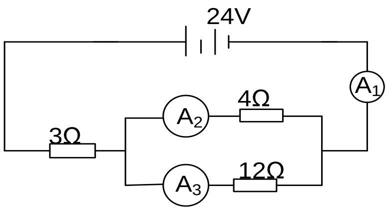
$$R = \frac{16x4}{16+4}$$

$$R = 3.2\Omega$$

$$I = \frac{2}{3.2}$$

$$I = 0.625\ A$$

iii)



Solution

$A_1 = A_2 + A_3$

A_1 reads current in the whole circuit

$V = IR$

$I_1 = \frac{V}{R}$

Total R= $\left[3 + \left(\frac{4x12}{4+12}\right)\right]$

$R = 3\Omega + 3\Omega$

$R = 6\Omega$

$I_1 = \frac{24}{6}$

$I_1 = 4A$

To find A_2 and A_3, we need to first find voltage across parallel combination

$V = IR_P$

I is the current through the parallel combination and $R_{\ P}$is total resistance of the parallel combination

$V = 4x\left(\frac{4x12}{4+12}\right)$

$V = 4x3$

$V = 12V$

Note : For any resistors in parallel, they have the same $p.d$

Current in A_2: $I_2 = \frac{V}{R}$

$I_2 = \frac{12}{4}$

$I_2 = 3A$

Current in A_3: $I_3 = \frac{V}{R}$

$I_3 = \frac{12}{12}$

$I_3 = 1A$

To quickly confirm the currents

Current in A_2: $I_2 = \frac{R_3}{R_2+R_3}xI$

$I_2 = \frac{12}{16}x4$

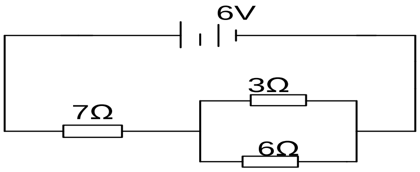
$I_2 = 3A$

Current in A_3: $I_3 = \frac{R_2}{R_2+R_3}xI$

$I_3 = \frac{4}{16}x4$

$I_3 = 1A$

2.



In the figure above find;

(i) Current through the circuit
(ii) Current across 3Ω and 6Ω resistor
(iii) $P.d$ across the 7Ω resistor
(iv) $P.d$ across the 3Ω and 6Ω resistor

Solution

i) Total resistance, R= $\left[7 + \left(\frac{6x3}{6+3}\right)\right]$

$R = 7\Omega + 2\Omega$

$R = 9\Omega$

$V = IR$

$I = \frac{V}{R}$

$I = \frac{6}{9}$

$I = 0.667\ A$

Current in the circuit is $0.667\ A$

ii) Voltage across the parallel combination

$V = IR_P$

$V = 0.667x\left(\frac{6x3}{6+3}\right)$

$V = 0.667x2$

$V = 1.334V$

Note : For any resistors in parallel, they have the same $p.d$

Current in $3\Omega\ resistor$: $I = \frac{V}{R}$

$I = \frac{1.334}{3}$

$I = 0.445A$

Current in $6\Omega\ resistor$: $I = \frac{V}{R}$

$I = \frac{1.334}{6}$

$I = 0.223A$

To quickly confirm the currents

Current in $3\Omega\ resistor$: $I = \frac{6}{6+3}x0.667$

$I = \frac{6}{9}x0.667$

$I = 0.445A$

Current in $6\Omega\ resistor$: $I = \frac{3}{6+3}x0.667$

$I = \frac{3}{9}x0.667$

$I = 0.223A$

iii) $P.d$ across the 7Ω resistor

$0.6A$ Passes through the 7Ω resistor

$V = I\ R$

$V = 7x\ 0.667$

$V = 4.669V$

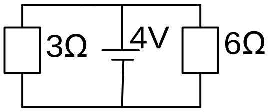
iv) $P.d$ across the 3Ω resistor and 6Ω resistor

$V = (6 - 4.669)V$

$V = 1.33V$

since the two resistors are in parallel therefore, they have the same $p.d$ of $1.33V$

3.



Two resistors of 3Ω and 6Ω are connected across a battery of emf of $4V$ as show, find

i) the combined resistance

ii) the current supplied by the battery

Solution

i) $R = \frac{product\ of\ resisstance}{sum\ of\ resistance}$

$R = \frac{6x3}{6+3}$

$R = 2\Omega$

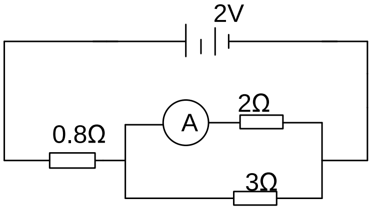
ii) $I = \frac{V}{R}$

$I = \frac{4}{2}$

$I = 2A$

Exercise

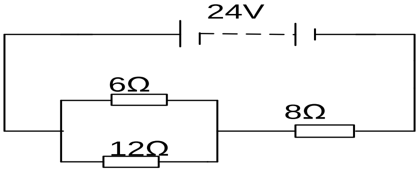
1.



Find the ammeter reading

[0.6A]

2. A $p.d$ of 24V from a battery is applied to a network of resistors as shown below

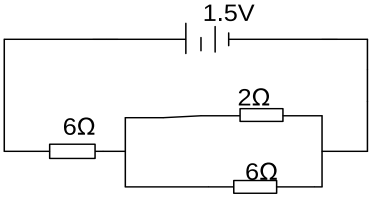


i) find the current through the circuit **[2A]**

ii) find the p.d across the 8Ω resistor **[16V]**

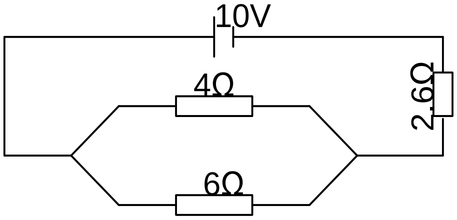
iii) find the current through the 6Ω resistor **[1.3A]**

3.



Find the current through the 2Ω resistor

4.



A battery of emf 10V and negligible internal resistance is connected across a network of resistors as shown above. calculate the current through the 6Ω resistor.

[0.8A]

Factors that affect resistance

a) Temperature

- Condution in metals is by free electorns. The drift electorns however are obstracted by atoms in their lattice positions.
- When temperature of the metal inceases, the atoms vibrate with a larger amplitude thus reducing the means free path of the free electorns reducing the drift velocity of free electorns hence increae in resistance

b) Length

The longer the conductor, the higher the resistance and the shorter the conductor the lower résistance. Free electrons collide morefrequently with atoms, at each collision they lose some kinetic energy to atoms vibrating at fixed mean positions. This leads to a decrease in the drift velocity of the electrons and hence an incease in resistance

c) Cross sectional area

The thinner the conductor, the higher the resistance and the thicker the conductor, the lower the resistance. When there is an increase in the cross sectional area the nuber of free electrons htat drift along the conductor also increases. This leads to an increase in current hence a decrease in ressitance.

The above factors can be combined as;

$$\rho \propto {}^{L}\!/_{A}$$

$$\boxed{R = \frac{\rho\, L}{A}}$$

Where ρ is resistivity

Definition

Electrical Resistivity is the resistance across opposite faces of a 1m-cube of 1a material
Resistivity is the electrical resistance across opposite faces of a 1m-cube of 1a material

Examples

1. A conductor of length $20cm$ has a cross sectional area of $2x10^{-4}\ m^2$. Its resistance at $20^\circ C$ is 0.6Ω. find the resistivity of the conductor at 20°C.

Solution

$$R = \frac{\rho\, L}{A}$$

$$\rho = \frac{RA}{L}$$

$$\rho = \frac{0.6\ x\ 2x10^{-4}}{0.2}$$

$$\rho = 6x10^{-4}\ \Omega\, m$$

2. A wire of diameter 14mm and length 50cm has its resistivity as $1.0\ x10^{-7}\ \Omega\ m$. What is the resistance of the wire at room temperature?

Solution

$$d = 14mm, r = \frac{14}{2} = 7mm$$

$$l = 50cm, l = 0.5m$$

$$A = \pi\, r^2$$

$$A = \frac{22}{7} x \left(\frac{7}{1000}\right)^2$$

$$A = 1.54x\ 10^{-4}\ m^2$$

$$R = \frac{\rho\, L}{A}$$

$$R = \frac{0.5\ x\ 1x10^{-7}}{1.54x\ 10^{-4}}$$

$$R = 3.25x10^{-4}\ \Omega$$

3. A steady uniform current of 5mA flows along a metal cylinder of cross sectional area of 0.2mm^2, length, 5m and resistivity 3x10^{-5}Ωm. find the p.d across the ends of the cylinder.

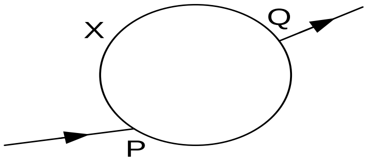
Solution

$$R = \frac{\rho\, L}{A}$$

$$R = \frac{5\,x\,3x10^{-5}}{2x\,10^{-7}}$$
$$R = 750\,\Omega$$

$$V = IR$$
$$V = 5x10^{-3}x750 = 3.75V$$

4. A wire of diameter ***d,*** length ***l*** and resistivity ***ρ*** forms a circular loop. Current enters and leaves at points **P** and **Q**.



Show that the resistance R of the wire is given by $R = \frac{4\,\rho X(l-X)}{\pi d^2 l}$

Solution

Let R_1 and R_2 be resistance of portion x and $l - x$ of the wire respectively.

$R_1 = \frac{\rho x}{A}$ and $R_2 = \frac{\rho(l-x)}{A}$

The two portions are in parallel, hence $R = \frac{R_1\,R_2}{R_1 + R_2}$

$$R = \frac{\left(\frac{\rho x}{A}\right)\left(\frac{\rho(l-x)}{A}\right)}{\left(\frac{\rho x}{A}\right) + \left(\frac{\rho(l-x)}{A}\right)} = \frac{\rho x(l-x)}{Al}$$

But $A = \frac{\pi d^2}{4}$

$$R = \frac{\rho x(l-x)}{Al} = \frac{4\,\rho x(l-x)}{\pi d^2 l}$$

Question

A p.d of 4.5V is applied to the ends of a 0.69m length of a wire of cross sectional area $6.6x10^{-7}m^2$. Calculate the drift velocity of electrons across the wire. (ρ of wire is $4.3x10^{-7}\Omega m$, number of electrons per m^3 is 10^{28} and electronic charge is $1.6x10^{-19}C$)

Temperature cofficient of resistance (α),

The temperature coefficient of resistance of a material is the fractional change in the resistance at 0 °C per degree celcius rise in temperature.

If a material has resistance R_o at 0 °C and its resistance increases to R_θ when heated through a temperature θ, then

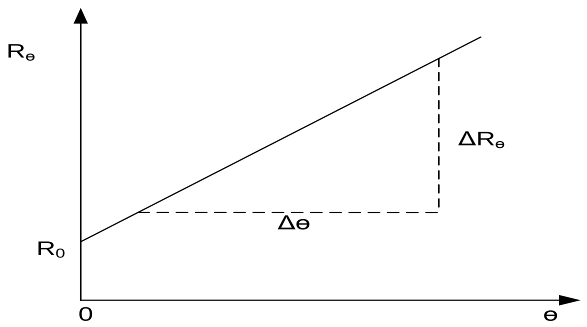
$$\alpha = \frac{R_\theta - R_0}{R_0 \theta}$$

The S.I unit of α is K^{-1}or $°C^{-1}$

The above equation can be rearranged;

$$\boxed{R_\theta = R_0(1 + \alpha\theta)}$$

A graph of R_θ against θ is a straight line whose intercept on the R_θ - axis is equal to R_0 and the slope = $\alpha\,R_0$.



$$S = \frac{\Delta R_\theta}{\Delta \theta} = \alpha\, R_0$$

$$\alpha = \frac{S}{R_0}$$

If R_1 and R_2 are the resistances of a conductor at temperatures θ_1 and θ_2, then

$R_1 = R_0(1 + \alpha\theta_1)$..............................i

$R_2 = R_0(1 + \alpha\theta_2)$..............................ii

Divide i by ii $\boxed{\frac{R_1}{R_2} = \frac{1+\alpha\theta_1}{1+\alpha\theta_2}}$

Note:

Super conductors are materails whose resistance vanishes when colled to about -273°C

Why metals have positive temperature coefficient of resistance

When the temperature of the metal increase, the amplitude of vibration of its atoms increase. This reduces the means free path for the conduction electrons. Thus fewer electrons now flow per second through the metal and hence less which increases the resistance.

Why semi-conductors have negative temperature coefficient of resistance

Semi-conductors have few electrons available for conduction at room temperature. When current is passed through it, the material heats is up. As the temperature increases, loosely bound electrons are released for conduction thus current increase hence resistance reduces

Examples

1. The resistivity of mild steel is $15 \times 10^{-8}\ \Omega$ m at 20 °C and its temperature coefficient of resistance is 50×10^{-4} K^{-1}. Calculate the resistivity at 60 °C.

Solutiom

Since the resistance of a conductor when heated is proportional to resistivity, it follows that the resistivity at temperature θ is given by

$$\rho_\theta = \rho_0(1 + \alpha\theta)$$

Where α is the temperature coefficient of resistivity.

At 20 °C,

$$15 \times 10^{-8} = \rho_0(1 + 50 \times 10^{-4} \times 20)$$

$$15 \times 10^{-8} = 1.1\rho_0$$

$$\rho_0 = \frac{15 \times 10^{-8}}{1.1} = 13.64 \times 10^{-8}\Omega\ m$$

At 60 °C,

$$\rho_{60} = 13.64 \times 10^{-8}(1 + 60 \times 50 \times 10^{-4})$$

$\rho_{60} = 17.7 \times 10^{-8}\ \Omega$ m

2. A coil of wire has resistances of 30 Ω at 20 °C and 34.5 Ω at 60 °C. Calculate
   (i) The temperature coefficient of resistance of the wire
   (ii) The resistance of the wire at 0 °C.

Solution:

(i) At 20 °C,

$R_{20} = 30\ \Omega$

$30 = R_o(1 + 20\alpha)$ 1

At 60 °C,

$R_{60} = 34.5\ \Omega$

$R_{60} = R_o(1 + 60\alpha)$ 2

Divide equation 1 by 2

$$\frac{30}{34.5} = \frac{R_o(1 + 20\alpha)}{R_o(1 + 60\alpha)}$$

$$\frac{30}{34.5} = \frac{1 + 20\alpha}{1 + 60\alpha}$$

$$30 + 1800\alpha = 34.5 + 690\alpha$$

$$1110\alpha = 4.5$$
$$\alpha = 4.05 \times 10^{-3} K^{-1}$$

(ii) Substitute for α in equation 1 or 2 to solve for R_o

$$30 = R_o + 20 \times 0.00405 \times R_o$$
$$R_o = 27.75\Omega$$

3. The resistance of a nichrome element of an electric fire is 50.9 Ω at 20 °C. When operating on at 240 V supply, the current flowing in it is 4.17 A. Calculate the steady temperature reached by the electric fire if the temperature coefficient of resistance of nichrome is $1.7 \times 10^{-4} K^{-1}$.

Solution:

At 20 °C, R_{20} = 50.9 Ω

From $R_\theta = R_o(1 + \alpha\theta)$

$$50.9 = R_o(1 + 20 \times 1.7 \times 10^{-4})$$
$$R_0 = \frac{50.9}{1.0034} = 50.7275\,\Omega$$

Let the unknown temperature to which the element heats up be, β.

At β °C,

$V = 240\,V, I = 4.14\,A$

$$R_\beta = \frac{240}{4.17} = 57.554\,\Omega$$
$$R_\beta = R_o(1 + \alpha\beta)$$
$$57.554 = 50.7275(1 + 1.7 \times 10^{-4}\beta)$$
$$6.8265 = 8.6237 \times 10^{-3}\beta$$

$\beta = 791.6$ °C

4. An electric heater consists of 5.0m of nichrome wire of diameter 0.58mm. When connected to a 240V supply, the heater dissipates $2.5kW$ and the temperature of the heater is found to be 1020°C. If the resistivity of the nichrome at 10°C is $1.02x10^{-6}\Omega m$. Calculate;
   (i) The resistance of nichrome at 10°C
   (ii) The mean temperature coefficient of reistance of nichrome between 10°C and 1020°C

Solution

(i) $R_{10} = \frac{\rho l}{A} = \frac{1.02x10^{-6}x5}{\pi\left(\frac{0.58x10^{-3}}{4}\right)} = 19.3\,\Omega$

(ii) **At** 1020°C, $P_{1020} = \frac{V^2}{R_{1020}} = 2.5kW$

$$R_{1020} = \frac{240^2}{2.5x10^3} = 23.04\,\Omega$$

$23.04 = R_o(1 + 1020\alpha)$ 1

At 10°C,

$19.3 = R_o(1 + 10\alpha)$ 2

Eq 2÷Eq 1

$$\frac{23.04}{19.3} = \frac{1 + 1020\alpha}{1 + 10\alpha}$$
$$\alpha = 1.92x10^{-4}K^{-1}$$

Exercise

3. The resistance of a nichrome element of an electric fire is 50 Ω at 20 °C. When operating on a 240 V supply, the current flowing in it is 4.A. Calculate the steady temperature reached by the electric fire if the temperature coefficient of resistance of nichrome is $2.0 \times 10^{-4} K^{-1}$. **An($1024.1$°C)**
4. The resistivity of a certain wire is $1.6 \times 10^{-7}\,\Omega$ m at 30 °C and its temperature coefficient of resistance is 6.0×10^{-3} K^{-1}. Calculate the resistivity at 80 °C. **An($2.01x10^{-7}\Omega m$)**
5. A nichrome wire of length 1.0m and diameter 0.72mm at 25° C, is made into a coil. The coil is immersed in 200cm^3 of water at the same temperature and a current of 5.0 A is passed through the coil for 8 minutes until when the water starts to boil at 100°C.

   Find:
   (i) the resistance of the coil at 25°C
   (ii) the electrical energy expended assuming all of it goes into heating the water.
   (iii) the mean temperature coefficient of resistance of nichrome between 0° C and 100°C.

5. Two wires A and B have lengths which are in the ratio $4 : 5$, diameters which are in the ratio $2 : 1$, and resistances in the ratio of $3 : 2$. If the wires are arranged in parallel and current of $1.0\ A$ flows through the combination, find the:
    (i) ratio of resistance of wire A to that of wire B
    (ii) current through wire A
6. A battery of e.m.f 12V and negligible internal resistance is connected to the ends of a metallic wire of length 50 cm and cross-sectional area 0.1 mm^2. If the resistivity of a material of the wire is $1.0\ x\ 10^{-6}\Omega$ m at what rate is heat generated in the wire?
7. The table below shows the resistance of a nichrome wire at various temperatures.

Temperature (°C)	75	120	150	250	300
Resistance (Ω)	103	103.8	104.4	105.9	106.8

Plot a suitable graph and use it to determine the temperature coefficient of resistance of nichrome

Electromotive force and internal resistance

The e.m.f of a cell is the energy supplied by the cell to transfer 1C of charge round a complete circuit in which the cell is connected.

It may also be defined as the p.d across the terminals of the cell when on an open circuit.

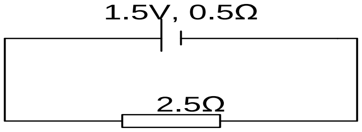
Internal resistance of a cell is the opposition in series with the external circuit which accounts for energy losses inside the cell when the cell is supplying current.
Internal resistance is represented by r.

$$\boxed{E = I\ (R + r)}$$

Examples

1. A battery of emf 1.5V and internal resistance 0.5Ω is connected in series with 2.5Ω resistor. Find;
    i) current through the circuit
    ii) $p.d$ of the 2.5Ω resistor

Solution



i) $E = I\ (R + r)$

$$I = \frac{E}{(R + r)}$$

$$I = \frac{1.5}{(2.5 + 0.5)}$$

$$I = \frac{1.5}{3}$$

$$I = 0.5\ A$$

ii) $V = IR$

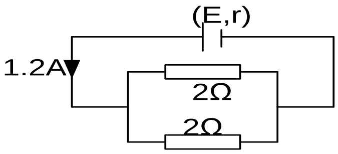
$$V = 0.5x2.5$$

$$V = 1.25V$$

2. A cell can supply a current of $1.2A$ through two 2Ω resistors connected in parralle. When they are connected in series the value of current is $0.4A$. Clculate the internal resistance and emf of the cell.

Solution

case 1



$$E = I\ (R + r)$$

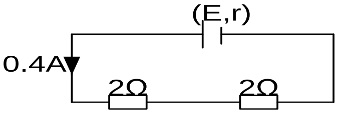
$$R = \frac{2x2}{2 + 2}$$

$$R = 1\Omega$$

$$E = 1.2\ (1 + r)$$

$E = 1.2 + 1.2\ r$……..[1]

Case 2



$$E = I\ (R + r)$$

$$R = 2 + 2$$

$R = 4\Omega$

$E = 0.4\ (4 + r)$

$E = 1.6 + 0.4\,r$........[2]

Equating 1 and 2

$1.2 + 1.2\,r = 1.6 + 0.4\,r$

$1.2\,r - 0.4\,r = 1.6 - 0.4$

$0.8r = 0.4$

$r = \frac{0.4}{0.8}$

$r = 0.5\Omega$

Also $E = 1.2 + 1.2\,r$

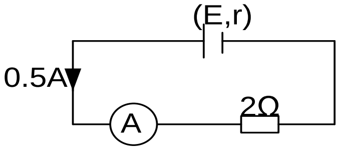
$E = 1.2 + 1.2\ x\ 0.5$

$E = 1.2 + 0.6$

$E = 1.8\ V$

3. An ammeter connected in series with a cell and a 2Ω resistor reads $0.5A$. When the 2Ω resistor is replaced by a 5Ω resistor, the ammeter reading drops to $0.25A$. Calculate the internal resistance and the emf of the cell.

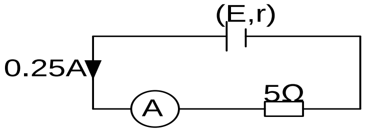
Solution



$E = I\ (R + r)$

$E = 0.5\ (2 + r)$

$E = 1 + 0.5\,r$........[1]



$E = I\ (R + r)$

$E = 0.25\ (5 + r)$

$E = 1.25 + 0.25\,r$........[2]

Equating 1 and 2

$1 + 0.5\,r = 1.25 + 0.25\,r$

$0.5\,r - 0.25\,r = 1.25 - 1$

$0.25r = 0.25$

$r = \frac{0.25}{0.25}$

$r = 1\ \Omega$

Also $E = 1 + 0.5\,r$

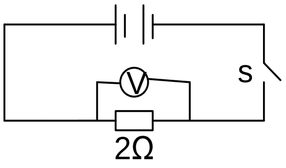
$E = 1 + 0.5\ x\ 1$

$E = 1 + 0.5$

$E = 1.5\ V$

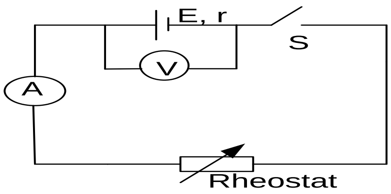
Exercise

1. A battery of e.m.f and internal resistance, r is connected across a variable resisto, when the resistor is set at 21Ω, the current through it is 0.48A. when it is set at 36Ω, the current is 0.30A. find E and r. **[4Ω, $12V$]**
2. A cell is joined in series with a resistance of 2Ω and a current of $0.25A$ flows through it. When a second resistance of 2Ω is connected in parallel with the first, the current through the cell is $0.3A$. Calculate the internal resistance and emf of the cell. **[4Ω, $1.5V$]**
3. Two cells each of e.m.f 1.5 V and internal resistance 0.5 Ω are connected in series with a resistor of 2 Ω as in the figure below.



The reading of the voltmeter V when S is closed is? **[2V]**

Measurement of E.m.f and internal resistance of a cell



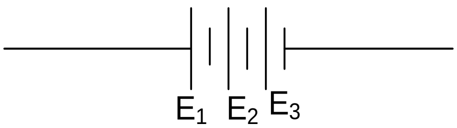
A-ammeter, V- voltmeter, R-resistor, S- switch

- Arrange the apparatus as shown above.
- Switch is closed and rheostat adjusted to a suitable value of I
- Ammeter reading I and voltmeter reading V are note and recorded

- The rheostat is varied for othe values of I and corresponding values of V are noted and recorded.
- A graph of V against I is plotted.
- The intercept on the V axis is noted which is the e.m.f of the cell.
- The slope s is obtained and the internal resistance of the cell $r = -s$

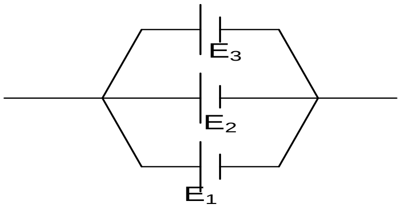
CELL ARRANGEMENTS

1. Series arrangement



Total emf E= $E_1 + E_2 + E_3$

2. Parallel arrangement

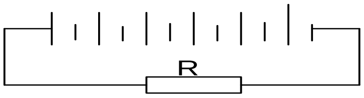


When cells of equal ***emf*** are connected in parallel
Total emf E= $E_1 = E_2 = E_3$

Example

1. Find the total emf in each of the following circuits if each cell is of emf $1.5V$

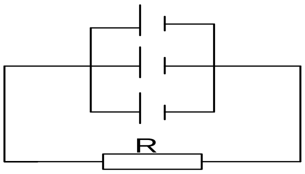
(i)



Solution

Total emf $E = 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5$
$= 9.0V$

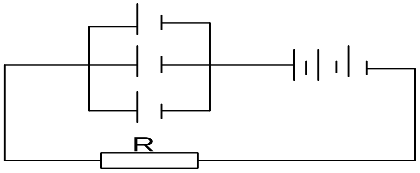
(ii)



Solution

Total emf $E = 1.5V$

(iii)



Solution

Total emf $E = 1.5 + 1.5 + 1.5 + 1.5$
$= 6.0V$

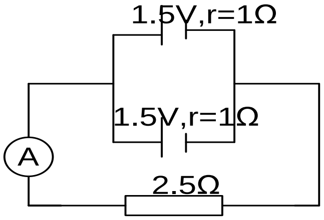
Note: If the cells are connected in parallel and have internal resistance, their resistance is calculated as resistors in parallel.

Advantages of series arrangement of cells over the paralle arrangement

In series arrangement the effective e.m.f is greater than the individual e.m.f of the cells and hence a greater current is drawn from the series com bination than in the parallel combination.
However the series aarrangement has a disadvantage of all the cells being drained at once thus the cells have a shorter life span.

Examples

1. Find the ammeter reading

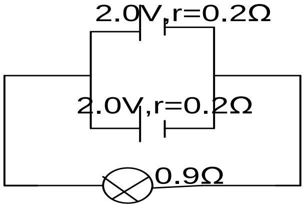


Solution

$$E = I\,(R + r)$$
$$r = \frac{1x1}{1+1}$$
$$r = 0.5\Omega$$
$$1.5 = I\,(2.5 + 0.5)$$
$$I = \frac{1.5}{3}$$
$$I = 0.5A$$

2. Two cells of emf 2.0V and internal resistance 0.2Ω each are connected together in parallel to form a battery. This battery is connected to a lamp of resistance0.9Ω. Calculate the current through the lamp and voltage across the lamp.

Solution



$$E = I\,(R + r)$$
$$r = \frac{0.2x0.2}{0.2 + 0.2}$$
$$r = 0.1\Omega$$

current through the lamp

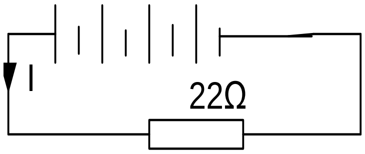
$$2 = I\,(0.9 + 0.1)$$
$$I = \frac{2}{1}$$
$$I = 2A$$

voltage across the lamp

$$V = IR$$
$$V = 2x0.9$$
$$V = 1.8V$$

3. Four cells each of emf $1.5V$ and internal resistance 0.5Ω are connected in series. What current will flow through an external resistor of 22Ω

Solution



Total emf E $= 1.5x4$

$$E = 6V$$

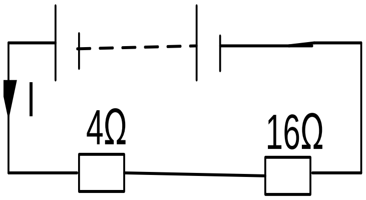
Total internal resistance r $= 0.5x4$

$$r = 2\Omega$$
$$E = I\,(R + r)$$
$$6 = I\,(22 + 2)$$
$$I = \frac{6}{24}$$
$$I = 0.25A$$

4. A battery containing 8 cells each of emf 1.5V and internal resistance 0.5Ω is connected to two other resistors of 4Ω and 16Ω . Calculate the minimum and maximum current that can flow through the battery.

Solution

For minimum current the resistors must be connected in series



Total emf E $= 1.5x8$

$E = 12V$

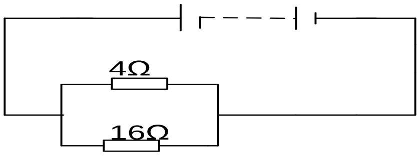
Total internal resistance r $= 0.5x8$

$r = 4\Omega$

Total external resistance R$= 4 + 16$

$$E = I\,(R + r)$$
$$12 = I\,(16 + 4 + 4)$$
$$I = \frac{12}{24}$$
$$I = 0.5A$$

For maximum current the resistors must be connected in parallel



Total emf E $= 1.5x8$

$E = 12V$

Total internal resistance r $= 0.5x8$

$r = 4\Omega$

Total external resistance R$= \frac{4x6}{4+6}$

$R = 3.2\Omega$

$E = I\,(R + r)$

$12 = I\,(3.2 + 4)$

$I = \frac{12}{7.2}$

$I = 1.67A$

WORK DONE BY AN ELECTRIC CURRENT(ELECTRICAL ENERGY)

If the $P.d, V$ is applied to the ends of a conductor and quantity of electricity, Q flows then

$work\ done = Q\,V$ but $Q = It$

$$W = It\,V$$

$$\boxed{W = IVt}$$

but $V = IR$

$$W = I\,(IR)\,t$$

$$\boxed{W = I^2\,R\,t}$$

but $I = \frac{V}{R}$

$$W = \left(\frac{V}{R}\right)^2 R\,t$$

$$\boxed{W = \frac{V^2\,t}{R}}$$

The work done is transferred into internal molecular energy accompanied by a rise in temperature subsequently, this energy may be given out in form of heat

ELECTRICAL POWER

This is the rate of doing work by an electric current.

$$power = \frac{work\ done}{time\ taken}$$

$$P = \frac{I\,V\,t}{t}$$

$$\boxed{P = I\,V}$$

Also

$$power = \frac{work\ done}{time\ taken}$$

$$P = \frac{I^2\,R\,t}{t}$$

$$\boxed{P = I^2\,R}$$

Also

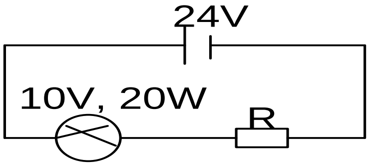
$$power = \frac{work\ done}{time\ taken}$$

$$P = \frac{V^2\,t}{R} \Big/ t$$

$$\boxed{P = \frac{V^2}{R}}$$

Examples

1. A battery of $emf\ 24V$ is connected in series with aresistance R and a lamp rated $10V, 20W$ as shown below.



if the bulb is operating normally . Find,

i) the p.d across the resistor

ii) the value of R

iii) power dissipated in the resistor

Solution

i) $p.d$ across the resistor

$= (24 - 10)V$

$= 14V$

ii) Current through the bulb

$I = \frac{P}{V}$

$I = \frac{20}{10}$

$I = 2A$

Bulb and resistor have the same current

$R = \frac{V}{I}$

$R = \frac{14}{2}$

$R = 7\Omega$

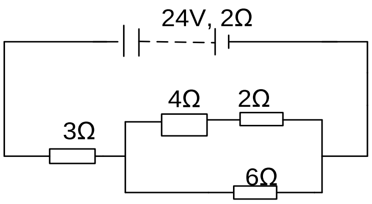
iii) power dissipated in the resistor

$P = I^2\,R$

$P = 2^2\,x\,7$

$P = 28W$

2. An accumulator of $emf\ 24V$ and internal resistance 2Ω is connected in a circuit as shown below.



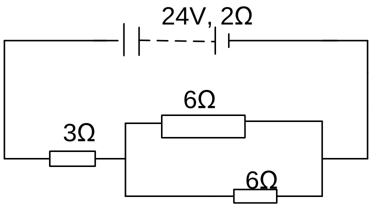
a) calculate the current through the 6Ω resistor
b) calculate the power expended in the 6Ω resistor
c) find the total power expended

Solution

a) for 4Ω and 2Ω resistors

total resistance $R = (4 + 2)$

$$R = 6\Omega$$



Total resistance= $\left[3 + \left(\frac{6x6}{6+6}\right)\right]$

$$= 6\Omega$$

$$E = I\,(R + r)$$
$$24 = I\,(6 + 2)$$
$$I = \frac{24}{8}$$
$$I = 3A$$

$p.d$ through parallel combination

$$V = IR$$
$$V = 3x\left(\frac{6x6}{6+6}\right)$$
$$V = 9V$$

Current through the 6Ω resistor

$$V = IR$$
$$I = \frac{V}{R}$$
$$I = \frac{9}{6}$$
$$I = 1.5A$$

b) power in 6Ω resistor

$$P = IV$$
$$P = 1.5\,x\,9$$
$$P = 13.5\,W$$

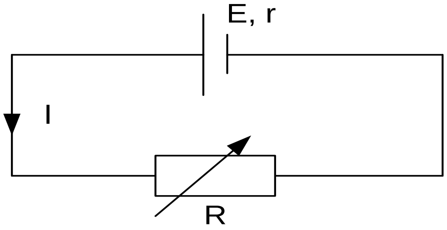
c) total power= $I\,E$

$$= 3x\,24$$
$$= 72W$$

Power out put and efficiency

Efficiency of the battery is the ratio of the useful power expended in the total external load to the power generated or supplied by the battery.

Usually efficiency is expressed in percentage and is denoted by ƞ



Power delivered to resistor, $P_{out} = IV$

Power supplied by cell, $P_{IN} = IE$

Eficiency,ƞ = $\frac{Power\ output}{Power\ input}x100\%$

$$ƞ = \frac{IV}{IE}x100\%$$
$$ƞ = \frac{V}{E}x100\%$$

but $E = I(R + r)$ and $V = IR$

$$ƞ = \left(\frac{R}{R + r}\right)x100\%$$

Example

A battery of e.m.f 18.0V and internal resistance 3.0Ω is connected to a resistor of resistance 8Ω. Calculate the

(i) power generated
(ii) efficiency
(iii)

Solution

$$P_{gen} = IE = \left(\frac{E}{R + r}\right)E$$
$$P_{gen} = \left(\frac{18x18}{8 + 3}\right) = 29.45W$$

$$ƞ = \frac{Power\ output}{Power\ input}x100\%$$
$$ƞ = \frac{IV}{IE}x100\%$$

but $E = I(R + r)$ and $V = IR$

$$ƞ = \left(\frac{R}{R + r}\right)x100\%$$

$$\eta = \left(\frac{8}{8+3}\right) x 100\%$$

$$\eta = \mathbf{72.7}\%$$

Maximum power output

Suppose the load resistance R is variable then the useful power expended in R will also vary and will be maximum when ***R = r.***

$$Power\ output = IV = \left(\frac{E}{R+r}\right) x \left(\frac{ER}{R+r}\right)$$

$$Power\ output, P_0 = \frac{E^2}{(R+r)^2}R$$

At maximum power, P$_m$, $\frac{dP_0}{dR} = 0$

$$\frac{dP_0}{dR} = E^2\frac{[(R+r)^2 - 2R(R+r)]}{(R+r)^4} = 0$$

$$R = r$$

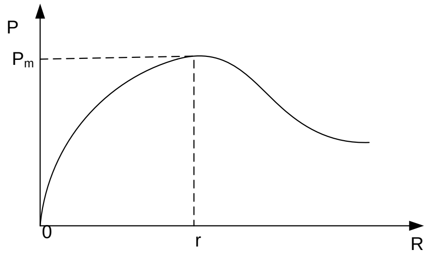
$$P_0 = \frac{E^2}{(r+r)^2}r$$

Power output, $\boxed{P_0 = \frac{E^2}{4r}}$

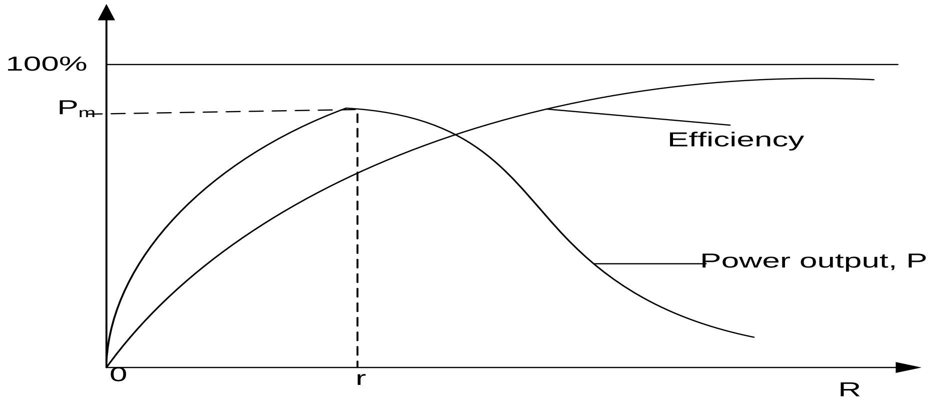
As R tends to zero, P tends to zero

As R tends to ∞, P tends to zero.

A graph of power out put P against load resistance R is shown below.



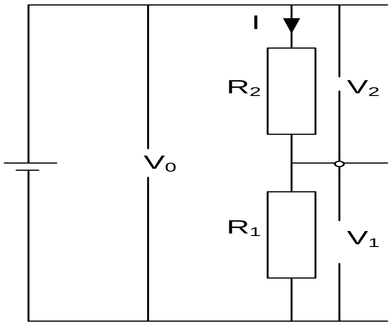
Variation of η and P with R



For values of R less than r, power output increases and attains a maximum value when R = r. For

The Potential Divider

The potential divider is used to obtain a fraction of a given p.d. The fraction can be fixed or variable.



Total resistance in the circuit, $R = R_1 + R_2$.

$$V_0 = I R$$

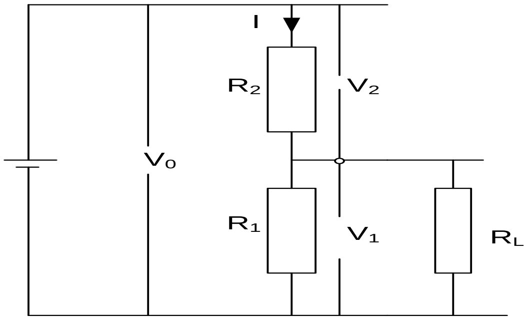
$$V_0 = I (R_1 + R_2)$$

$$I = \frac{V_0}{R_1 + R_2}$$

$$V_1 = IR_1$$

$$V_1 = \left(\frac{V_0}{R_1 + R_2}\right) R_1$$

Suppose a load of resistance R_L is connected in parallel with R_1.



Let the effective resistance of R_1 and R_L be R_P.

$$R_P = \frac{R_1 R_L}{R_1 + R_L}$$

Effective resistance in the circuit, $R = R_P + R_2$

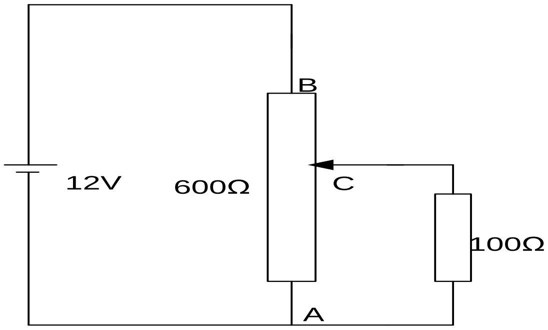
Total current in the circuit, $I = \frac{V_O}{R_P + R_2}$

Hence $V_1 = IR$

$$V_1 = \left(\frac{V_O}{R_P + R_2}\right) R_P$$

Example

1. A 12V battery is connected across a potential divider of resistance 600Ω as shown below. If the load of 100Ω is connected across the terminals A and C when the slider is half way up the divider, find:



(i) p.d across the load
(ii) p.d across a and c when the load is removed.

Solution

(i) Effective resistance

$$R = \frac{300x100}{300 + 100} + 300 = 375\Omega$$

current supplied by the battery, $I = \frac{V}{R}$

$$I = \frac{12}{375} = 0.032A$$

hence current through parallel combination of resistors = $0.032A$

p.d across parallel combination of resistors,

$$V = IR = 0.032x\left(\frac{300x100}{300+100}\right) = 2.4V$$

Hence the p.d across the load is $2.4V$.

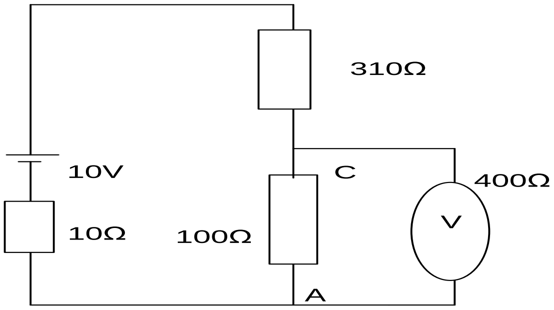
(ii) when the load is removed

$$I = \frac{12}{600} = 0.02A$$

Hence p.d across AC = $0.02x300 = 6V$

Exercise

1.



(i) Find the reading of the voltmeter

(ii) Calculate the power dissipated in the 10Ω resistor

2. A resistor of 500Ω and one of 200Ω are placed in series with a 6V supply. What will be the reading on a voltmeter of internal resistance 2000Ω when placed across

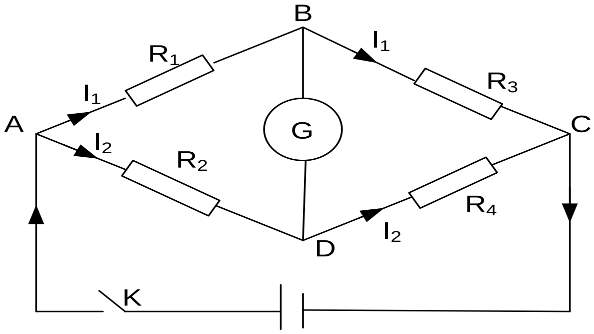
(i) the 5000Ω resistor

(ii) 2000Ω resistor.

WHEASTONE BRIDGE

It is a bridge circuit consisting of four resistances R₁, R₂, R₃, R₄ and a sensitive centre - zero galvanometer, G.

Balaance condition for a wheatstone



Switch K is closed and the reisistance R_1 , R_2 , R_3 and R_4 adjusted until the galvanometer shows <u>no deflection</u>.

At balance condition B and D are at the <u>same potential.</u>

P.d across AB, = p.d across AD,

$$I_1R_1 = I_2R_2 \dots\dots\dots\dots\dots\dots\dots..(1)$$

Current flowing through R_3 is therefore I_1 and that through R_4 is I_2

P.d across BC, = p.d across DC,

$$I_1R_3 = I_2R_4 \dots\dots\dots\dots\dots\dots\dots..(2)$$

Eqn(1) $\div$ (2)

$$\frac{I_1R_1}{I_1R_3} = \frac{I_2R_2}{I_2R_4}$$

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}}$$

Equation in the box is called the balance condition of a Wheatstone bridge.

NOTE :

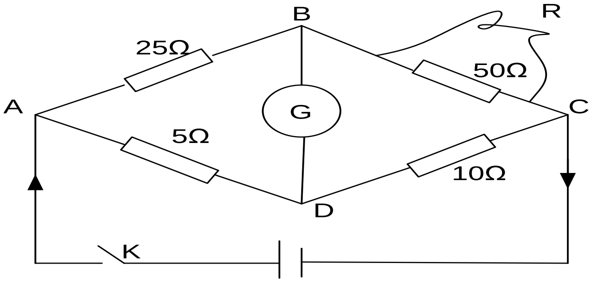
A wheastone bridge s not suitable for measureing very low and very high resistances

Explanation

- When the resistances are very low, resistances of connecting wires become comparable to the test resistances. Errors in the measurd values therefore become significant
- When the reistances are very high, the current flowing becomes very small. The galvanometer becomes less sensitive hence difficulty in determining the balance value

Examples

(g)



When the switch is closed the galvanometer shows no deflection when the 50Ω resistor is shunted with a resistance R, find the value of R

Solution

Let R_P be the total resistance of R Ω and 50Ω that are in parallel.

$$\therefore R_P = \frac{50R}{50+R}\Omega$$

Then at balance;

$\frac{R_1}{R_3} = \frac{R_2}{R_4} \Rightarrow \frac{25}{R_P} = \frac{5}{10}$ $\quad \therefore R_P = 12.5\Omega$

$$\therefore 12.5 = \frac{50R}{50+R}\Omega$$

$$\therefore R = 16.67\Omega$$

(h) In a Wheatstone bridge, the ratio arms R_1 and R_2 are approximately equal. When R_3 = 500Ω , the bridge is balanced. On interchanging R_1 and R_2, the value of R_3 for balancing is 505Ω. Find the value of R_4 and the ratio $R_1 : R_2$. **An(502.5Ω, 1:1.005)**

The Metre Bridge or Slide Wire

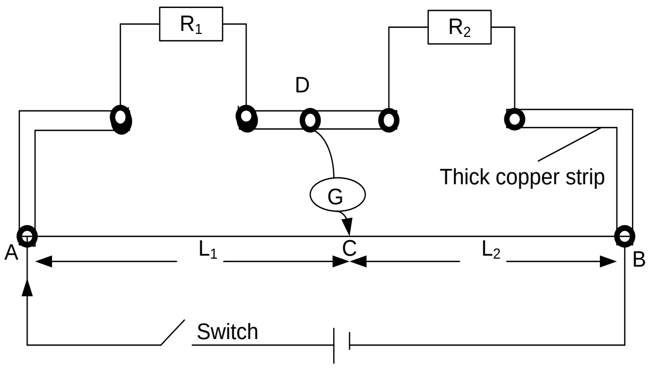
The simple metre bridge is the practical form of a Wheatstone bridge.

It consists of a uniform resistance wire 1 m long lying alongside or on a metre rule scale.

Thick copper or brass strips of low resistance connect various parts of the metre bridge.

Since the wire is uniform its resistance per cm is constant. The resistance of wire between any two points on the wire is proportional to the length separating them.

$$R_{AC} = kl_1$$



Balance condition

On closing switch K, the is jockey is tapped along AB until a point is found when the galvanometer shows <u>no deflection</u>. At balance point, D and C are at the <u>same potential</u>.

p.d across R_1 =p.d across l_1

p.d across R_2 =p.d across l_2

but current through R_1 = current through $R_1 = I_1$ and current through AB= I_2

$I_1R_1 = I_2rl_1$............................... 1

$I_1R_2 = I_2rl_2$...............................2

Where r is resistance per cm of wire AB.

Divide equation 1 by 2

$\frac{I_1R_1}{I_1R_2} = \frac{I_2kl_1}{I_2kl_2}$...................................3

$$\boxed{\frac{R_1}{R_2} = \frac{l_1}{l_2}}$$

<u>End – errors</u>

R_2 should be chosen such that the balance point, C is fairly near the centre of the wire (between 30 and 70 cm). This minimizes errors in the result and gives a more accurate value because the end –errors from both ends will be evenly distributed. A better result can be obtained by interchanging R_1 and R_2 and obtaining a second pair of values of l_1 and l_2. An average value of R_1 can then be taken.

If either l_1 or l_2 is very small, the resistance of the end connections is not negligible and must be added to R_{AC} or R_{CB}.

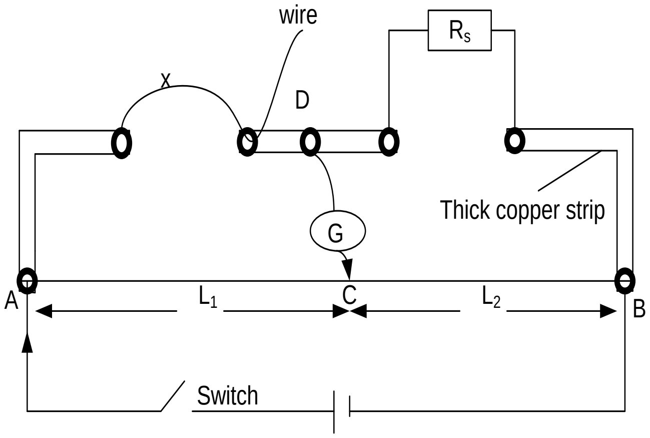
Let the end connection errors be equivalent to lengths e_1 and e_2 from A and B respectively.

Equation 3 then becomes $\boxed{;\ \frac{R_1}{R_2} = \frac{l_1+e_1}{l_2+e_2}}$

Notes

The metre bridge is therefore unsuitable for very low resistances because the contact resistances become comparable to the test resistances. It is equally not suitable for very high resistances because the galvanometer becomes insensitive

<u>To determine the resistivity of a material inform of a wire</u>



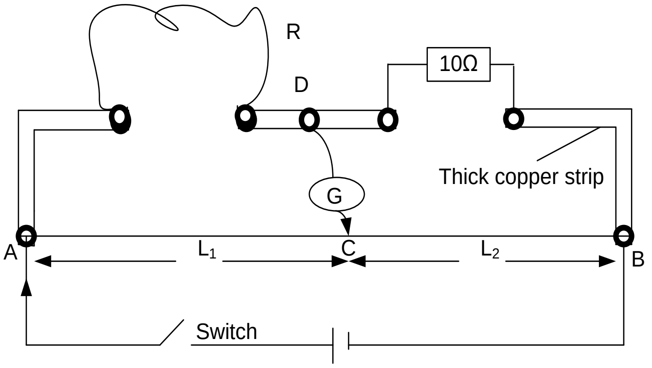
- ❖ With a micrometer screw gauge measure the diameter *d*, of the wire and the cross-sectional area ***A***, of the wire determined from $A = \frac{\pi d^2}{4}$.
- ❖ Connect a length *x* of the wire across the left hand gap and a standard resistance R_s in the right hand gap as shown above.
- ❖ Close switch K and tap the jockey along AB until you locate a point for which the galvanometer shows no deflection.
- ❖ Measure and record the balance lengths l_1 and l_2
- ❖ Determine the resistance R of the wire from

$$R_x = R_s \frac{l_1}{l_2}$$

- Repeat the procedure for different values of x and tabulate the results in a suitable table.
- Plot a graph of R_x against x and determine the slope s of the graph.
- Determine the resistivity of the wire from $\rho = SA$

Examples

1. A 110cm length of wire of diameter 0.85mm is placed in the left hand gap of a metre bridge and standard 10Ω coil being placed in the right hand gap.



Balance length obtained was 46.7cm from end of the bridge. Calculate the resistivity of the wire.

Solution

At balance; $\frac{R}{46.7} = \frac{10}{53.6}$

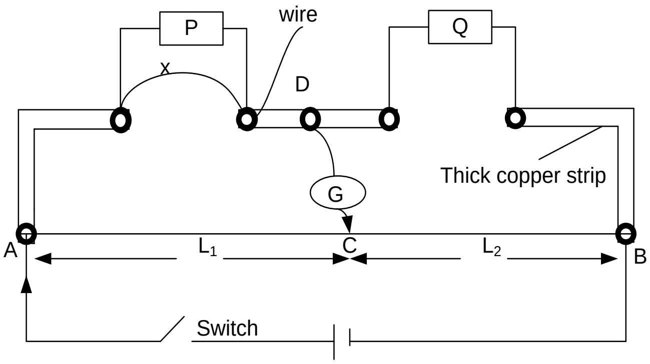
$\therefore R = 8.713\Omega$

$$\therefore \rho = \frac{RA}{L} = \frac{R\pi d^2}{L\,x4}$$

$$\rho = \frac{8.713 \times 3.14 \times (0.85 \times 10^{-3})^2}{1.1x4}$$

$$\rho = 4.5 \times 10^{-6}\Omega m$$

2.



In the diagram below, resistors P and Q are 5 Ω and 2Ω respectively. A wire X of length 60.0 cm and diameter 0.02 mm is connected across P so that the balance point is 66.7 cm from A. Calculate the resistivity of the wire.

Solution:

Let the effective resistance of P and X be R

When G shows no deflection, there is balance, hence

$$\frac{R}{Q}=\frac{l_1}{l_2}$$

$$\frac{R}{Q}=\frac{66.7}{33.3}$$

$$\frac{R}{2}=\frac{66.7}{33.3}$$

$$R=4\,\Omega$$

But $R=\frac{PX}{P+X}$

$$\therefore 4=\frac{5X}{5+X}$$

$$X=20\,\Omega$$

Length, *l* of X = 60 cm = 0.60 m and diameter, *d* = 0.02 mm

Cross sectional area of X, $A=\frac{\pi d^2}{4}$

$\mathrm{A}=\frac{\pi}{4}\times(0.02\times10^{-3})^2=3.14\times10^{-10}$ m²

From $R=\frac{\rho l}{A}$, it follows that $\rho=\frac{RA}{l}$

$$\rho=\frac{20\times3.14\times10^{-10}}{0.6}$$

$\rho=1.05\times10^{-8}\,\Omega$ m

3. A material of a wire of length 120cm and crossectional area $0.04cm^2$ has a resistance as 0.5Ω at 0°C. Find the resistivity of metal at 300°C, given temperature coefficient of resistance $as\ 7.5x\ 10^{-3}K^{-1}$.

Solution

$$R_\theta=R_0(1+a\theta)$$

$$R_{300}=0.5(1+7.5\times10^{-3}\times300)$$

$$=1.625\Omega$$

$$\therefore \rho=\frac{RA}{L}=\frac{1.625\times0.04\times10^{-4}}{1.2}$$

$\rho=5.42\times10^{-6}\Omega$ m

4. Find length of a wire of diameter 1.5mm and resistivity $2x10^{-6}$ at 30°C needed to make a coil of resistance 4Ω at 125°C, if temperature coefficient of resistance is $2.5\times10^{-3}\ K^{-1}$.

Solution:

$$R_\theta=R_0(1+a\theta)$$

Resistance at 30°C:

$$R_{30}=R_0(1+2.5\times10^{-3}\times30).$$

$$R_{30}=1.075R_O\ldots\ldots\ldots\ldots(i)$$

Resistance at 125°C:

$$R_{125}=R_0(1+2.5\times10^{-3}\times125)$$

$$R_{125}=1.3125R_0\ldots\ldots\ldots(ii)$$

But $R_{125}=4\Omega$

$$4=1.3125R_0$$

$$R_0=3.048\Omega$$

$$R_{30}=1.075R_O$$

$$R_{30}=1.075x3.048=3.28\Omega$$

$$R=\frac{\rho L}{A}$$

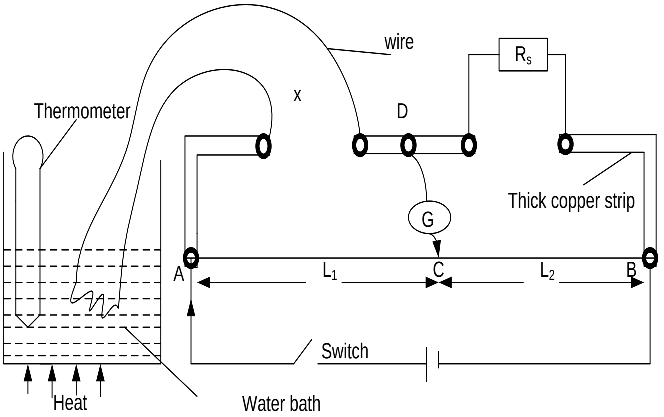
We are required to find length at 30°C

$L=\frac{RA}{\rho}=\frac{R_{30}A}{\rho_{30}}$ and $A=\frac{\pi d^2}{4}$

$$L=\frac{3.28\times\pi(1.5\times10^{-3})^2}{2\times10^{-6}x\,4}$$

$$L=2.9m$$

To Determine the Temperature Coefficient of Resistance of a Material



- A specimen fine wire is made into a coil and immersed in a water bath placed.
- The ens of the coil are connected in the left hand gap of the meter bridge and a standard resistance R_S in the right hand gap.
- The water bath is heated to a suitable temperature θ and stirred to ensure uniform temperature.
- Switch K is closed and the jockey tapped at different points on a uniform wire AB until a point is found where the galvanometer shows no deflection.
- The balance lengths l_1 and l_2 are measured and recorded.

- The resistanceR_θ of the coil at temperature θ is determined from

$$R_\theta = R_s\frac{l_1}{l_2}$$

- The experiment is repeated for different values of temperature, θ and the results tabulated.
- A graph of $R_\theta$against θ is plotted.
- The intercept R_0 on the R_θ - axis is read and the slope, S of the graph is determined.
- The temperture coeffiecnt of ressitance of the material is calculated from $\alpha = \frac{S}{R_o}$.

Examples:

1. A resistance coil is connected across the left hand gap of a metre bridge. When a 5.0 Ω standard resistor is connected across the right hand gap and the coil is immersed in an ice – water mixture, the balance point is at a point 45 .0 cm from the left hand end. When the coil is immersed in a steam bath at $100\ °\mathrm{C}$, the balance point shifts to a point 52.8 cm from the left hand end of the bridge. Find the temperature coefficient of the material of the coil.

Solution:

At $0°\mathrm{C}$; Using the balance condition

$$\frac{R_o}{R_s} = \frac{l_1}{l_2}$$

$$R_o = \frac{45}{55}\times 5 = 4.09\Omega$$

At $100°\mathrm{C}$: resistance at 100°C be R_{100}

$$\frac{R_{100}}{R_s} = \frac{l_1}{l_2}$$

$$R_{100} = \frac{52.8}{47.8}\times 5 = 5.59\Omega$$

$$\alpha = \frac{R_{100} - R_o}{\Delta\theta R_o}$$

$$\alpha = \frac{5.59-4.09}{4.09\ x\ (100-0)}$$

$$\alpha = 3.6\times 10^{-3}\mathrm{K}^{-1}$$

2. A resistance coil consists of a nichrome wire of diameter $4\times 10^{-4}m$ and length $\frac{\pi}{2}$. The coil is connected across the left hand gap of a metre bridge. When a 10 Ω standard resistor is connected across the right hand gap and the coil is immersed in an ice – water mixture, the balance point is at a point 60 .0 cm from the left hand end.
   (i) Find the resistivity of nichrome wire and its resistance at 0°C
   (ii) What would the balance length be when the coil is immersed in a steam bath at $100\ °\mathrm{C}$
   (temperature coefficient of the nichrome wire between 0°C and 100°C is $1.7x10^{-4}\mathrm{K}^{-1}$)

Solution:

(i) At $0°\mathrm{C}$;

Using the balance condition we have;

$$\frac{R_o}{R_s} = \frac{l_1}{l_2}$$

$$R_o = \frac{60}{40}\times 10 = 15\Omega$$

$\rho = \frac{RA}{L}$ and $A = \frac{\pi d^2}{4}$

$$\rho_0 = \frac{15\times\pi(4\times 10^{-4})^2}{4x\,{}^{\pi}/_{2}}$$

$$\rho_0 = 1.2\times 10^{-6}\Omega\,\mathrm{m}$$

(ii) $R_{100} = R_0(1 + 100\alpha)$

$$R_{100} = 15(1 + 1.7x10^{-4}x100)$$

$$R_{100} = 15.26\Omega$$

At $100°\mathrm{C}$

$$\frac{R_{100}}{R_s} = \frac{l_1}{l_2}$$

$$\frac{15.26}{10} = \frac{l}{100 - l}$$

$$l = 60.4cm$$

3. A nickle wire and a 10 Ω standard resistor are connected across the gaps of a meter bridge. When the nickel wire was at $0°\mathrm{C}$, balance point was found 40 .0 cm from the end of the bridge wire adjacent to the nickel wire. When the nickel wire was at $100\ °\mathrm{C}$, the balance point shifts to a point 50.0 cm. Find the temperature of the nickel wire when the balance point was at 42.0cm and the resistivity of nickel at this temperature. (length of the wire is 150cm and cross-sectional area is $2.5\times 10^{-4}cm^2$)

Solution:

At 0°C ; Using the balance condition

$$\frac{R_o}{R_s} = \frac{l_1}{l_2}$$

$$R_o = \frac{40}{60} \times 10 = 6.67\Omega$$

At 100°C: resistance at 100°C be R_{100}

l_1 = 50 cm, l_2 =100 – 50 = 50 cm

$$\frac{R_{100}}{R_s} = \frac{l_1}{l_2}$$

$$R_{100} = \frac{50}{50} \times 10 = 10\Omega$$

$$\alpha = \frac{R_{100} - R_o}{\Delta\theta R_o}$$

$$\alpha = \frac{10-6.67}{6.67x\,(100-0)}$$

$$\alpha = 5.0 \times 10^{-3}\text{K}^{-1}$$

when the balance length is 42cm let the resistance be R_θ

$$R_\theta = \frac{42}{58} \times 10 = 7.24\Omega$$

$$R_\theta = R_0(1 + \theta\alpha)$$

$$7.24 = 6.67(1 + 5.0 \times 10^{-3}\theta)$$

$$\boldsymbol{\theta = 17.24}\text{°C}$$

$$\rho = \frac{RA}{L}$$

$$\rho_\theta = \frac{7.24 \times 2.5 \times 10^{-4}x10^{-4}}{1.5}$$

$$\rho_\theta = 1.21 \times 10^{-7}\Omega\,\text{m}$$

4. When a coil x is connected across the Left hand gap of a metre bridge and heated to a temperature of 30°C, the balance point is found to be 51.5°C from the left hand side of the slide wire. when the temperature is raised to 100°C, the balance point is 54.6cm from the left hand side. Find the temperature coefficient of resistance of x.

Solution

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

$$R_{30} = \frac{51.5}{100 - 51.5}R_x = 1.06R_x$$

$$R_{100} = \frac{54.6}{100 - 54.6}R_x = 1.203R_x$$

$$R_{30} = R_0(1 + 30\alpha) \ldots\ldots\ldots\ldots\ldots.(i)$$

$$R_{100} = R_0(1 + 100\alpha) \ldots\ldots\ldots\ldots\ldots.(i)$$

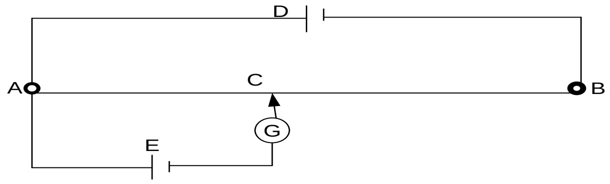
$$\frac{R_{30}}{R_{100}} = \frac{R_0(1 + 30\alpha)}{R_0(1 + 30\alpha)} = \frac{1.06R_x}{1.203R_x}$$

$$\alpha = 2.01x10^{-3}K^{-1}$$

Exercise

1. Two resistance coils P and Q are placed in the gaps of a metre bridge. A balance point is found when the movable contact touches the bridge wire at a distance of 35.5cm from the end joined to end P. When the coil Q is shunted with a resistance of 10Ω, the balance point is moved through a distance of 15.5cm. Find the values of the resistances P and Q.
2. In a metre bridge when a resistance in left gap is 2Ω and unknown resistance in right gap, the balance point is obtained from the zero end at 40cm on the bridge wire. On shunting the unknown resistance with 2Ω, find the shift of the balance point on the bridge wire. **An (22.5cm)**
3. With a certain resistance in the left gap of a slide wire, the balancing point is obtained when a resistance of 10Ω is taken out from the resistance box. On increasing the resistance from the resistance box by 12.5Ω, the balancing point shifts by 20cm. Find the value of unknown resistance. **An(15Ω)**

PRINCIPLE OF POTENTIOMETER SLIDE- WIRE

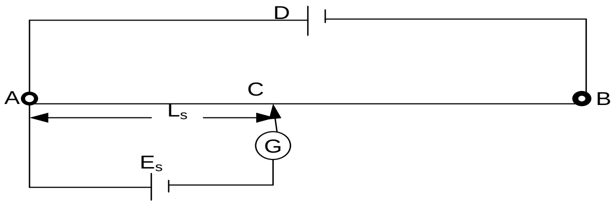


- A driver cell **D**, maintains a constant current through the slide wire.
- The wire is uniform hence has a constant resistance per cm and therefore the p.d per cm is also constant.
- Knowing the p.d per cm of the slide wire any p.d can be determined by balancing it against a known length of the wire
- If p.d per cm is k and balance elenght is l then the required p.d is $V = kl$

NB: The slider or jockey must not be scrapped on the potentiometer wire otherwise the wire will become non-uniform when scrapped.

Standardization (calibration) of a potentiometer wire

This refers to determining the p.d per cm of a potentiometer wire using a standard cell so that it can be used to measure p.ds.



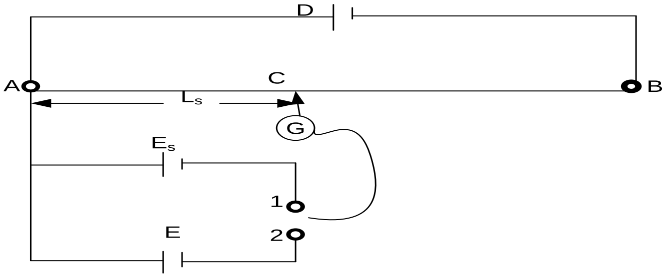
- ❖ Connect a standard cell of e.m.f E_S as shown above.
- ❖ The slidding contact is moved along the uniform wire AB until a point is found where the galvanometer G shows no deflection.
- ❖ The balance length l_S is measured.
- ❖ At balance point, $E_S =$ p.d across AC.

$$\therefore E_S = kl_S$$

$$\Rightarrow\ k = \frac{E_S}{l_S}$$ where ***k*** =calibration constant

Applications of the potentiometer

To Measure e.m.f of a cell by comparison



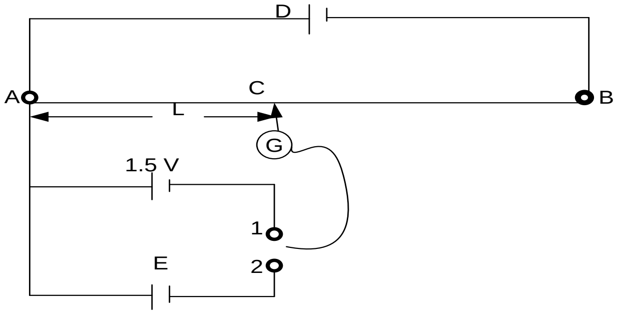
- ❖ Connect a standard cell of e.m.f E_S and the cell of unknown e.m.f E as shown above.
- ❖ With galvanometer connected to position 1, the jockey is tapped at different points along wire AB until a point is found where the galvanometer G shows no deflection.
- ❖ Measure the balance length l_S.
- ❖ With galvanometer connected to position 1, the jockey is tapped at different points along wire AB until a point is found where the galvanometer G shows no deflection.
- ❖ Measure the balance length l.
- ❖ The e.m.f of the test cell is got from

$$E = \left(\frac{l}{l_S}\right)E_S$$

Example:

A standard cell of e.m.f $1.5\ V$ is balanced on a potentiometer wire by a length of 60.0 cm. Another cell of unknown e.m.f, E is balanced on the same potentiometer wire by a length of 75.0 cm. Calculate the value of E.

Solution:



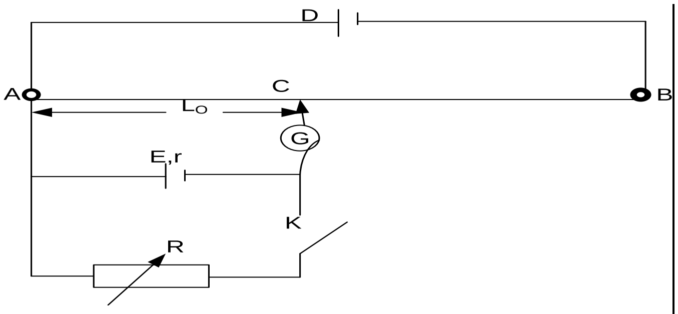
When G is at 1, the standard cell is balanced. At balance, p.d across AC = e.m.f of the standard cell

$$E = \left(\frac{l}{l_S}\right)E_S$$

$$E = \left(\frac{75}{60}\right)x1.5$$

$$E = 1.88V$$

To measure the internal resistance, *r* of a cell.



- Connect the cell and a resistance box *R*, as shown above.
- With switch K open, tap the jockey at different positions along the slide wire AB until you locate a point at which the galvanometer shows no deflection.
- Measure the balance length l_O
- Set the resistance box to a suitable value *R* and then close switch K.
- Tap the jockey at different positions along AB until a point C at which the galvanometer shows no deflection is located.
- Measure and record the balance length AC = l.
- Internal resistance of cell, $r\ = R\left(\frac{l_0}{l} - 1\right)$

Theory of experiment

At balance when K is open, the cell is on an open circuit and p.d across AC is equal to e.m.f of cell.

$E\ =\ klo$ 1

When K is closed the cell supplies current to R and is now on a closed circuit.

At balance p.d across R = p.d across AC

$V\ =\ kl$ 2

Divide equation 1 by 2

$\frac{E}{V} = \frac{l_0}{l}$...............................3

$E\ =\ I(R\ +\ r)$ and $V\ =\ IR$

Substitute for *E* and *V* in equation 3

$$\frac{I(R\ +\ r)}{IR} = \frac{l_0}{l}$$

$$\boxed{r\ = R\left(\frac{l_0}{l} - 1\right)}$$

Example:

1. A dry cell gives a balance length of 84.8 cm on a potentiometer wire. When a resistor of resistance 15Ω is connected across the terminals of the cell, a balance length of 75.0 cm is obtained. Find the internal resistance of the cell.

Solution

$$\frac{E}{V} = \frac{l_0}{l}$$

$$E\ =\ I(R\ +\ r) \text{ and } V\ =\ IR$$

$$\frac{I(R\ +\ r)}{IR} = \frac{l_0}{l}$$

$$r\ = R\left(\frac{l_0}{l} - 1\right)$$

$$r\ = 15\left(\frac{84.8}{75} - 1\right)$$

$$r = 1.96\Omega$$

2. The e.m.f of a battery A is balanced by a length of 75.0cm on a potentiometer wire. The e.m.f of a standard cell of 1.02V is balanced by a length of 50.0cm. Find;
   (i) E.m.f of battery A
   (ii) The new balance length if A has internal resistance of 2Ω and a resistor of 8Ω is connected across its terminals

Solution

$$E = \left(\frac{l}{l_s}\right)E_s$$

$$E = \left(\frac{75}{50}\right)x1.02$$

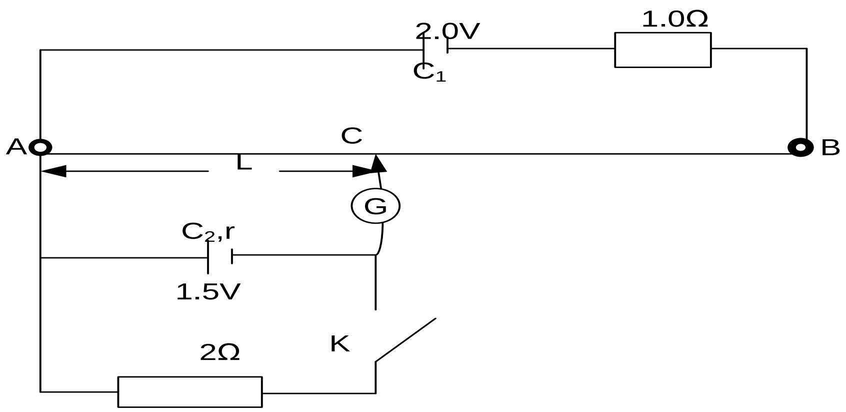
$$E = 1.53V$$

$$r\ = R\left(\frac{l_0}{l} - 1\right)$$

$$2\ = 8\left(\frac{75}{l} - 1\right)$$

$$l = 60cm$$

3.



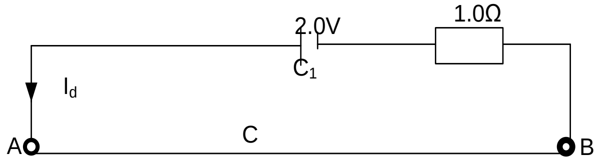
In the circuit above, AB is a uniform wire of length 1 m and resistance 4.0 Ω. C_1 is an acummulator of e.m.f 2 V and negligible internal resistance. C_2 is a cell of e.m.f 1.5 V.

(a) Find the balance length AC when the switch is open

(b) If the balance length is 75.0 cm when the switch is closed, find the internal resistance of C_2.

Solution:

(a) Consider the driver cell circuit only.



$$2 = I_d(R_{AB} + 1)$$
$$2 = I_d(4 + 1)$$
$$I_d = \frac{2}{5} = 0.4\ A$$

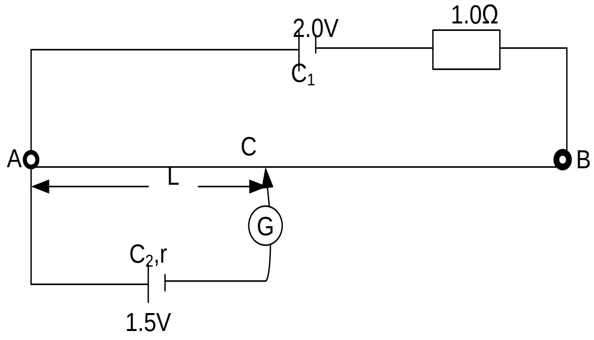
P.d across the whole wire, $V_{AB} = I_d R_{AB}$ where R_{AB} is the resistance of wire AB.

$$V_{AB} = 0.4 \times 4 = 1.6\ V$$

P.d per cm, k of AB is given by:

$$k = \frac{V_{AB}}{AB} = \frac{1.6}{100} = 0.016\ \text{V cm}^{-1}$$

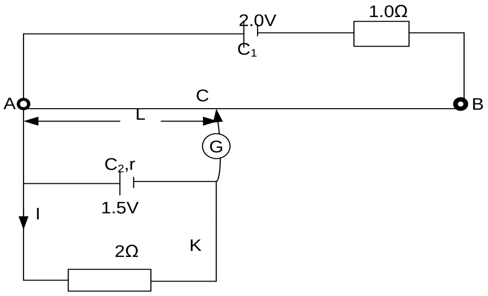
With S open cell C_2 is now on an open circuit and at balance; p.d across AC = e.m.f of C_2.

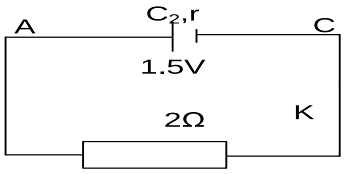


$$V_{AD} = kl = 1.5$$

$$l = \frac{1.5}{k} = \frac{1.5}{0.016} = 93.75\ \text{cm}$$

(b) With S closed, current is drawn from C_2 and the cell is now on a closed circuit and supplies current I to the 2.0 Ω resistor.





At balnce p.d across AC = p.d across the 2.0 Ω resistor, V

$V = k.l$ where l = 75.0 cm

$$V = 0.016 \times 75 = 1.2 \text{ V}$$

But V = IR, hence $I = \frac{V}{R} = \frac{1.2}{2} = 0.6 \text{ A}$

Since at balance the current through G is zero, we can now only consider the lower circuit shown above.

From $E = I(R + r)$we have;

$$1.5 = 0.6(2 + r)$$

$$1.5 = 1.2 + 0.6r$$

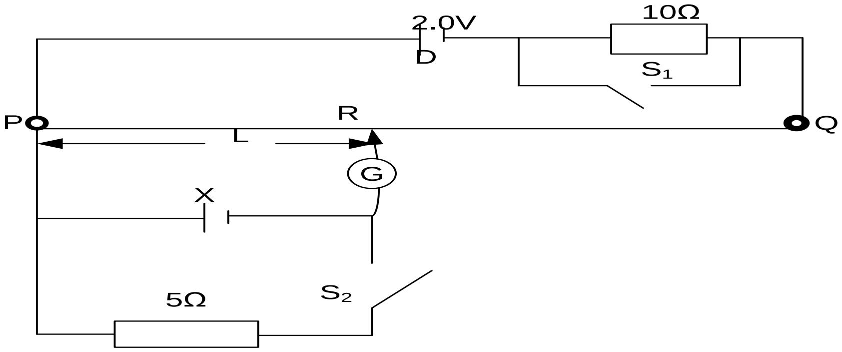
$r = \frac{0.3}{06}$ = 0.5 Ω

Or

$$r = R\left(\frac{l_0}{l} - 1\right)$$

$$r = 2\left(\frac{93.7}{75} - 1\right) = 0.5\ \Omega$$

4. The figure below shows a cell D of negligible internal resistance with e.m.f $2V$. PQ is a uniform slide wire of length 1.00 m and resistance 50Ω.



With both switches S_1 and S_2 open, the balance length PR is 0.90 m. When S_2 is closed and S_1 left open, the balance length changes to 0.75 m. Determine the

(i) e.m.f of cell X.
(ii) internal resistance, r, of X
(iii) balance length when both S_1 and S_2 are closed

solution

(i) with both switches open: $2 = I_d(R_{AB} + 10)$

$$2 = I_d(50 + 10)$$

$$I_d = \frac{1}{30}A$$

P.d across the wire AB: $V_{AB} = \frac{1}{30} \times 50 = \frac{5}{3}\ V$

P.d per cm, k of AB is given by:

$$k = \frac{5}{300}\ \text{V cm}^{-1}$$

e.m.f of cell x = kl

$$E = \frac{5}{300}x90 = 1.5V$$

(ii) With S_2 closed:

p.d across x;

$$V = \frac{5}{300}x75 = 1.25V$$

current supplied by x:

$$I = \frac{V}{R} = \frac{1.25}{5} = 0.25A$$

But $E = I(r + R)$

$$1.5 = 0.25(r + 5)$$

$$r = 1\Omega$$

Or

$$\frac{E}{V} = \frac{l_0}{l}$$

$E = I(R + r)$ and $V = IR$

$$\frac{I(R + r)}{IR} = \frac{l_0}{l}$$

$$r = 5\left(\frac{90}{75} - 1\right) = 1.0\ \Omega$$

(iii) When both switches are closed

10Ω is out of the circuit, current passes through the switch

$$2 = I_d(R_{AB})$$
$$2 = I_d(50)$$
$$I_d = \frac{1}{25}A$$

P.d across the wire AB: $V_{AB} = \frac{1}{25} \times 50 = 2\ V$

P.d per cm, k of AB is given by:

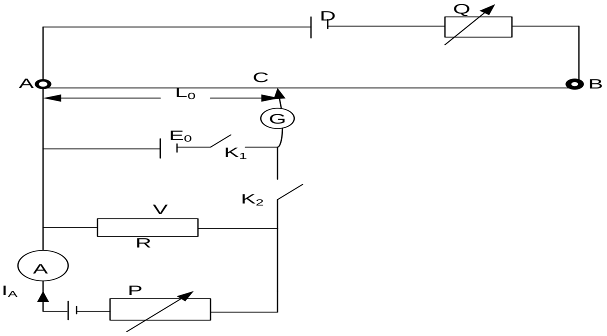
$$k = \frac{2}{100}\ \mathrm{V\ cm^{-1}}$$

e.m.f of cell x $= kl$

$$1.25 = \frac{2}{100}xl$$
$$l = 62.5cm$$

Calibration of an ammeter and Current Measurement

Current can be measured on a potentiometer by measuring the p.d *V* it sets up across a standard resistance *R*, and then using $I = \frac{V}{R}$.



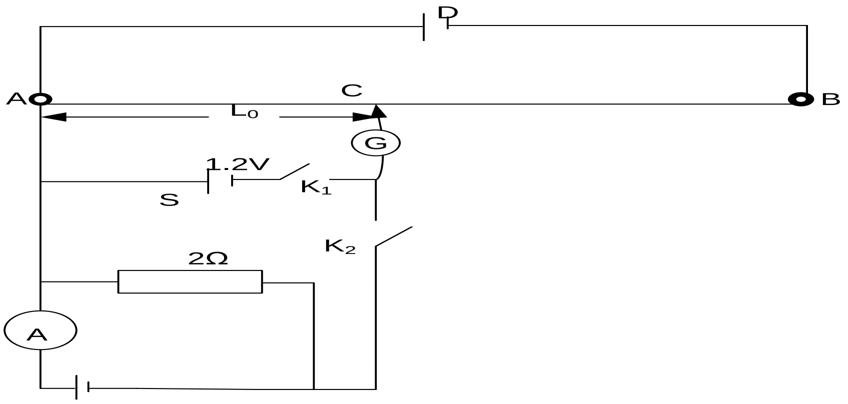
- Connect a standard cell of e.m.f *E₀* and a standard low resistance *R* as shown above.
- With switches K₁ closed and keep K₂ open, tap the jockey at different positions along AB until the galvanometer shows no deflection.
- Measure and record the balance length *l₀*
- P is adjusted so that the ammeter records the smallest current I_r. With switch K₁ open and K₂ closed, the balance length *l* is obtained and recorded.
- Determine the actual current I_A, $\left(I_A = \frac{E_0}{l_0}\ x \frac{l}{R}\right)$ and error *e*, in the ammeter reading, $e = I_A - I_r$ and
- The experiment is repeated for different adjustments of P and hence for different readings of the ammeter I_r.
- Tabulate the results including values of I_r and *e.*
- Plot a calibration graph of e against I_r

NB: The percentage error in the ammeter reading $= \frac{e}{I_a} \times 100\%$.

By calculating *I_a* the potentiometer is being used to measure current.

The nature of the graph shows that the errors in *I_r* occur randomly.

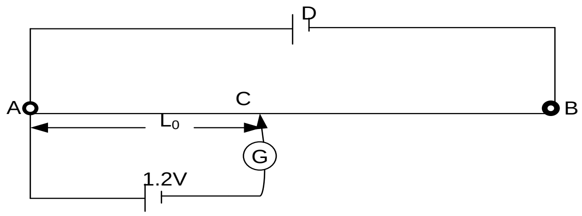
Example:



In the circuit above, S is a standard cell of e.m.f $1.2\ V$. When switch K_1 is closed and K_2 is open, a balance length AC= $30.2\ cm$ is obtained. When K_1 is opened and K_2 is closed, the balance length is $26.8\ cm$ and the ammeter, A reads $0.4\ A$. Calculate the percentage error in the ammeter reading.

Solution

K_1 is closed and K_2 is open;

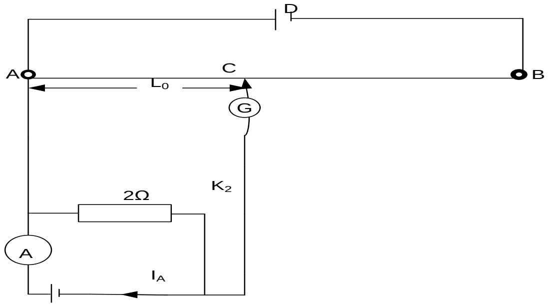


At balance, p.d across AC = 1.2

$30.2 \times k = 1.2$

$k = \frac{1.2}{30.2} = 0.0397$ V cm^{-1}

K_1 is opened and K_2 is closed;



At balance, p.d acrossAC= p.d across 2 Ω

$V = AC \times k$

$V = 26.8 \times 0.0397 = 1.064$ V

But $V = I_a R$

$I_a = \frac{V}{R} = \frac{1.064}{2} = 0.532$ A

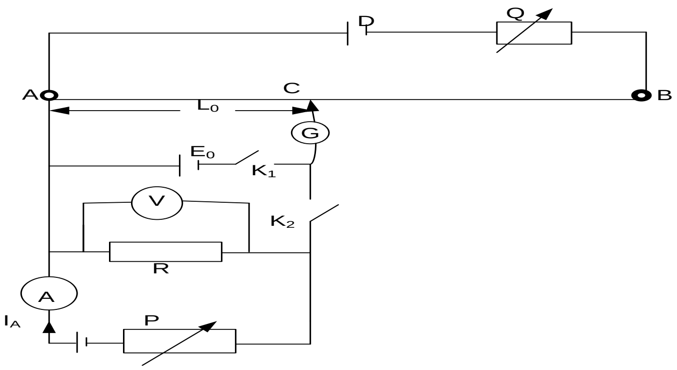
Error in the ammeter reading, $e = I_a - I_r$

$e = 0.532 - 0.4 = 0.132$ A

$$percentage\ error = \frac{e}{I_a} \times 100\%$$

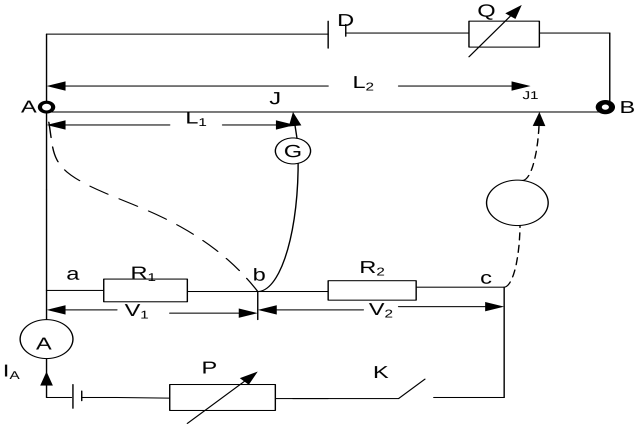
$$= \frac{0.132}{0.532} \times 100 = 24.8\%$$

Calibration of Voltmeter



- A standard cell of e.m.f E_0 and a rheostat P are connected as shown above.
- Switch K_1 is closed while K_2 is left open. A balance point is located where the galvanometer indicates zero deflection. The balance length l_0 is measured and recorded..
- The p.d per cm, k of the slide wire is calculated from
  $k = \frac{E_0}{l_0}$ 1
- P is adjusted so that the voltmeter records the smallest p.d it possible to read. K_1 is then closed and a point is located where the galvanometer register zero current.
- The balance length l is obtained and recorded.
- The experiment is repeated for different adjustments of P and hence for different readings, V_r of the voltmeter. Balance length l is determined.
- The results are tabulated including values of $V_a = kl$ and the error $e = (V_a - V_r)$.
- A graph of e against V_r is plotted and constitutes the calibration curve for the voltmeter.

Comparison of two Resistances (measurement of resistance)



- Connect the two resistances R_1 and R_2 in series with an ammeter A and rheostat P as shown above.
- Close switch K. With the galvanometer at ***a*** and ***b***, the balance length AJ = l_1 is measured and recorded. Hence $IR_1 = kl_1 \dots\dots\dots\dots (i)$
- Connections at ***a*** and ***b*** are removed and replaced by those at ***b*** and ***c***(dotted lines).
- The balance length AJ' = l_2 is measured and recorded. Hence $IR_2 = kl_2 \dots\dots\dots\dots (ii)$
  Equation (i) divide by (ii)

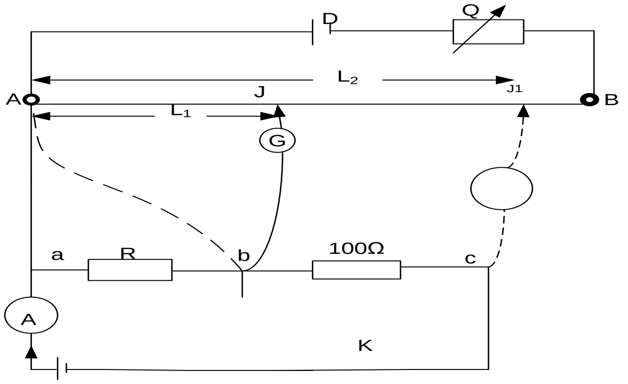
$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

Practical points to note

1. If the balance point cannot be obtained with both resistances, then either I is too large or the p.d V_{AB} across AB is very small. Balance can be achieved by adjusting rheostats P and Q.
2. If balance can be obtained with R_1 and not with R_2, then R_2 is much greater than R_1. The method is thus suitable for resistances that do not differ much in magnitude.
3. If the balance lengths are very small, an end – error (correction) due to the resistance of the contact at the zero end must be added to the balance lengths.

Example

1. The circuit below is used to compare the resistance R of an unknown resistor with a standard 100 Ω resistor.



The distances l from one end of the slide wire of the potentiometer to the balance point are 40.0 cm and 58.8 cm, respectively when G is connected to b and then to c respectively. If the slide wire is 1.00 m long, find the value of R.

Solution

Let the current in the lower circuit be I

With G connected at b,

the p.d across $100\,\Omega$ = p.d across l

$$100I = kl_1$$

where k is the p.d per cm and l_1 = 40.0 cm

$$100I = 40k \text{ ------------------- } 1$$

With G at c p.d 100 and R = p.d across the new balance length,

$$(100 + R)I = kl_2$$

$$(100 + R)I = 58.8k \text{ ------------------------- } 2$$

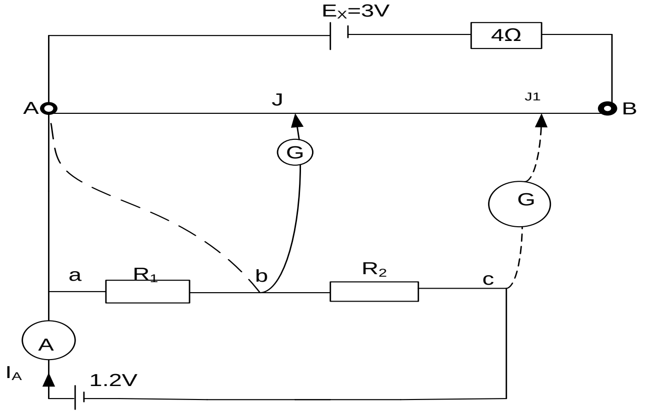
Divide equation 2 by equation 1

$$\frac{100 + R}{100} = \frac{58.8}{40.0}$$

$$100 + R = 147$$

$$R = 47\,\Omega$$

2. An accumulator of e.m.f 3V with negligible internal resistance is connected in series to a 4 Ω resistor and a potentiometer wire AB of length 1.0m as shown below.



The accumulator supplies a steady current of 0.25A through the wire AB. With the galvanometer connected at b, the balance length $AJ = 46cm$ and when the galvanometer is at c, the balance length $AJ^1 = 75cm$. Find;

(i) The value of R_1 and R_2

(ii) The reading of the ammeter A

Solution

(i) The current along AB $I = 0.25A$

p.d across 4 $\Omega = 0.25x4 = 1.0V$

P.d across AB $V_{AB} = 3 - 1 = 2V$

P.d per cm along AB, $k = \frac{2}{100} = 0.02\ Vcm^{-1}$

With G connected at b,

the p.d across R_1 = p.d across l

$$R_1 I = kl_1$$

where k is the p.d per cm and l_1 = 46.0 cm

$$R_1 x0.25 = 0.02x46$$

$$R_1 = 3.68\Omega$$

With G at c, p.d R_1 and R_2 = p.d across the new balance length,

$$(R_1 + R_2)I = kl_2$$

$$(3.68 + R_2)x0.25 = 0.02x75$$

$$R_2 = 2.32\Omega$$

(ii)

$$(R_1 + R_2)I_A = 1.2$$

$$\frac{1.2}{3.68 + 2.32} = I_A$$

$$I_A = 0.2A$$

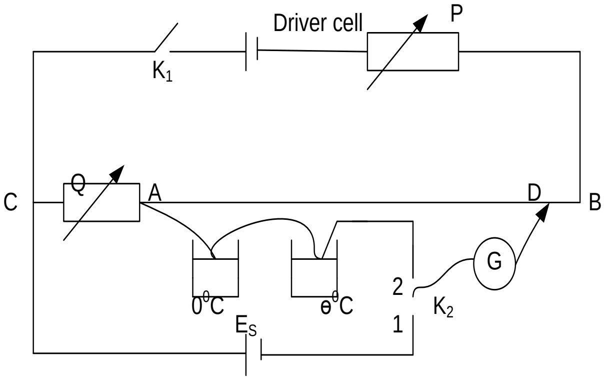
MEASUREMENT OF THERMOELECTRIC E.M.F

The e.m.fs of thermo junctions (thermocouples) are very small; of the order of a millivolt.

If such an e.m.f is measured on a simple potentiometer the balance – point is found to be very near end A and results in the end –error being serious, thus affecting the value obtained.

<u>How to modify the simple potentiometer to measure small e.m.fs or p.ds</u>

To obtain measurable balance lengths a suitable high resistance R is connected in series with the slide wire so that the driver cell sets up a small p.d across AB. This helps in producing a small p.d per cm.



- The standard cell of emf E_s is connected across Q and the slide wire.
- K₁ is closed and K₂ is connected to position 1. Tap the jockey at different positions along wire AB until the galvanometer shows no deflection.
- The balance length l_s is measured and recorded.
- While K₁ is closed, K₂ is connecteds to position 2 and the point on AB when the galvanometer registers zero current is found. The balance length l is measured.
- E is found using the formula $E = \frac{E_s r\ l}{Q + r\ l_s}$ where r is the resistance per cm of the slide wire.

Theory of experiment

Current through the wire AB $i_p = \frac{V_O}{P + Q + r\ l}$

where L is the length of the wire.

Hence p.d per cm, $k = i_p r = \frac{V_O\ r}{P + Q + r\ l}$

When K₂ is in position 1, $E_s = i_p Q + k\ l_s$

$$E_s = \frac{V_O}{P + Q + r\ l}(Q + r\ l_s) \ldots\ldots\ldots\ldots\ldots(i)$$

When K₂ is in position 2, $E = kl$

Where E is the emf of the thermocouple.

$$E = \frac{V_O\ r\ l}{P + Q + r\ l} \ldots\ldots\ldots\ldots\ldots(ii)$$

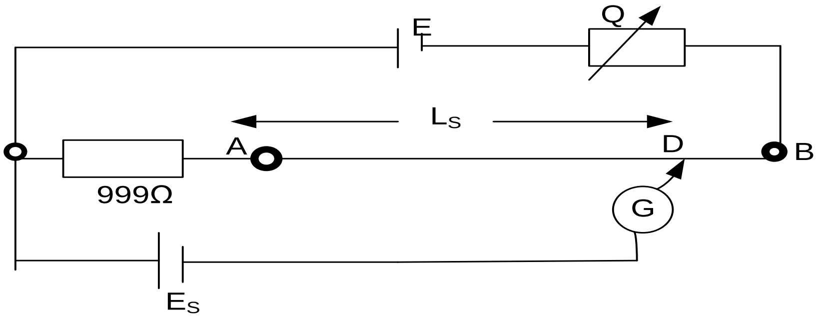
equation (ii) divide by (i)

$$\frac{E}{E_s} = \frac{r\ l}{Q + r\ l_s}$$

Hence $E = \frac{E_s r\ l}{Q + r\ l_s}$

Examples:

1.



In the figure above, E is a driver cell of e.m.f 2 V and negligible internal resistance. Es is a standard cell of e.m.f 1.00 V and AB is a uniform wire of resistance 10 Ω and length 100 cm. the galvanometer G shows no deflection when $ls = 10.0\ cm$. Find

(i) The current in the driver circuit

(ii) The resistance of the rheostat

(iii) The e.m.f of a thermocouple which is balanced by a length of 60 cm of the slide wire AB.

Solution:

(i) Let the resistance per cm of AB be β

$$\beta = \frac{R_{AB}}{100} = \frac{10}{100} = 0.1\ \Omega\ cm^{-1}$$

$$R_{AD} = \beta l_s$$

$$R_{AD} = 0.1 \times 10 = 1\ \Omega$$

But, $E_s = V_{999} + V_{AD}$

$$E_s = I_d(R + R_{AD})$$

$$I_d = \frac{E_s}{R+R_{AD}}$$

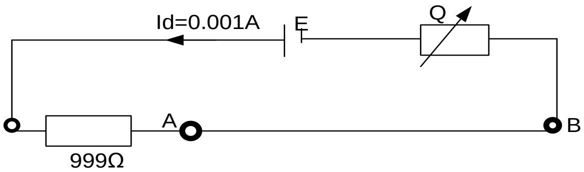
$$I_d = \frac{1}{999+1} = \frac{1}{1000} = 0.001\ \text{A}$$

(ii) $E = I_d(999 + 10 + R)$

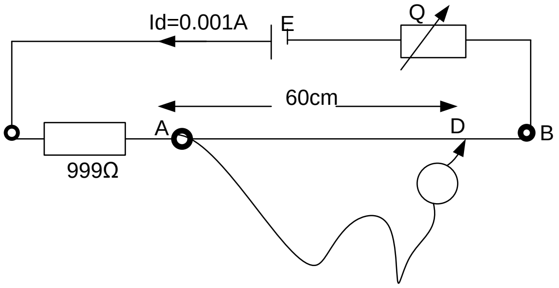
$$2 = 0.001(1009 + R)$$

$$= 1.009 + 0.001R$$

$$R = 991\ \Omega$$



(iii)



$$V_{AB} = I_d R_{AB}$$

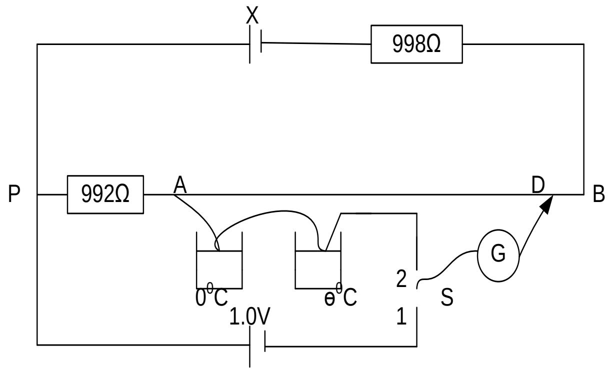
$$V_{AB} = 0.001 \times 10 = 0.01\ \text{V}$$

$$p.d\ per\ cm, k = \frac{V_{AB}}{100} = \frac{0.01}{100}$$

$k = 0.0001$ V cm^{-1}

$$E = kl = 0.0001 \times 60 = 0.006\ \text{V}$$

2.



In the figure above X is an accumulator of negligible internal resistance. AB is a uniform wire of length 1.0m and diameter $3.57x10^{-4}m$ and resisitivity $1.0x10^{-6}\ \Omega m$. G is a galvnomeer connected to a sliding contact D. When s is in position 1, G shows no deflection when AD is 80cm. When s is in position 2, G shows no deflection when AD is 40cm. find;

(i) The resistance of AB

(ii) E.m.f of the thermocouple

(iii) The e.m.f of the accumulator x

Solution

(i) $R = \rho\frac{l}{A}$

$$R_{AB} = 1.0x10^{-6}x\frac{1}{\pi\left(\frac{3.57x10^{-4}}{2}\right)^2}$$

$$R_{AB} = 10\Omega$$

(ii) When s is in position 1, the 1.0V cell is being balanced

For the driver cell; Let the resistance per cm of AB be β

$$\beta = \frac{R_{AB}}{100} = \frac{10}{100} = 0.1\ \Omega\ cm^{-1}$$

$$R_{AD} = \beta l_s$$

$$R_{AD} = 0.1 \times 80 = 8\ \Omega$$

At balance: $1.0 = V_{PA} + V_{AD}$

$$1.0 = I_d(R + R_{AD})$$

$$I_d = \frac{1}{992+8} = \frac{1}{1000} = 0.001\text{ A}$$

$$V_{AB} = I_d R_{AB}$$

$$V_{AB} = 0.001x\ 10$$

$$V_{AB} = 0.01V$$

p.d per cm along AB; $k = \frac{0.01}{100} = 1x10^{-4}Vcm^{-1}$

When s is in position 2, the thermocouple is being balanced

$$E_T = kl$$

$$E_T = 1x10^{-4}x40$$

$$E_T = 4mV$$

(iii) $E_x = I_d(998 + 992 + 10)$

$$E_x = 0.001(998 + 992 + 10)$$

$$E_x = 2.0V$$

Advantages of a potentiometer

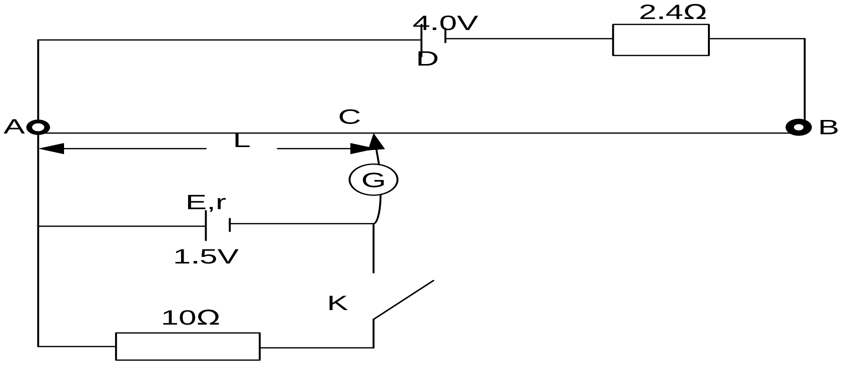
1. It does not draw any current from the p.d being measured and therefore gives accurate results. Resistance of the connecting wires and the galvanometer does not affect the results.
2. It can measure a wide range of p.ds since the length of the slide wire can be adjusted.
3. It gives accurate results since they depend only on measurements of lengths, standard resistances and standard e.m.fs.

Disadvantages

1. It does not give direct results and is therefore time consuming;
2. It requires technical skills to operate;
3. It is cumbersome.

Exercise

1. A dry cell gives a balance length of 85.0cm on a potentiometer wire. When a resistor of resistance 16Ω is connected across the terminals of the cell, a balance length of 76.0cm is obtained. Find the internal resistance of the cell. **An(1.89Ω)**
2. A dry cell gives a balance length of 0.75m on a potentiometer wire. When the cell is shunted by a resistance of 14Ω, the balance length of 0.70m is required. Find the internal resistance of the cell. **An(1.0Ω)**
3. A 1Ω resistor is in series with an ammeter m in a circuit. The p.d across the resistor is balanced by a length of $60cm$ on a potentiometer wire. A standard cell of emf $1.02V$ is balanced by a length of $50cm$. If m reads $1.1A$, what is the error in the reading? $\boldsymbol{An(0.124A)}$
4. A potentiometer wire of length $1m$ and resistance 1Ω is used to measure an emf of the order mV. A battery of emf $2V$ and negligible internal resistance is used as a driver cell. Calculate the resistance to be in series with potentiometer so as to obtain a potential drop of $5mV$ across the wire. $\boldsymbol{An(399\Omega)}$
5. In a potentiometer, a cell of emf **x** gave a balance length of **a** cm and another cell of emf **y** gave a balance length of **b** cm. When the cells are connected in series, a balance length of **c** cm was obtained. It was also discovered that **a + b** ≠ **c.** Show that the true ratio $\frac{x}{y}=\frac{c-b}{c-a}$.
6.

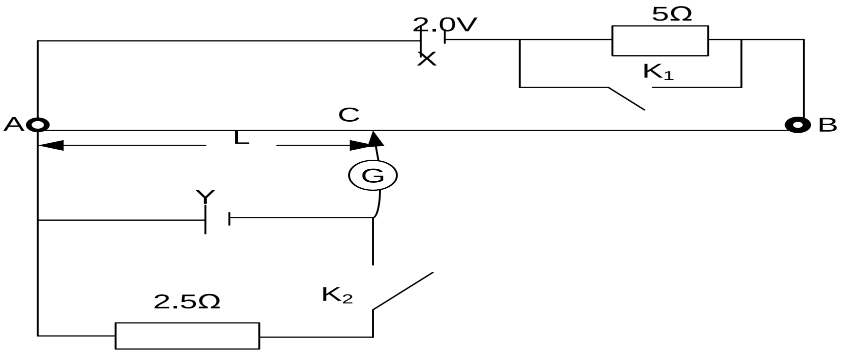


In the circuit above, AB is a uniform wire of length 1 m and resistance 2.0 Ω. D is a cell of e.m.f 4.0 V and negligible internal resistance. E is a cell of e.m.f 1.5 V.

(i) Find the balance length AC when the switch is open. **An(82.5cm)**

(ii) If the balance length is 71.5 cm when the switch is closed, find the internal resistance of E. **An(1.54Ω)**

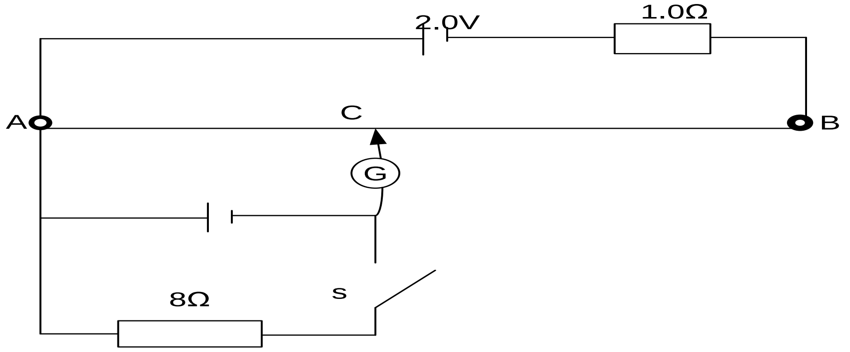
7.



In the circuit above, **x** has negligible internal resistance and length AB is 100cm and resistance of AB is 20Ω. When K_1 and K_2 is open, the balance length $AC = 80cm$. When K_2 is closed and K_1 open, the balance length $AC = 65cm$. Find

(i) the emf of cell y
(ii) internal resistance of cell y.
(iii) the balance length when K_1 and K_2 are closed.

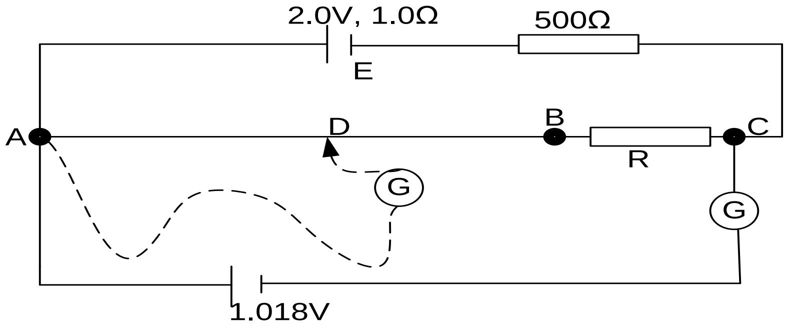
8.



In the figure above the slide wire AB is 1 m long and has a resistance of 4Ω . When switch S is:

(i) open, the balance length AC is 88.8 cm. Find the value of the e.m.f of the cell
(ii) closed, the balance length is found to be 82.5 cm. Calculate the internal resistance of this cell

9. In the figure below, AB is a uniform resistance wire of length 1.00 m and resistance 10.0Ω E. is an accumulator of e.m.f 2.0 V and internal resistance 1.0 Ω

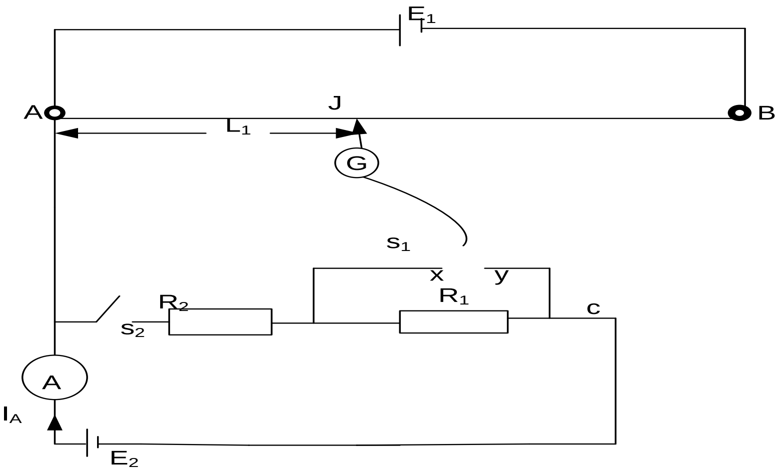


When a standard cell of e.m.f 1.018 V is connected in series with a galvanometer, G across AC, the galvanometer shows no deflection. When the standard cell is removed and a thermocouple connected via the galvanometer, G, as shown by the dotted line, G shows no deflection when AD = 41.0 cm.

Calculate the:

(i) value of R,
(ii) e.m.f of the thermocouple. **An(509.3Ω, 8.04mV)**
(iv)

10.



The circuit above shows a uniform slide wire AB of length 100 cm. The e.m.f. of cell E_1 is 2.0 V and its internal resistance is negligible. E_2 is a cell of e.m.f 1.1 V and its internal resistance is 1.0 Ω, R_1 = 1.0 Ω and R_2 = 2.0 Ω. The switch S_1 enables the galvanometer G to be connected to X or Y. Calculate the balance length for each position of S_1 when the switch S_2 is

(a) open and

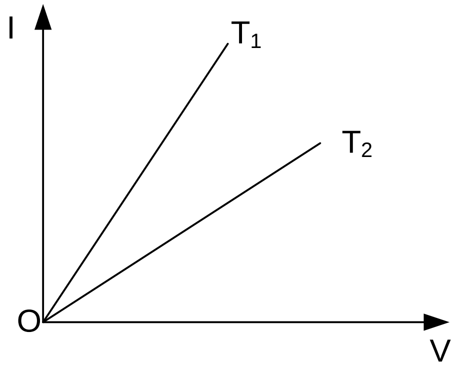
(b) Closed.

Uneb 2016

(a) **(i)** Define **electrical resistivity** (01mark)

(ii) Explain how length and temperature of a conductor affect its resistance. (04marks)

(iii) Figure below shows the current- voltage graphs for a metallic wire at two different tempeatures T_1 and T_2

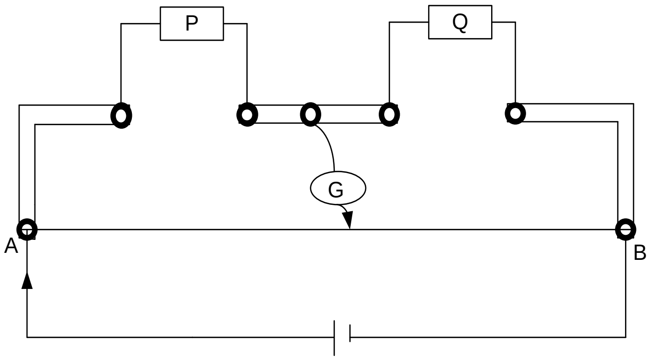


State which of the two temperatue is greater and explain your answer. (03marks)

(b) (i) Derive the balance condition when using a meter bridge to measure resistance. (04marks)

(ii) State two precautions taken to achieve an accurate measurement. (02marks)

(c) Figure below show two resistors P and Q of resistance 5Ω and 2Ω repectively connected in the two gaps of the meter bridge.



A resistance X of cross-sectional area $1mm^2$ is connected across P so that the balance point is 66.7 cm from A. If the resistivity of the wire X is $1.0x10^{-5}\Omega m$ and the resistance wire AB of the meter bridge is 100cm long, calculate the length of X. (04marks)

(d) Explain how electrons attain a steady drift velocity when current flows through a conductor. (02marks)

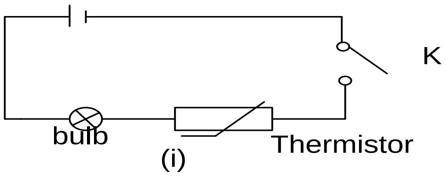
Uneb 2015

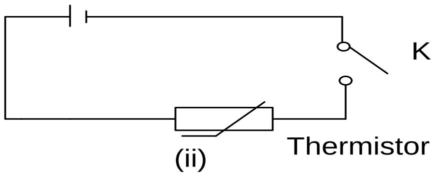
(a) **(i)** Define **temperature coefficient** of resistance (01mark)

(ii) Explain the origin of the heating effect of electric current in a metal conductor. (03marks)

(iii) Describe with the aid of an I-V sketch the variation of current with p.d a cross a semiconductor. (02marks)

(b) A cell, a bulb, a swtich and a thermistor with negative temperature coefficient of resistance are connected as shown below



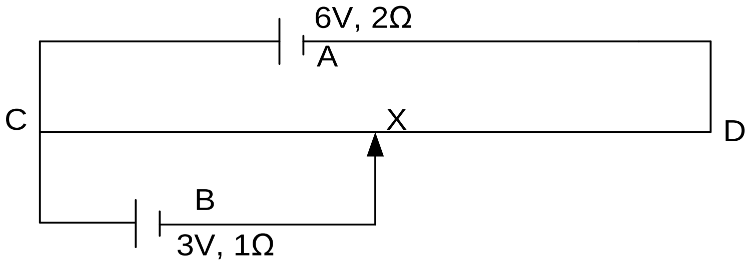


(i) Explain what would happen when in figure (i) switch K is closed (04marks)

(ii) If the bulb in figure (i) is removed and circuit connected as shown in figure (ii), explain what happens when switch K is closed. (03marks)

(c) State the **law of conservation of current at a junction**. (01mark)

(d) Two cells A, of e.m.f 6V and internal resistance 2Ω and B of e.m.f 3V and internal resistance 1Ω respectively are connected across a uniform resistance wire CD of resistance 8Ω as shown below



If X is exactly in the middle of the wire CD, calculate the;

(i) Power dissipated in CX (04marks)

(ii) P.d across the terminals of cell A (02marks)