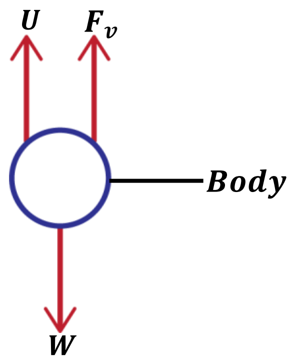
MOTION IN FLUIDS

A fluid is a substance which can flow e.g. liquids and gases.
When a body falls through a fluid, it will be acted upon by the following forces.



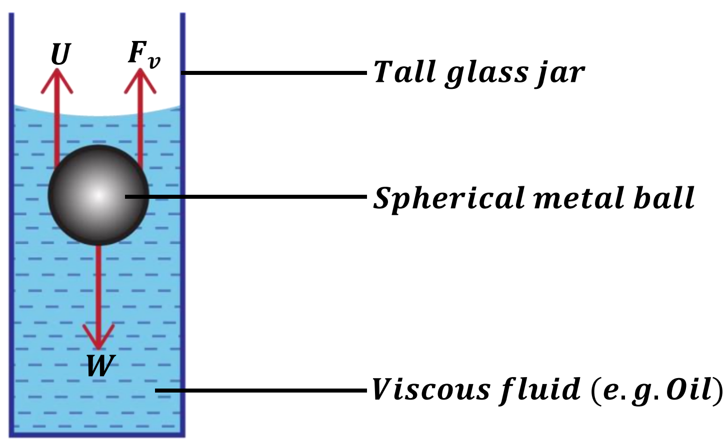
- Upthrust (U) acting upwards.
- Viscous drag or Viscous force (F_v) acting upwards.
- Weight of the body (W) acting downwards.

Upthrust (Buoyancy):
This is the upward force that a fluid exerts on a body falling through it.
For example;

- When pushing a jerrycan into water, our fingers experience an upward force.
- A balloon filled with air or hydrogen rises up due to upthrust.

Viscous drag (Viscous force):
This the force that opposes motion of a body in a fluid.
Viscous drag increases with an increase in velocity or speed of the body in a body.

Describing motion of a body falling in a viscous fluid (e.g. oil)



When the ball falls through a fluid, it first accelerates downwards until it attains a constant velocity called **terminal velocity**. At this velocity, the weight of the ball is equal to sum of upthrust and viscous drag.

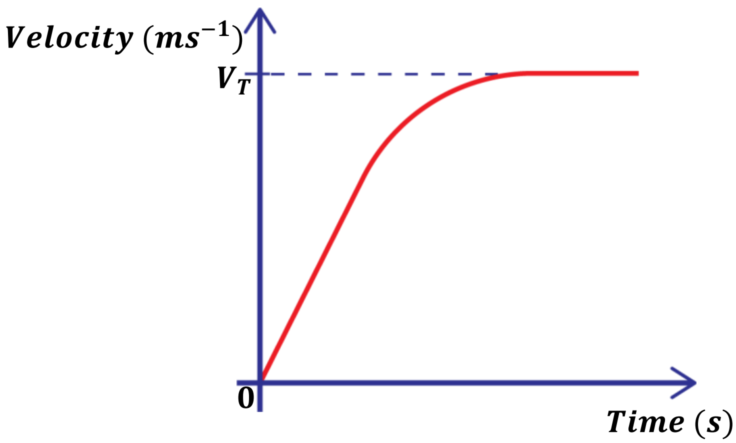
$$i.e.\ \boldsymbol{Weight = Upthrust + Viscous\ drag}$$
$$\boldsymbol{W = U + F_v}$$

The ball continues with this constant velocity until it hits the bottom of the tall glass jar.

NOTE:
As the body accelerates downwards, the viscous drag continues to increase with the increasing velocity and eventually the body can no longer accelerate. Therefore, it has a constant velocity.

Terminal velocity:
This is a constant velocity attained by the body falling in a fluid when the resultant force on the body is zero.
i.e. when upward forces (viscous drag and upthrust) = downward forces (weight of the body).

Velocity-time graph for a body falling in a fluid

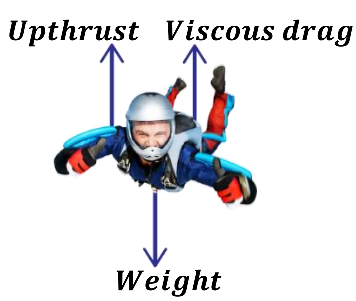


$V_T - Terminal\ velocity$

Note: In case the object is moving in air (e.g. a balloon floating in air), the viscous drag is composed of the air resistance.

Question:
Explain what happens to a parachutist diving from an aero plane.

- ***At first, the parachutist accelerates downwards as he/she begins to fall.***
- ***As the parachutist's velocity (speed) increases, the viscous drag also increases until the parachutist is unable to accelerate any more. At this point the parachutist attains a constant velocity called terminal velocity.***
- ***At terminal velocity, weight of the parachutist is equal to the upthrust and viscous drag. Therefore, the resultant force on the parachutist is zero.***



FLUID FLOW

Fluid flow describes how fluids move and how they behave and interact with the surrounding environment. The flow of a liquid may either be steady (orderly) or unsteady (unorderly).
Flow of a fluid depends on three factors namely;

- Characteristics of the fluid (i.e. density, compressibility and viscosity)
- Speed or velocity of flow.
- Shape of surface on which the fluid is flowing.

Types of fluid flow:

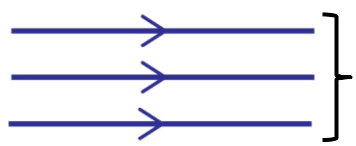
There are two types of fluid flow namely;

- Streamline flow (Laminar flow).
- Turbulent flow.

STREAMLINE FLOW (LAMINAR FLOW):

Streamline flow is the type of fluid flow where all the fluid particles that pass any point follow the same path at the same velocity (uniform velocity).

In streamline flow, the fluid particles move or travel in the same direction and with the same speed. Therefore, streamline flow is a steady, orderly and uniform flow of a fluid.
Streamline flow occurs when the fluid is moving at a low speed.



Streamlines

Definition:

A **streamline** is a line showing particles of a fluid having streamline flow.

TURBULENT FLOW:

Turbulent flow is the type of fluid flow where the speed (velocity) and the direction of fluid particles passing any point vary with time.

In turbulent flow, the fluid particles travel in different directions with different speeds. Therefore, turbulent flow is an unsteady, disorderly and non-uniform flow of the fluid.
Turbulent flow occurs when the fluid is moving with a high speed.

Practical example:

Streamline flow

Turbulent flow

When a tap is opened slightly, the water flows out slowly in form of a thin smooth orderly stream. At this point, the type of fluid flow is streamline flow.
As the tap is opened further, the water flows out fast in a disorderly way. At this point, the type of fluid flow is turbulent flow.

BERNOULLI'S PRINCIPLE

Bernoulli's principle states that when the speed of a moving fluid increases, the pressure in the fluid decreases and vice versa.

This relationship between speed and pressure was formulated by a scientist called **Daniel Bernoulli**.

Rate of flow of a fluid:

This is the volume of the fluid that passes a point of a tube in a given time.

$$Rate\ of\ fluid\ flow = \frac{Volume\ of\ fluid}{Time}$$

$$Rate\ of\ fluid\ flow = \frac{Cross\ sectional\ Area\ of\ tube\ \times distance\ moved\ by\ fluid}{Time}$$

$$Rate\ of\ fluid\ flow = \frac{A \times d}{t}$$

$$But\ speed, v = \frac{d}{t}$$

$$Rate\ of\ fluid\ flow = A \times v$$

$$Therefore, \quad \boxed{Rate\ of\ fluid\ flow = Av}$$

$where\ A - Cross\ sectional\ area.$

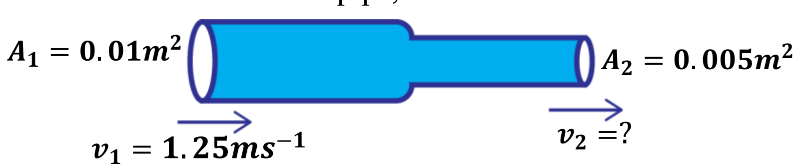
$v - Velocity\ or\ speed\ of\ the\ fluid.$

NOTE:

- The rate of flow at any section of the pipe is the same.
- In streamline flow of a fluid, the larger the pipe, the lower speed of the fluid and vice versa

Examples:

1. Water flows in through a horizontal pipe of cross-sectional area $0.01m^2$. At the outlet section, the cross-sectional area is $0.005m^2$. If the velocity of water at the larger cross-section is $1.25ms^{-1}$, find;
   i) rate of flow of water in the larger pipe.
   ii) Speed of water in the smaller pipe,



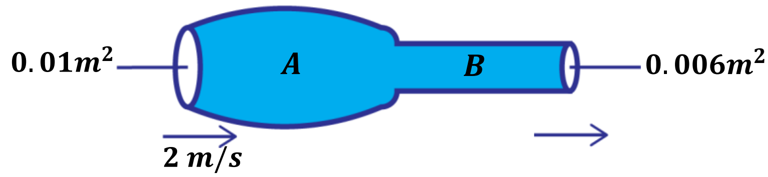
i)

$$Rate\ of\ flow = A_1v_1$$
$$Rate\ of\ flow = 0.01 \times 1.25$$
$$Rate\ of\ flow = 0.0125m^3s^{-1}$$

ii)

$$Rate\ of\ flow = A_2v_2$$
$$0.0125 = 0.005 \times v_2$$
$$v_2 = \frac{0.0125}{0.005}$$
$$v_2 = 2.5ms^{-1}$$

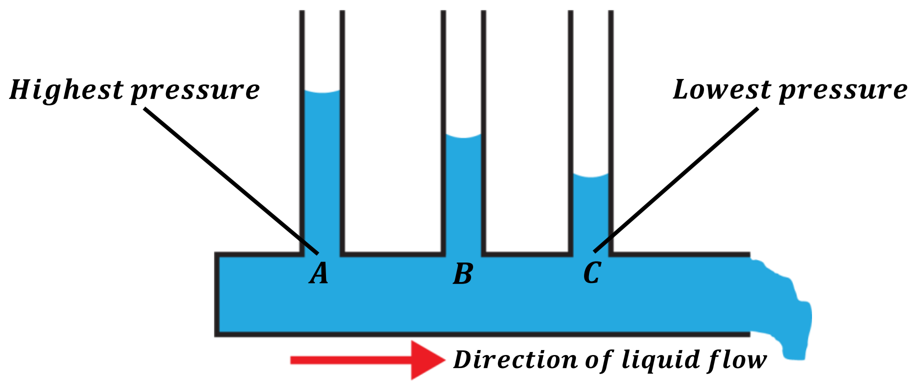
2. Water flows through a horizontal pipe of varying cross-section area as shown in the figure below.the velocity of water in pipe **A** is $2m/s$. Determine the velocity of water in pipe **B**.



$$\textit{since the rate of flow is the same throught the pipe;}$$
$$\textit{Rate of flow in } A = \textit{Rate of flow in } B$$
$$A_1v_1 = A_2v_2$$
$$0.01 \times 2 = 0.006 \times v_2$$
$$v_2 = \frac{0.01 \times 2}{0.006}$$
$$v_2 = 3.33m/s$$

Demonstrating Bernoulli's principle in liquids

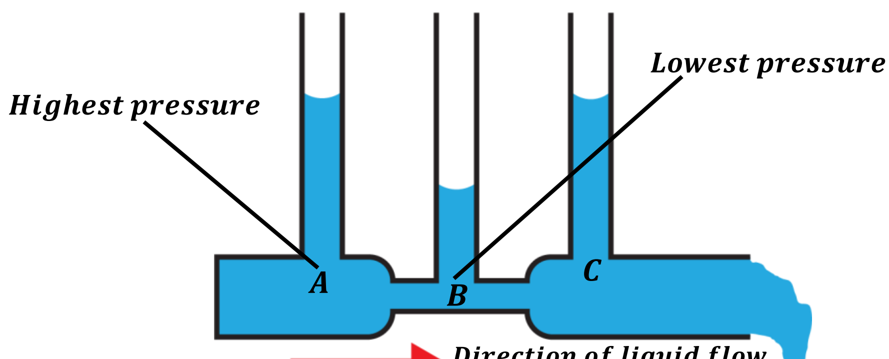
Consider a liquid flowing through a horizontal uniform tube from left to right as shown below.



Height of the liquid in the tube goes on decreasing as water flow. This indicates that the liquid pressure decreases from left to right.

This explains that liquids flow from places with higher pressure to place with lower pressure.

However, if a venturi tube (non-uniform tube) is used where the diameter at **B** is made smaller than **A** and **C**, the liquid level become lowest at **B** and water level rises again at **C**.

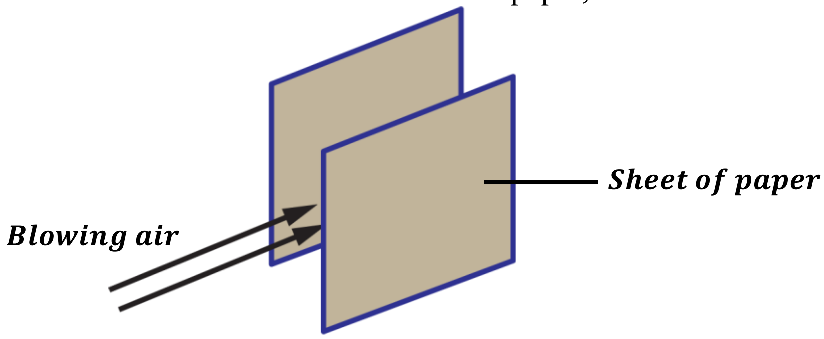


The liquid level falls at B indicating a decrease in pressure.

This is because the liquid flows fastest at B and according to Bernoulli's principle, the faster the liquid flow, the lower the liquid pressure.

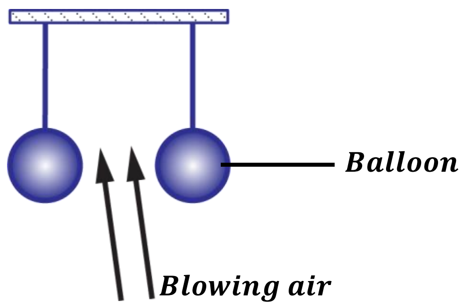
Demonstrating Bernoulli's principle in gases

- When air is blown between two sheets of paper;



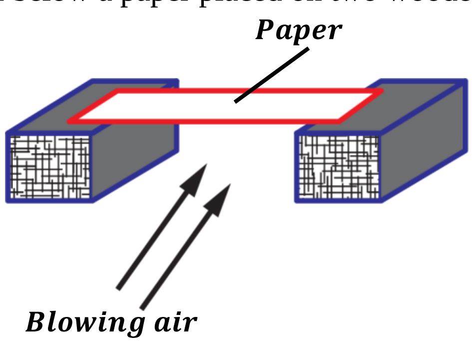
Observation: The two sheets come together.
Explanation: When air is blown between them, the air molecules moves faster resulting to a decrease in pressure in between. Therefore, the external pressure out exceeds the inside pressure and forces the sheets to come closer.

- When air is blown between two balloons filled with a gas;



Observation: The two balloons come together.
Explanation: When air is blown between them, the air molecules moves faster resulting to a decrease in pressure in between. Therefore, the external pressure out exceeds the inside pressure and forces the balloons to come closer.

- When air is blown below a paper placed on two wooden blocks.;

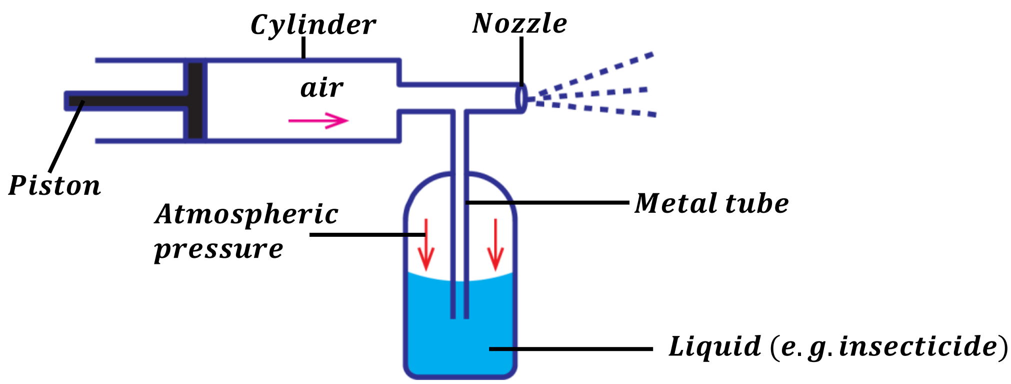


Observation: The paper curves upwards.
Explanation: When air is blown below the paper, the air molecules under the paper move faster resulting to a decrease in pressure. Therefore, the external pressure on top of the paper exceeds the the pressure below and forces the paper down thus curving upwards.

APPLICATIONS OF BERNOULLI'S PRINCIPLE

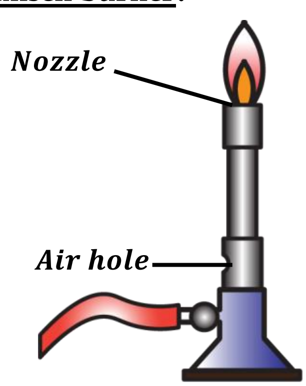
Bernoulli's principle is applied in:

Spray guns:



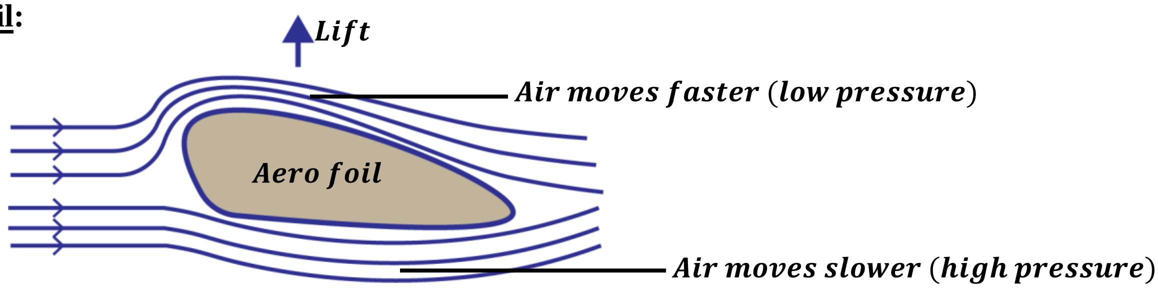
- When the piston is pushed in, it forces the air in the cylinder to move with a high velocity through the nozzle.
- The movement of air with a high velocity causes a decrease in pressure inside the cylinder.
- Since the pressure in the cylinder is less than the pressure acting on the liquid (atmospheric pressure), the atmospheric pressure forces the liquid to rise through the metal tube.
- The rising liquid is sprayed out through the nozzle.

Bunsen burner:



- When a Bunsen burner is connected to a gas supply, the gas is made to move with a high velocity inside the burner through the nozzle. This creates a region of low pressure inside the burner.
- Since the atmospheric pressure outside the burner is now more than the pressure inside, it forces air from outside atmosphere to enter in the burner and mixes with the gas.
- The mixture of air and gas enables the gas to burn completely and produce a clean, hot and smokeless flame.

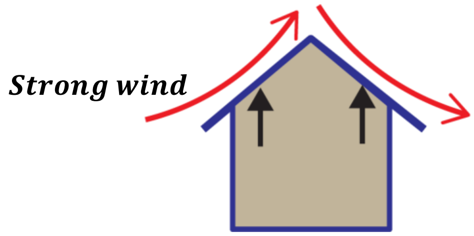
Aero foil:



- The wing of an aero plane is the form of an aero foil i.e. the upper surface is curved and the surface below is flat. This makes the air passing over the top of the wing to move a longer distance thus the air travels faster at top than at the bottom.
- Since the speed of air is higher on the top surface than the bottom, the pressure is lower than that at the bottom surface. This causes a pressure difference which causes a net upward lifting force which helps the plane to rise.

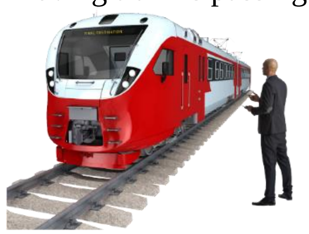
Practical examples:

1. Explain why a thatched roof of a house can be completely lifted off the house by a strong wind.



A strong wind moves over the roof top with a high speed thus creating a lower pressure above the roof top than the pressure below the roof.
Since the pressure below the roof is higher than that at the top, it causes an upward force which lifts up the roof resulting into blowing of the roof.

2. Explain using Bernoulli's principle why it is dangerous to stand near the edge of a platform in a railway station, when a fast-moving train is passing by.



This is because a person standing near a fast-moving train will tend to fall towards it according to Bernoulli's principle.
The speed of air molecules between the fast-moving train and the person is high thus creating a region of low pressure. Since the pressure behind the person is now greater than pressure in front of the person, it pushes the person towards the train.

VISCOSITY:

When water is poured on a person's head, it runs through his/her hair and then flows over the face quickly. But when honey is poured on the person's head, it takes a lot of time to flow through the person's head. This is because of a property of fluids called **viscosity**.

Definition:

Viscosity is the measure of fluid's resistance to flow.

Therefore, honey is thicker than water so it has a high viscosity than water.

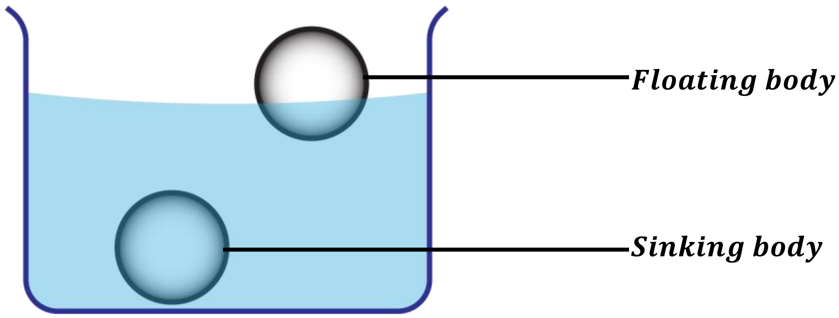
Viscous fluid:

This is a fluid with a high viscosity. Therefore, a viscous fluid doesn't flow easily.

Examples of viscous fluids include;

- Honey
- Oil
- Glues
- Syrups

FLOATING AND SINKING



- ❖ If you lift a bucket of water from a tank, the bucket appears to be lighter inside the water and suddenly heavy when it comes out of water.
- ❖ When we go swimming, we feel a little weightless in the water than our actual weight.
- ❖ A large ship made of metal (steel) floats on water while a small steel pin sinks in water.

Therefore, an object weighs less in water than it does in air. This loss of weight is due to the upthrust of water acting on the object.
All the above experiences can be explained by Archimedes' principle formulated by a Greek mathematician called **Archimedes**.

ARCHIMEDES' PRINCIPLE:

It states that when a body is wholly or partially immersed in a fluid, it experiences an upthrust equal to the weight of the fluid displaced.

$$\boxed{\boldsymbol{Upthrust = Weight\ of\ the\ fluid\ displaced}}$$

Terms used in Archimedes' principle:

- **Upthrust (Buoyancy):**
  This is the upward force that a fluid exerts on a body falling through it.
- **Actual weight:**
  This is the weight of the body in air.
- **Apparent weight:**
  This is the weight of the body when completely immersed or submerged in a fluid.
  Apparent weight is always less than the actual weight.
- **Apparent loss in weight:**
  This is the difference between actual weight of a body and apparent weight of the body.

$$\boldsymbol{Apparent\ loss\ in\ weight = Actual\ weight - Apparent\ weight}$$

Apparent loss in weight is also equal to upthrust.
Therefore,

$$\boldsymbol{Upthrust = (actual\ weight - apparent\ weight)}$$

Also, $\boxed{\boldsymbol{Apparent\ weight = Actual\ weight - Upthrust}}$

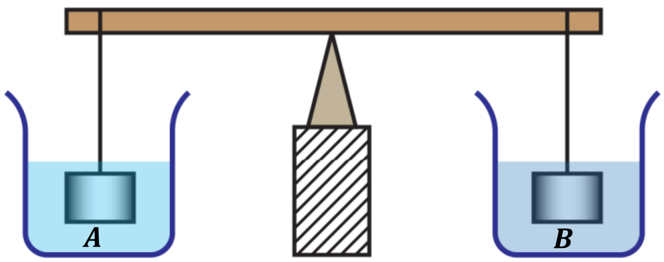
Factors affecting upthrust acting on the body

Density of the fluid:
Denser liquids exert greater upthrust on a body immersed in it than less dense liquids.
E.g. salty water is denser than fresh water therefore, an object immersed in salty water displaces a greater weight of the salty water (upthrust) than when in fresh water. Thus, the body feels weightless.

Volume of body immersed the fluid:
A body experiences a greater upthrust when fully or wholly immersed in a fluid than when it is partially immersed.

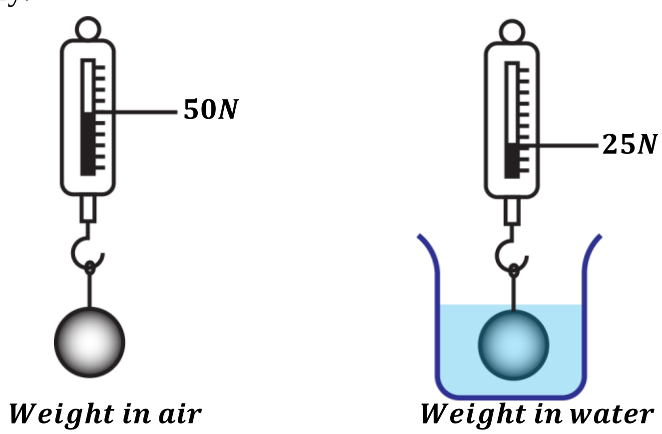
Practical examples:

- ❖ The figure below shows a uniform bar in equilibrium with two equal masses suspended at an equal from the pivot from either ends.



Salt water is added into beaker **A** and fresh water in beaker **B** until the masses are fully submerged. It is observed the bar tips towards beaker **B**. Explain this observation.
Salt water is denser than fresh water so it exerts a greater upthrust on the mass immersed in beaker A. Therefore, the mass in beaker A displaces a greater weight of the salty water.
The apparent weight of mass in beaker A is therefore lower than the apparent weight of mass in beaker B. This causes the bar to tilt towards B.

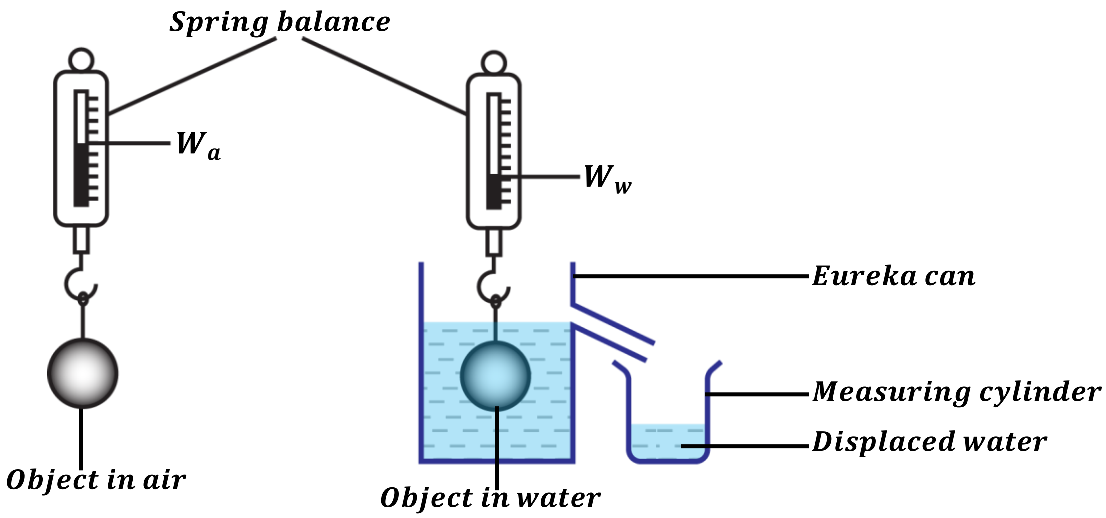
- ❖ The figure below shows weights of a ball suspended on a spring balance when weighed in air and water respectively.



Explain why the weight of the ball in water is less than the weight of the ball in air.
This is because when the ball is submerged in water, it experiences an upthrust thus lowering the apparent weight of the ball.

Recall, $\boxed{Apparent\ weight = Actual\ weight - Upthrust}$

EXPERIMENT TO VERIFY ARCHIMEDES' PRINCIPLE



- An object is weighed in air using a spring balance and its weight in air, W_a is recorded.
- An eureka can (displacement can) is completely filled with water up to its spout.
- An empty beaker of known weight, W_b is placed under the spout of the eureka can.
- The object is then weighed when completely immersed in water using a spring balance and its weight in water, W_w is recorded.
- The weight of the beaker and displaced water, W_{b+d} is measured and recorded.
- The weight of displaced water is then calculated from $W_d = W_{b+d} - W_b$.
- Since upthrust is equal to apparent loss in weight, it is calculated from $U = W_a - W_w$.
- It is found out that upthrust is equal to weight of displaced water (i.e. $U = W_d$) thus verifying Archimedes' principle.

CALCULATIONS INVOLVING ARCHIMEDES' PRINCIPLE

❖ When a body is immersed in a fluid, it experiences an apparent loss in weight which is equal to the upthrust acting on the body.

Therefore, $Upthrust = Apparent\ loss\ in\ weight$

$$Upthrust = weight\ in\ air - weight\ in\ fluid$$

$$\boxed{\mathbf{Upthrust} = \boldsymbol{W_a - W_f}}$$

❖ According to Archimedes' principle;

$$Upthrust = weight\ of\ displaced\ fluid$$

$$Upthrust = mass\ of\ displaced\ fluid \times acceleration\ due\ to\ gravity$$

$$Upthrust = m_f \times g$$

$But\ mass = density\ \times volume$

$$Upthrust = \rho_f \times V_f \times g$$

$$\boxed{\mathbf{Upthrust} = \boldsymbol{V_f \rho_f g}}$$

Where V_f – density of water.

ρ_f – volume of displaced water.

NOTE:

The volume of displaced fluid is equal to the volume of the body immersed in the fluid.

Recall: Displacement of finding volume of an irregular object.

Examples:

$(Where\ necessary\ use\ density\ of\ water\ as\ 1000kgm^{-3})$

1. A block weighs 25N in air. When completely immersed in water, it weighs 10N. Calculate;
   i) upthrust on the block.
   ii) volume of water displaced.

$W_a = 25N, \quad W_w = 10N$

i)

$Upthrust = W_a - W_w$

$Upthrust = 25 - 10$

$Upthrust = 15N$

ii)

$Upthrust = weight\ of\ displaced\ water$

$Upthrust = V_w \rho_w g$

$15 = V_w \times 1000 \times 10$

$V_w = \frac{15}{10000}$

$V_w = 0.0015m^3$

2. A metal weighs 20N in air and 15N when fully immersed in water. Calculate;
   i) upthrust.
   ii) weight of displaced water.
   iii) volume of displaced water.

$W_a = 20N, \quad W_w = 15N$

i)

$Upthrust = W_a - W_w$

$Upthrust = 20 - 15$

$Upthrust = 5N$

ii)

$Weight\ of\ displaced\ water = Upthrust$

$Weight\ of\ displaced\ water = 5N$

iii)

$Upthrust = weight\ of\ displaced\ water$

$Upthrust = V_w \rho_w g$

$5 = V_w \times 1000 \times 10$

$V_w = \frac{5}{10000}$

$V_w = 0.0005m^3$

3. A concrete block of mass $3.0 \times 10^3 kg$ and volume $1.2m^3$ is totally immersed in a fluid of density $2.0 \times 10^3 kgm^{-3}$. Find;
   i) weight of the block in air.
   ii) Weight of the block in the fluid.

$m_b = 3.0 \times 10^3 kg, \quad V_b = V_f = 1.2m^3 \quad \rho_f = 2.0 \times 10^3 kgm^{-3}$

i)

$W_a = m_b g$

$W_a = (3.0 \times 10^3) \times 10$

$W_a = 30000N$

ii)

$Upthrust = weight\ of\ displaced\ fluid$

$Upthrust = V_f \rho_f g$

$Upthrust = (1.2) \times (2.0 \times 10^3) \times (10)$

$Upthrust = 24000N$

$Upthrust = W_a - W_f$

$24000 = 30000 - W_f$

$W_f = 6000N$

4. An object weighs 30N in air and 25N when immersed in water. Calculate;
   i) upthrust.
   ii) volume of the object.
   iii) density of the object.

$$W_a = 30N, \qquad W_w = 25N$$

i)

$$Upthrust = W_a - W_w$$
$$Upthrust = 30 - 25$$
$$Upthrust = 5N$$

ii)

$$Upthrust = weight\ of\ displaced\ water$$
$$Upthrust = V_w \rho_w g$$
$$5 = V_w \times 1000 \times 10$$
$$V_w = \frac{5}{10000}$$
$$V_w = 0.0005m^3$$

$$volume\ of\ object = volume\ of\ displaced\ water$$
$$V_o = V_w$$
$$V_o = 0.0005m^3$$

iii) *Density of object*

$$W_a = V_o \rho_o g$$
$$30 = 0.0005 \times \rho_o \times 10$$
$$\rho_o = \frac{30}{0.005}$$
$$\rho_o = 6000kgm^{-3}$$

OR

$$\rho_o = \frac{mass\ of\ object\ (m_o)}{volume\ of\ object\ (V_o)}$$
$$From\ W_a = m_o g$$
$$m_o = \frac{30}{10} = 3kg$$
$$\rho_o = \frac{3}{0.0005}$$
$$\rho_o = 6000kgm^{-3}$$

5. A body weighs 50N in air and 30N when fully immersed in water. Calculate the mass of water displaced.

$$W_a = 50N, \qquad W_w = 30N$$

$$Upthrust = W_a - W_w$$
$$Upthrust = 50 - 30$$
$$Upthrust = 20N$$

$$Upthrust = weight\ of\ displaced\ water$$
$$Upthrust = m_w g$$
$$20 = m_w \times 10$$
$$m_w = \frac{20}{10}$$
$$m_w = 2kg$$

6. A piece of metal of density $2500kgm^{-3}$ weighs 1N in air. Find the weight of the metal when completely submerged in water.

$$W_a = 1N, \qquad W_w =? \qquad \rho_m = 2500kgm^{-3}$$

Volume of metal

$$W_a = V_m \rho_m g$$
$$1 = V_m \times 2500 \times 10$$
$$V_m = \frac{1}{25000}$$
$$V_m = 0.00004m^3$$

$$volume\ of\ metal = volumeof\ displaced\ water$$
$$V_w = V_m = 0.00004m^3$$

$$Upthrust = weight\ of\ displaced\ water$$
$$Upthrust = V_w \rho_w g$$
$$Upthrust = 0.00004 \times 1000 \times 10$$
$$Upthrust = 0.4N$$

$$Upthrust = W_a - W_w$$
$$0.4 = 1 - W_w$$
$$W_w = 0.6N$$

7. An object weighs 50N in air and 30N when wholly submerged in water. Calculate;
   i) Buoyant force on the object.
   ii) volume of the object.
   iii) density of object.

$W_a = 50N,\qquad W_w = 30N$

i)

$Upthrust = W_a - W_w$

$Upthrust = 50 - 30$

$Upthrust = 20N$

ii)

$Upthrust = weight\ of\ displaced\ water$

$Upthrust = V_w\rho_w g$

$20 = V_w \times 1000 \times 10$

$V_w = \dfrac{20}{10000}$

$V_w = 0.002m^3$

$volume\ of\ object = volume\ of\ displaced\ water$

$V_o = V_w$

$V_o = 0.002m^3$

iii) *Density of object*

$W_a = V_o\rho_o g$

$50 = 0.002 \times \rho_o \times 10$

$\rho_o = \dfrac{50}{0.02}$

$\rho_o = 2500kgm^{-3}$

OR

$\rho_o = \dfrac{mass\ of\ object\ (m_o)}{volume\ of\ object\ (V_o)}$

$From\ \ W_a = m_o g$

$m_o = \dfrac{50}{10} = 5kg$

$\rho_o = \dfrac{5}{0.002}$

$\rho_o = 2500kgm^{-3}$

8. A solid of volume $800cm^3$ is totally immersed in oil of density $0.8gcm^{-3}$. Calculate the
   i) mass of oil displaced.
   ii) upthrust on the solid.

i)

$V_s = 800cm^3\qquad V_o = 800cm^3\qquad \rho_o = 0.8gcm^{-3}$

$mass\ of\ oil = density\ of\ oil \times volume\ of\ oil$

$m_o = \rho_o \times V_o$

$m_o = 0.8 \times 800$

$m_o = 640g$

ii)

$Upthrust = weight\ of\ displaced\ oil$

$Upthrust = m_o g$

$Upthrust = (\dfrac{640}{1000}) \times 10$

$Upthrust = 6.4N$

NOTE:

When a body is partially immersed in a fluid, it displaces a volume of a fluid equal to the fraction of its volume that is immersed in the fluid.

i.e. $Volume\ of\ displaced\ fluid = fraction\ of\ the\ volume\ of\ body\ immersed\ in\ the\ fluid.$

9. An iron cube of mass $480g$ and density $8gcm^{-3}$ is suspended by a string so that it is half immersed in oil of density $0.9gcm^{-3}$. Find
   i) upthrust acting on the cube.
   ii) the tension in the string.

$$m_c = 480g = \frac{480}{1000} = 0.48kg,$$
$$\rho_c = 8gcm^{-3} = (8 \times 1000) = 8000kgm^{-3}$$
$$\rho_o = 0.9gcm^{-3} = (0.9 \times 1000) = 900kgm^{-3}$$

$\underline{volume\ of\ iron\ cube}$

$$V_c = \frac{m_c}{\rho_c}$$
$$V_c = \frac{0.48}{8000} = 0.00006m^3$$

$\underline{volume\ of\ displaced\ oil}$

$$V_o = \frac{1}{2} \times volume\ of\ iron\ cube$$
$$V_o = \frac{1}{2} \times 0.00006 = 0.00003m^3$$

i)

$$Upthrust = weight\ of\ displaced\ oil$$
$$Upthrust = V_o \rho_o g$$
$$Upthrust = 0.00003 \times 900 \times 10$$
$$Upthrust = 0.27N$$

ii)

$\underline{weight\ of\ cube\ in\ air}$

$$W_a = m_c g = 0.48 \times 10$$
$$W_a = 4.8N$$

$$Tension = Apparent\ weight\ (W_o)$$
$$Apparent\ weight = weight\ of\ cube\ in\ oil$$
$$Upthrust = W_a - W_o$$
$$0.27 = 4.8 - W_o$$
$$W_o = 4.53N$$

EXERCISE:
($Where\ necessary\ use\ density\ of\ water\ as\ 1000kgm^{-3}$)

1. A body weighs 100N in air and 80N in water. Calculate.
   i) upthrust on the body.
   ii) volume of displaced water.
   iii) density of the body.
   iv) mass of the body.

2. A string supports a solid block of mass $1kg$ and density $9000kgm^{-3}$ which is completely immersed in water. Calculate the tension in the string.

3. A stone of volume $200cm^3$ and density $2.7gcm^{-3}$ is completely immersed in Kerosene.
   a) Determine the upthrust exerted on the stone.
   b) Determine how much it will weigh in kerosene ($density\ of\ kerosene = 0.8gcm^{-3}$)

4. A glass block of mass $2.0 \times 10^3 kg$ and volume $2.4m^3$ is totally immersed in a fluid of density $1.0 \times 10^3 kgm^{-3}$. Find;
   iii) weight of the block in air.
   iv) Weight of the block in the fluid.

5. An iron cube of volume $800cm^3$ is totally immersed in
   a) Water b) oil of density $0.8gcm^{-3}$
   Calculate the upthrust in each case.

DETERMINING RELATIVE DENSITY OF A SOLID USING ARCHIMEDES' PRINCIPLE

- The solid is weighed in air and its weight, W_a is recorded.
- The solid is then weighed when completely immersed in water and its weight, W_w is recorded.

$$Relative\ density\ of\ solid = \frac{Weight\ of\ solid\ in\ air}{Apparent\ loss\ of\ weight\ in\ water}$$

$$Relative\ density\ of\ solid = \frac{Weight\ of\ solid\ in\ air}{Upthrust}$$

$$\boxed{R.D = \frac{W_a}{W_a - W_w}}$$

Recall:

$$Relative\ density = \frac{Density\ of\ substance}{Density\ of\ equal\ volume\ of\ water}$$

Examples:

($Where\ necessary\ use\ density\ of\ water\ as\ 1000kgm^{-3}$)

1. A solid weighs 25N. It weighs 15N when completely immersed in water. Calculate;
   i) relative density of a solid.
   ii) density of a solid.

$$W_a = 25N, \quad W_w = 15N$$

i)

$$R.D = \frac{W_a}{W_a - W_w}$$

$$R.D = \frac{25}{25 - 15}$$

$$R.D = 2.5$$

ii)

$$R.D = \frac{Density\ of\ solid}{Density\ of\ water}$$

$$R.D = \frac{\rho_s}{\rho_w}$$

$$2.5 = \frac{\rho_s}{1000}$$

$$\rho_s = 2500kgm^{-3}$$

2. A metallic solid weighs 24N and 16N when completely immersed in water. Calculate;
   i) relative density of the metal.
   ii) density of the metal.

$$W_a = 24N, \quad W_w = 16N$$

i)

$$R.D = \frac{W_a}{W_a - W_w}$$

$$R.D = \frac{24}{24 - 16}$$

$$R.D = 3$$

ii)

$$R.D = \frac{Density\ of\ metallic\ solid}{Density\ of\ water}$$

$$R.D = \frac{\rho_s}{\rho_w}$$

$$3 = \frac{\rho_s}{1000}$$

$$\rho_s = 3000kgm^{-3}$$

3. An object has a relative density of 7 and weighs 70N in air. Find its weight when it is fully immersed in water.

$$W_a = 70N, \quad W_w = ? \quad R.D = 7$$

$$R.D = \frac{W_a}{W_a - W_w}$$

$$7 = \frac{70}{70 - W_w}$$

$$490 - 7W_w = 70$$

$$W_w = \frac{490 - 70}{7}$$

$$W_w = 60N$$

4. A body weighs 600g in air and 400g in water. Calculate;
   a) upthrust on the body.
   b) volume of the body.
   c) relative density of the solid.

$$From\ \boldsymbol{weight = mg}$$

$$W_a = \frac{600}{1000} \times 10 = 6N \qquad W_w = \frac{400}{1000} \times 10 = 4N$$

a) $Upthrust = W_a - W_w$

$Upthrust = 6 - 4$

$Upthrust = 2N$

b) $Upthrust = weight\ of\ displaced\ water$

$$Upthrust = V_w \rho_w g$$

$$2 = V_w \times 1000 \times 10$$

$$V_w = \frac{2}{10000}$$

$$V_w = 0.0002m^3$$

c)

$$R.D = \frac{W_a}{W_a - W_w}$$

$$R.D = \frac{6}{6 - 4}$$

$$R.D = 3$$

<u>DETERMINING RELATIVE DENSITY OF A LIQUID USING ARCHIMEDES' PRINCIPLE</u>

- The solid is weighed in air and its weight, W_a is recorded.
- The solid is then weighed when completely immersed in water and its weight, W_w is recorded.
- The solid is then weighed when completely immersed in another liquid and its weight, W_l is recorded.

$$Relative\ density\ of\ solid = \frac{Apparent\ loss\ of\ weight\ in\ liquid}{Apparent\ loss\ of\ weight\ in\ water}$$

$$Relative\ density\ of\ solid = \frac{Upthrust\ in\ liquid}{Upthrust\ in\ water}$$

$$\boxed{R.D = \frac{W_a - W_l}{W_a - W_w}}$$

Examples:

($Where\ necessary\ use\ density\ of\ water\ as\ 1000kgm^{-3}$)

1. A metal weighs 25N in air and 20N in water and 15N in a liquid. Calculate;
   a) relative density of the liquid.
   b) density of the liquid.

$$W_a = 25N, \quad W_w = 20N \quad W_l = 15N$$

a)

$$R.D = \frac{W_a - W_l}{W_a - W_w}$$

$$R.D = \frac{25 - 15}{25 - 20}$$

$$R.D = \frac{10}{5}$$

$$R.D = 2$$

b)

$$R.D = \frac{Density\ of\ liquid}{Density\ of\ water}$$

$$R.D = \frac{\rho_l}{\rho_w}$$

$$2 = \frac{\rho_l}{1000}$$

$$\rho_l = 2000kgm^{-3}$$

2. A solid weighs 55N in air. When in a liquid, it weighs 25N and it weighs 30N when in water. Calculate;
   i) relative density of the liquid.
   ii) density of the liquid.

$$W_a = 55N, \quad W_w = 25N \quad W_l = 30N$$

c)

$$R.D = \frac{W_a - W_l}{W_a - W_w}$$

$$R.D = \frac{55 - 25}{55 - 30}$$

$$R.D = \frac{30}{25}$$

$$R.D = 1.2$$

d)

$$R.D = \frac{Density\ of\ liquid}{Density\ of\ water}$$

$$R.D = \frac{\rho_l}{\rho_w}$$

$$1.2 = \frac{\rho_l}{1000}$$

$$\rho_l = 1200kgm^{-3}$$

3. An object weighs 100N in air and 40N in kerosene of relative density 0.8. Find its weight in water.

$$W_a = 100N, \quad W_l = 40N, \quad W_w = ? \quad R.D = 0.8$$

$$R.D = \frac{W_a - W_l}{W_a - W_w}$$

$$0.8 = \frac{100 - 40}{100 - W_w}$$

$$80 - 0.8W_w = 60$$

$$W_w = \frac{80 - 60}{0.8}$$

$$W_w = 25N$$

4. A body weighs 20g in air, 18.2g in milk and 18g in water. Calculate;
   a) the relative density of the body.
   b) the relative density of the milk.

$$From\ \mathbf{weight = mg}$$

$$W_a = \frac{20}{1000} \times 10 = 0.2N \quad W_l = \frac{18.2}{1000} \times 10 = 0.182N \quad W_w = \frac{18}{1000} \times 10 = 0.18N$$

a) relative density of body

$$R.D = \frac{W_a}{W_a - W_w}$$
$$R.D = \frac{0.2}{0.2 - 0.18}$$
$$R.D = 10$$

b) relative density of milk

$$R.D = \frac{W_a - W_l}{W_a - W_w}$$
$$R.D = \frac{0.2 - 0.182}{0.2 - 0.18}$$
$$R.D = 0.9$$

5. When a metal is completely immersed in liquid A, its apparent weight is 20N. When immersed in another liquid B, the apparent weight is 16N. If the density of the liquid is $\frac{9}{8}$ times that of A, calculate the weight of metal in air.

$$W_{lA} = 20N, \quad W_{lB} = 16N$$

$$R.D = \frac{Density\ of\ liquid\ A}{Density\ of\ liquid\ B}$$
$$R.D = \frac{\rho_A}{\rho_B}$$
$$R.D = \frac{\rho_A}{\frac{9\rho_A}{8}}$$
$$R.D = \frac{8}{9}$$

$$R.D = \frac{Upthrust\ in\ liquid\ A}{Upthrust\ in\ liquid\ B}$$
$$R.D = \frac{W_a - W_{lA}}{W_a - W_{lB}}$$
$$\frac{8}{9} = \frac{W_a - 20}{W_a - 16}$$
$$8W_a - 128 = 9W_a - 180$$
$$W_a = 52N$$

EXERCISE

1. A piece of glass weighs 0.5N in air and 0.3N in water and 0.32N in benzene. Calculate;
   i) relative density of glass.
   ii) density of glass.
   iii) relative density of liquid.
   iv) density of benzene.

2. An object weighs 5.6N in air, 4.8N in water and 4,6N when immersed in a liquid. Find the relative density of the liquid.

3. A piece of iron weighs 555N in air. When completely immersed in water, it weighs 530N and weighs 535N when completely immersed in alcohol. Calculate the relative density of alcohol.

4. A glass block weighs 43N in air. When wholly immersed in water, the block weighs 23N. Calculate the;
   a) upthrust on the glass block.
   b) density of the glass block.
   c) volume of the glass block.

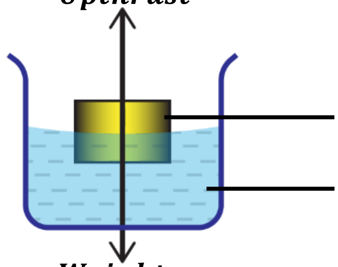
5. A solid weighs 0.50N in air. It weighs 0.30N when fully immersed in water and 0.32N when fully submerged in a liquid. Calculate the;
   i) upthrust on the body due to water.
   ii) volume of the solid.
   iii) density of the solid
   iv) relative density of the liquid.
   v) density of the liquid.

6. A solid weighs 600g in air, 450g in water and 480g in a liquid. Find the;
   a) relative density of the liquid.
   b) density of the liquid.

FLOATATION

Recall: A body floats in a fluid if its average density is less than the density of the fluid.

When an object is placed in liquid, it is acted upon by the upthrust and its weight.



- The object sinks in the liquid if its weight is greater than upthrust.
- The object floats in the liquid if its weight is equal to upthrust. Therefore, the apparent weight (resultant force on the object) must be zero for a body to float in the liquid.

However, when a cork is held below the liquid surface (e.g. water), and released, it rises because its upthrust is greater than its weight.

Note: By Archimedes' principle, $Upthrust = weight\ of\ displaced\ fluid$. Therefore, for a floating body, its weight is equal to weight of displaced fluid.

LAW OF FLOATATION:

It states that a floating body displaces its own weight of the fluid in which its floats.

$$\boxed{\boldsymbol{Weight\ of\ floating\ body = Weight\ of\ displaced\ fluid}}$$

Mathematically;

$$\boldsymbol{W_b = W_f}$$
$$\boldsymbol{m_b g = m_f g}$$
$$\boldsymbol{m_b = m_f} \quad [mass\ of\ floating\ body = mass\ of\ displaced\ fluid]$$
$$\boldsymbol{\rho_b V_b = \rho_f V_f}$$

Examples:

1. A piece of wood of density $2.5gcm^{-3}$ and volume $100cm^3$ floats on a liquid of density $4gcm^{-3}$. Calculate the volume of liquid displaced.

   $\boldsymbol{\rho_w = 2.5gcm^{-3}}$, $\boldsymbol{V_w = 100cm^3}$, $\boldsymbol{\rho_l = 4gcm^{-3}}$, $\boldsymbol{V_l} = ?$

   $Weight\ of\ floating\ wood = weight\ of\ displaced\ liquid$

$$\boldsymbol{W_w = W_l}$$
$$\boldsymbol{m_w g = m_l g}$$
$$\boldsymbol{m_w = m_l}$$
$$\boldsymbol{\rho_w V_w = \rho_l V_l}$$
$$\boldsymbol{2.5 \times 100 = 4 \times V_l}$$
$$\boldsymbol{V_l = \frac{250}{4}}$$
$$\boldsymbol{V_l = 62.5cm^3}$$

2. A piece of cork of density $0.15gcm^{-3}$ and volume $200cm^3$ floats in water of density $1gcm^{-3}$. Calculate the volume of the cork out of the water.

$\rho_c = 0.15gcm^{-3}$, $V_c = 200cm^3$, $\rho_w = 1gcm^{-3}$, $V_w = ?$

$Weight\ of\ floating\ cork = weight\ of\ displaced\ water$

$$W_c = W_w$$
$$m_c g = m_w g$$
$$m_c = m_w$$
$$\rho_c V_c = \rho_w V_w$$
$$0.15 \times 200 = 1 \times V_w$$
$$V_w = \frac{30}{1}$$
$$V_w = 30cm^3$$

$Volume\ of\ cork\ out\ of\ water = volume\ of\ cork - volume\ of\ displaced\ water$

$Volume\ of\ cork\ out\ of\ water = 200 - 30$

$Volume\ of\ cork\ out\ of\ water = 170cm^3$

3. A piece of cork of volume $100cm^3$ is floating on water. If the density of cork is $0.25gcm^{-3}$. Calculate the volume of the cork immersed in water. (density of water is $1gcm^{-3}$).

$\rho_c = 0.25gcm^{-3}$, $V_c = 100cm^3$, $\rho_w = 1gcm^{-3}$, $V_w = ?$

$Weight\ of\ floating\ cork = weight\ of\ displaced\ water$

$$W_c = W_w$$
$$m_c g = m_w g$$
$$m_c = m_w$$
$$\rho_c V_c = \rho_w V_w$$
$$0.25 \times 100 = 1 \times V_w$$
$$V_w = \frac{25}{1}$$
$$V_w = 25cm^3$$

$Volume\ of\ cork\ immersed\ in\ water = volume\ of\ displaced\ water$

$Volume\ of\ cork\ immersed\ in\ water = 25cm^3$

4. A glass block of density $5gcm^{-3}$ and volume $200cm^3$ floats on a liquid of density $8000kgm^{-3}$. Calculate the volume of liquid displaced.

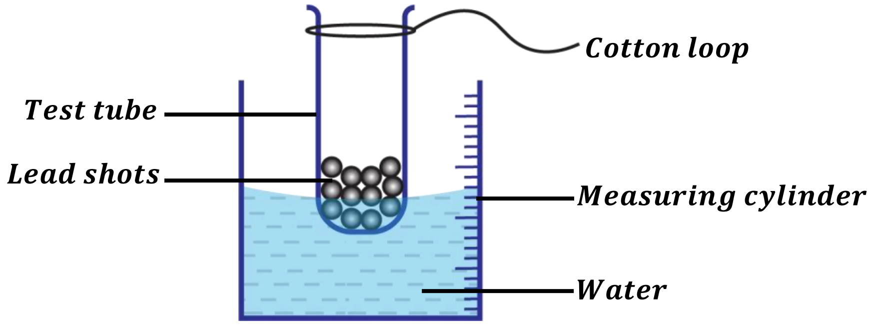
$\rho_g = 5gcm^{-3} = (5 \times 1000) = 5000kgm^{-3}$, $\rho_l = 8000kgm^{-3}$

$V_g = 200cm^3 = \dfrac{200}{1000000} = 0.0002m^3$, $V_l = ?$

$Weight\ of\ floating\ glass = weight\ of\ displaced\ liquid$

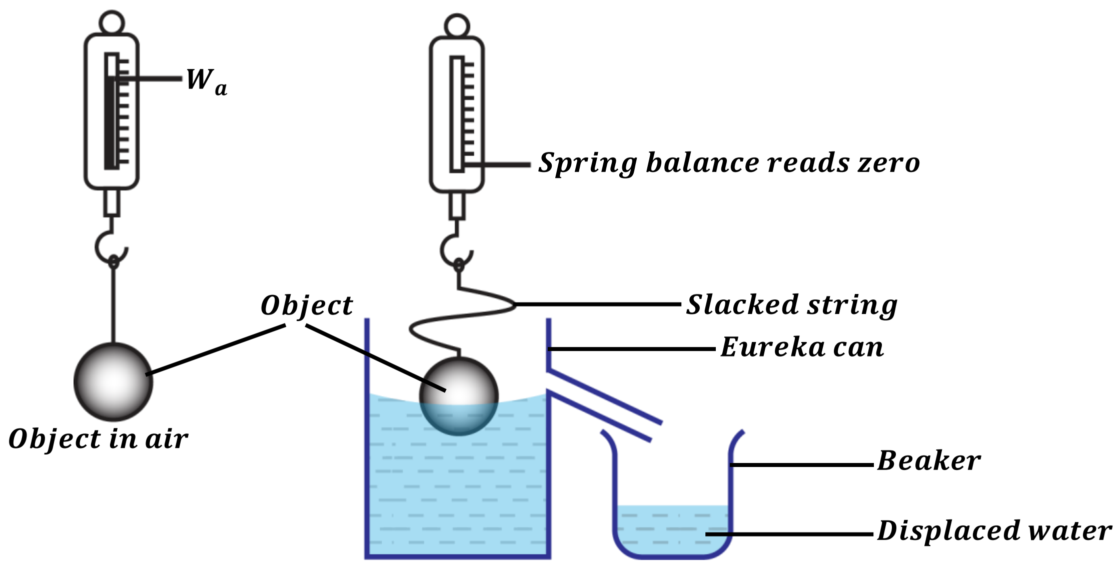
$$W_g = W_l$$
$$m_g g = m_l g$$
$$m_g = m_l$$
$$\rho_g V_g = \rho_l V_l$$
$$5000 \times 0.0002 = 8000 \times V_l$$
$$V_l = \frac{1}{8000}$$
$$V_l = 0.000125m^3$$

EXPERIMENT TO VERIFY THE LAW OF FLOATATION



- A measuring cylinder is filled with some water and its initial volume V_1 is recorded.
- A test tube with a cotton loop is then placed in the measuring cylinder.
- Lead shots are then added to the test tube until the test tube floats vertically.
- The reading of the new water level V_2 is recorded.
- Volume of water displaced by test tube is calculated as $(V_2 - V_1)$.
- The weight of displaced water $= \rho_w(V_2 - V_1)g$.
- The test tube with lead shots is then removed from water, dried and weighed and its weight is recorded. (The cotton loop helps to attach the test tube to the spring balance)
- It is found that $(weight\ of\ test\ tube + lead\ shots)$ is equal to the weight of displaced water thus verifying the law of floatation.

OR



- An object is weighed in air using a spring balance and its weight in air, W_a is recorded.
- An eureka can (displacement can) is completely filled with water up to its spout.
- An empty beaker of known weight, W_b is placed under the spout of the eureka can.
- The object is made to float on water in the Eureka can (displacement can) and displaced water is collected in the beaker.
- The weight of the beaker and displaced water, W_{b+d} is measured and recorded.
- The weight of displaced water is then calculated from $W_d = W_{b+d} - W_b$.
- It is found out that weight of the object in air is equal to the weight of displaced water $(W_a = W_d)$, thus verifying the law of floatation.

Fraction of a floating body submerged in a fluid:

$$weight\ of\ floating\ body = weight\ of\ displaced\ fluid$$

$$W_b = W_f$$

$$m_b g = m_f g$$

$$m_b = m_f \quad [mass\ of\ floating\ body = mass\ of\ displaced\ fluid]$$

$$\rho_b V_b = \rho_f V_f$$

Therefore,

$$Fraction\ \ of\ body\ submerged = \frac{\rho_b}{\rho_f} = \frac{V_f}{V_b}$$

$$\boldsymbol{Fraction\ \ of\ body\ submerged = \frac{Density\ of\ floating\ body}{Density\ of\ fluid}}$$

OR

$$\boldsymbol{Fraction\ \ of\ body\ submerged = \frac{Volume\ of\ displaced\ fluid}{Volume\ of\ floating\ body}}$$

Examples:

1. A piece of wood floats with $\frac{4}{5}$ of its volume under a liquid of density $800kgm^{-3}$. Find the density of the wood.

$$\boldsymbol{Fraction\ \ of\ wood\ submerged = \frac{Density\ of\ floating\ wood}{Density\ of\ liquid}}$$

$$\boldsymbol{Fraction\ \ of\ wood\ submerged = \frac{\rho_w}{\rho_l}}$$

$$\boldsymbol{\frac{4}{5} = \frac{\rho_w}{800}}$$

$$\boldsymbol{\rho_w = \frac{800 \times 4}{5}}$$

$$\boldsymbol{\rho_w = 640kgm^{-3}}$$

2. An object of volume $240cm^3$ floats with three quarters of its volume under water. Calculate the
   i) density of the object if the density of water is $1000kgm^{-3}$.
   ii) volume of displaced water

i)

$$\boldsymbol{Fraction\ submerged = \frac{3}{4}, \qquad \rho_w = 1000kgm^{-3}}$$

$$\boldsymbol{Fraction\ \ of\ object\ submerged = \frac{Density\ of\ floating\ object}{Density\ of\ water}}$$

$$\boldsymbol{Fraction\ \ of\ object\ submerged = \frac{\rho_o}{\rho_w}}$$

$$\boldsymbol{\frac{3}{4} = \frac{\rho_o}{1000}}$$

$$\boldsymbol{\rho_o = \frac{1000 \times 3}{4}}$$

$$\boldsymbol{\rho_o = 750kgm^{-3}}$$

ii)

$$V_o = 240cm^3 = \frac{240}{1000000} = 0.00024m^3$$

$$\text{Fraction of object submerged} = \frac{\text{Volume of displaced water}}{\text{Volume of floating object}}$$

$$\text{Fraction of object submerged} = \frac{V_w}{V_o}$$

$$\frac{3}{4} = \frac{V_w}{0.00024}$$

$$V_w = \frac{0.00024 \times 3}{4}$$

$$V_w = 0.00018m^3$$

OR

$$\text{Weight of floating object} = \text{Weight of displaced water}$$

$$W_o = W_w$$

$$m_o g = m_w g$$

$$m_o = m_w$$

$$\rho_o V_o = \rho_w V_w$$

$$750 \times 0.00024 = 1000 \times V_w$$

$$V_w = \frac{0.18}{1000}$$

$$V_w = 0.00018m^3$$

3. The mass of a piece of cork of density $0.25gcm^{-3}$ is $20g$.
   a) What fraction of the cork is immersed when it floats in water? (Density of water is $1gcm^{-3}$)
   b) What is the volume of displaced water?

a)

$$m_c = 20g, \quad \rho_c = 0.25gcm^{-3}, \quad \rho_w = 1gcm^{-3}$$

$$\text{Fraction of cork submerged} = \frac{\text{Density of floating cork}}{\text{Density of water}}$$

$$\text{Fraction of object submerged} = \frac{\rho_c}{\rho_w}$$

$$\text{Fraction of object submerged} = \frac{0.25}{1}$$

$$\text{Fraction of object submerged} = \frac{1}{4}$$

b)

<u>volume of cork</u>

$$V_c = \frac{m_c}{\rho_c}$$

$$V_c = \frac{20}{0.25} = 80cm^3$$

$$\text{Fraction of cork submerged} = \frac{\text{Volume of displaced water}}{\text{Volume of floating cork}}$$

$$\text{Fraction of cork submerged} = \frac{V_w}{V_c}$$

$$\frac{1}{4} = \frac{V_w}{80}$$

$$V_w = \frac{80}{4}$$

$$V_w = 20cm^3$$

OR

$$\text{Weight of floating cork} = \text{Weight of displaced water}$$

$$W_c = W_w$$

$$m_c g = m_w g$$

$$m_c = m_w$$

$$\rho_c V_c = \rho_w V_w$$

$$0.25 \times 80 = 1 \times V_w$$

$$V_w = \frac{20}{1}$$

$$V_w = 20cm^3$$

4. A cube made of oak and of height 15cm floats in water with 10.5cm of its height below the surface. Calculate;
   a) fraction of oak immersed in water.
   b) density of oak. (Density of water is $1gcm^{-3}$)

a)

$$\textbf{Fraction of object submerged} = \frac{10.5}{15}$$

$$\textbf{Fraction of object submerged} = \frac{7}{10}$$

b)

$$\textbf{Fraction of oak submerged} = \frac{\textbf{Density of floating oak}}{\textbf{Density of water}}$$

$$\textbf{Fraction of oak submerged} = \frac{\rho_o}{\rho_w}$$

$$\frac{7}{10} = \frac{\rho_o}{1}$$

$$\rho_o = \frac{1 \times 7}{10}$$

$$\rho_o = 0.7gcm^{-3}$$

5. An object of Relative density 0.8 floats in water. Find
   i) fraction of object immersed in water.
   ii) fraction of object exposed.
   (Density of water is $1000kgm^{-3}$)

i)

$$R.D = \frac{Density\ of\ object}{Density\ of\ water}$$

$$R.D = \frac{\rho_o}{\rho_w}$$

$$0.8 = \frac{\rho_o}{1000}$$

$$\rho_o = 800kgm^{-3}$$

$$\textbf{Fraction of object submerged} = \frac{\rho_o}{\rho_w}$$

$$\textbf{Fraction of object submerged} = \frac{800}{1000}$$

$$\textbf{Fraction of object submerged} = \frac{4}{5}$$

ii)

$$\textbf{Fraction exposed} = 1 - \frac{4}{5}$$

$$\textbf{Fraction exposed} = \frac{1}{5}$$

EXERCISE:

1. A solid of volume $10m^3$ floats in water of density $10^3 kgm^{-3}$ with $\frac{3}{5}$ of its volume submerged. Calculate;
   a) the mass of the solid.
   b) the density of the solid.

2. A piece of wood of density $5gcm^{-3}$ and volume $200cm^3$ floats on a liquid of density $8gcm^{-3}$. Calculate the volume of wood immersed in the liquid.

3. A piece of wood of volume $40cm^3$ floats in water with only half of volume submerged. If the density of water is $1000kgm^{-3}$, calculate the density of wood.

4. A piece of wood of volume $30cm^3$ on a liquid of density $0.8g/cm^3$ with $\frac{2}{3}$ of its volume immersed in the liquid. Determine the weight of the piece of wood.

5. A solid of volume $2.0 \times 10^{-4}m^3$ floats in water of density $10^3 kgm^{-3}$ with $\frac{3}{4}$ of its volume submerged. Find the mass of the solid.

6. A block of wood of volume $100cm^3$ and density $500kg/m^3$ floats in a liquid of density $800kg/m^3$. Calculate the volume of wood submerged in the liquid.

7. A slab of ice of volume $800cm^3$ and density $0.9gcm^{-3}$ floats in water of density $1.1gcm^{-3}$. What fraction of ice slab is above the salt water?

APPLICATIONS OF THE LAW OF FLOATATION

1. **Ships**:

A ship is able to float whenever the upthrust is equal to its weight. Thus, it displaces an amount of water equal to its weight.

Plimsoll lines

Why ships float on water

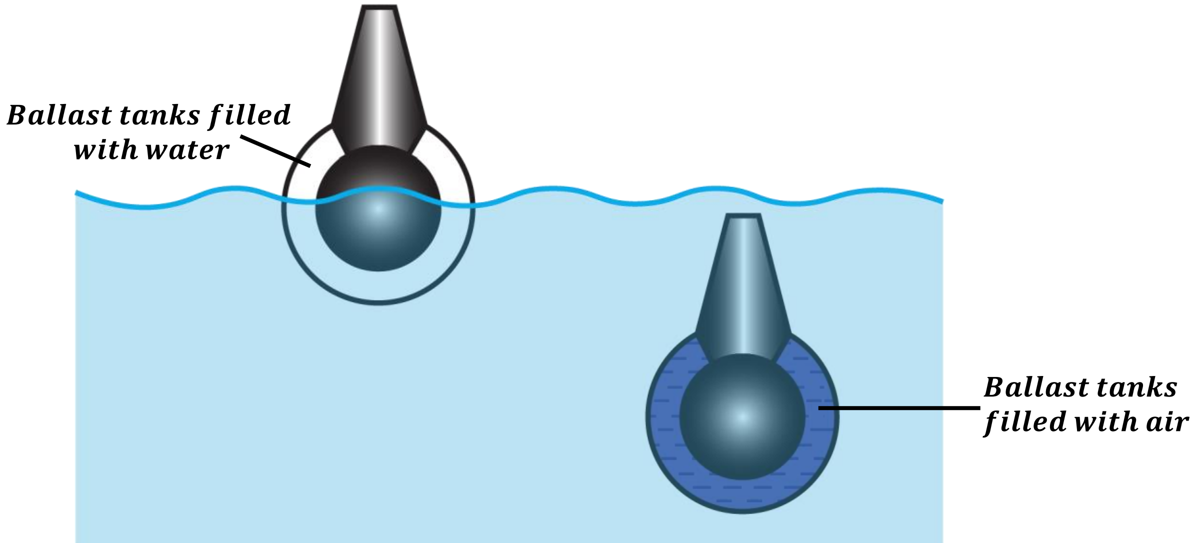
Ships float on water although they are made from iron and steel (metals) which are denser than water. This is because the ship is made hollow and contains air so that the average density of the ship is less than that of water.
If a hole develops on one side of a ship, the ship will take in water making its average density more than that of water thus it sinks.

NOTE:

The plimsoll lines (loading lines) on the sides of the ship show the level to which the ship can be safely loaded so that it can float on water.

2. **Submarines:**

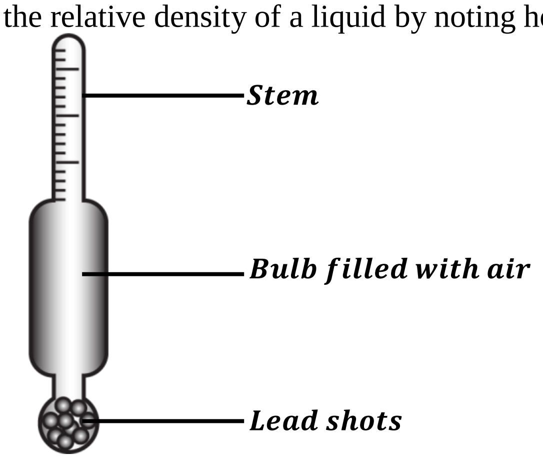
Submarines can float on water and can sink in water. The average density of a submarine is varied by the ballast tanks.



- ❖ For submarines to float, the ballast tanks are filled with air so that the average density of the submarine is less than that of water.
- ❖ For submarines to sink, the ballast tanks are filled with water so that the average density of the submarine is greater than that of water.

3. **Hydrometer:**

This is a device used to find the relative density of a liquid by noting how far it floats in a liquid.



It consists of;

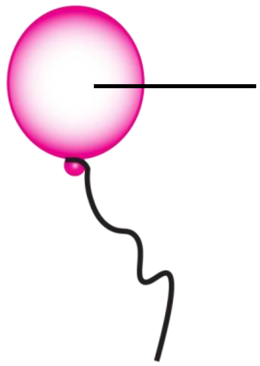
- a long and thin stem which makes the hydrometer more sensitive.
- a bulb filled with air with lead shots at the bottom. The lead shots keep the hydrometer upright when it floats in a liquid. Lead shots lower the centre of gravity of the hydrometer which increases its stability.

How to increase sensitivity of a hydrometer.

- By making the stem very thin (narrow)
- By making the bulb large.

Uses of a hydrometer:
- It is used to test the purity of milk.
- It is used to determine the level of sugar in some drinks (Lactometer).
- It used to determine the relative density of a car battery acid (Car battery tester).

4. Balloons:



A balloon filled with hydrogen rises in air because the density of hydrogen is less than the density of air in the atmosphere. Therefore, the upthrust acting on the balloon is greater than the weight of the balloon hence causing it to rise.
The balloon rises until when it becomes stationary. At this point, the weight of the balloon is equal to upthrust hence it starts to float.

NOTE: Balloons that carry passengers control their weight by blowing in hot gases into the gas bag to make them rise and letting out gases out of the gas bag to make them go down.

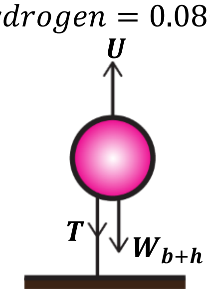
When the balloon is floating in air,

$$\boxed{\begin{aligned} &\textit{Upthrust} = \textit{weight of displaced air} \\ &\textit{Upthrust} = \textit{Weight of balloon} + \textit{Weight of hydrogen gas} + \textit{weight of load} \end{aligned}}$$

Examples:

1. A balloon of mass $0.005kg$ is inflated with hydrogen gas and held stationary on the ground by a string. If the volume of inflated balloon is $0.005m^3$.
   a) Calculate the upthrust acting on the balloon (Lifting force of the balloon)
   b) Calculate the tension in the string.
   ($Density\ of\ hydrogen = 0.080kgm^{-3}$, $Density\ of\ air = 1.150kgm^{-3}$)

a)

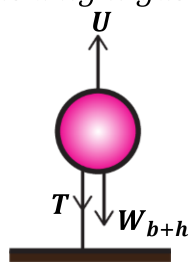


$$\boldsymbol{m_b = 0.005kg}$$
$$\boldsymbol{V_h = V_a = 0.005m^3}$$
$$\boldsymbol{\rho_h = 0.080kgm^{-3}}$$
$$\boldsymbol{\rho_a = 1.150kgm^{-3}}$$

$$\boldsymbol{Upthrust = Weight\ of\ displaced\ air}$$
$$\boldsymbol{Upthrust = m_a g}$$
$$\boldsymbol{Upthrust = \rho_a \times V_a \times g}$$
$$\boldsymbol{Upthrust = 1.150 \times 0.005 \times 10}$$
$$\boldsymbol{Upthrust = 0.0575N}$$

b) $Upthrust = Weight\ of\ balloon + Weight\ of\ hydrogen + Tension\ of\ string$
$Upthrust = m_b g + m_h g + T$
$Upthrust = m_b g + \rho_h V_h g + T$
$0.0575 = (0.005 \times 10) + (0.080 \times 0.005 \times 10) + T$
$0.0575 = 0.054 + T$
$T = 0.0035N$

2. A balloon of mass 5g is inflated with hydrogen and held stationary by a string. If the volume of the balloon is $0.005m^3$, find the tension in the string.
$(Assume\ hydrogen\ is\ a\ light\ gas\ and\ density\ of\ air = 1.25kgm^{-3})$



$m_b = 5g = \frac{5}{1000} = 0.005kg$
$V_h = V_a = 0.005m^3$
$m_h = 0kg\ (negligable)$
$\rho_a = 1.25kgm^{-3}$

$Upthrust = Weight\ of\ displaced\ air$
$Upthrust = m_a g$
$Upthrust = \rho_a \times V_a \times g$
$Upthrust = 1.25 \times 0.005 \times 10$
$Upthrust = 0.0625N$

But also;

$Upthrust = Weight\ of\ balloon + Weight\ of\ hydrogen + Tension\ of\ string$
$Upthrust = m_b g + m_h g + T$
$0.0625 = (0.005 \times 10) + 0 + T$
$0.0625 = 0.05 + T$
$T = 0.0125N$

3. A balloon has a capacity $10m^3$ and is filled with hydrogen. The balloon's fabric and container have a mass of $1.25kg$. Calculate the maximum weight in the container the balloon can lift.
$(Density\ of\ hydrogen = 0.089kgm^{-3}, Density\ of\ air = 1.29kgm^{-3})$

$m_b = 1.25kg$
$V_h = V_a = 10m^3$
$\rho_h = 0.089kgm^{-3}$
$\rho_a = 1.29kgm^{-3}$

$Upthrust = Weight\ of\ displaced\ air$
$Upthrust = m_a g$
$Upthrust = \rho_a V_a g$

But also;

$Upthrust = Weight\ of\ balloon + Weight\ of\ hydrogen + Weight\ of\ load$
$\rho_a V_a g = m_b g + m_h g + W_L$
$\rho_a V_a g = m_b g + \rho_h V_h g + W_L$
$(1.29 \times 10 \times 10) = (1.25 \times 10) + (0.089 \times 10 \times 10) + W_L$
$129 = 21.4 + W_L$
$W_L = 107.6N$

4. Explain what happens to a balloon;
   i) When it is filled with air and its open end tied and released.
   When a balloon is filled with air, the upthrust is less than the weight of the fabric of the balloon and the weight of air inside the balloon. The resultant force causes the balloon to move slowly downwards.

   ii) When it is filled with hydrogen gas and its open end tied and released.
   When the balloon is filled with hydrogen (or helium), which is less dense than air, the upthrust is greater than the weight of the balloon and its contents. Therefore, the balloon rises until when upthrust is equal to the weight of balloon and its content thus making the balloon to float.

EXERCISE:

1. A balloon of capacity $20m^3$and is filled with hydrogen. The balloon's fabric and container have a mass of $2.5kg$. Calculate the maximum mass of the load the balloon can lift
   $(\,Density\ of\ hydrogen = 0.089kgm^{-3}, Density\ of\ air = 1.29kgm^{-3})$

2. A hot air balloon is made from a very light material. It displaces $360kg$ of air and contains $300m^3$ of hydrogen gas of density $0.08kgm^{-3}$. Find the maximum load the balloon can lift.

3. The envelope of a hot-air balloon contains $1500m^3$ of hot air of density $0.8kgm^{-3}$. The mass of the balloon (not including the hot air) is $420kg$. The density of the surrounding air is $1.3kgm^{-3}$. Calculate the lifting force of the balloon.

4. A weather forecasting balloon is made of a fabric of mass $40kg$. Calculate the volume of hydrogen in the balloon which would just support an additional load of mass 80kg when floating in air.
   $(\,Density\ of\ hydrogen = 0.09kgm^{-3}, Density\ of\ air = 1.29kgm^{-3})$

5. A weather forecasting balloon of volume $15m^3$ contains hydrogen of density $0.09kgm^{-3}$. The volume of container carried by the balloon is negligible. The mass of empty balloon alone is 7.15kg. The balloon is floating in air of density $1.29kgm^{-3}$. Calculate;
   a) the mass of hydrogen in the balloon.
   b) the mass of hydrogen and the balloon.
   c) the mass of air displaced by the balloon