PRINCIPAL MATH CBC PRACTICE ITEMS

ITEM 1

The Ministry of Gender, Labour, and Social Development is organizing the "National Youth Skills Expo" in Jinja. A steering committee of 5 members must be formed from a pool of 8 qualified young innovators from the Central region and 6 from the Northern region. To ensure fairness, the organizers have decided that the committee must have at least 3 members from the Central region. Additionally, the committee needs to assign specific roles: A Chairperson, A Secretary, and A Treasurer, while the remaining two will serve as general members.

Tasks:

a) Help the Ministry determine the total number of different ways the 5-member committee can be selected to meet the regional representation requirement.
b) If the 5 members have already been chosen, calculate the number of ways the specific executive roles (Chairperson, Secretary, and Treasurer) can be assigned among them.
c) Determine the probability that a randomly selected committee of 5 (with no regional restrictions) would consist entirely of innovators from only one region.

ITEM 2

Okello is a dairy farmer in Mbarara who wants to construct an open rectangular milk cooling tank. Due to space constraints, the base of the tank must have a length twice its width (x). He has 12 square meters of sheet metal to fabricate the tank. He wants the tank to hold the maximum volume of milk possible to save on cooling costs. After construction, he plans to install a tap that releases milk at a rate modeled by $R(t) = 4t - t^2$ litres per minute, where t is the time in minutes after opening the tap.

Tasks:

a) Formulate an expression for the total surface area and the volume of the tank in terms of the width x, and show that the volume is maximized when the width is approximately 1.15 meters.
b) Calculate the maximum volume of milk this tank can hold when constructed with these optimal dimensions.
c) Using the flow rate model $R(t)$, determine the total amount of milk that will have flowed out of the tank during the first 3 minutes.
d) Advise Okello on whether the tank will be completely empty after 4 minutes if it initially contained 15 litres of milk, giving a mathematical reason for your answer.

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ITEM 3

A coffee processing factory in Masaka produces "Premium Arabica" coffee bags. From previous quality checks, it is known that 10% of the bags are slightly underweight due to machine errors. A quality assurance officer, Mrs. Namono, randomly selects 20 bags from a large batch for inspection. She also models the weight of the bags as a continuous random variable X with a probability density function

$$f(x) = \begin{cases} k(x-1)(3-x), & 1 \leq x < 3 \\ 0 \quad , & elsewhere \end{cases}$$

Tasks:

a) Using the machine error data, determine the probability that Namono finds exactly 2 underweight bags in her sample of 20.
b) Calculate the probability that at least 3 bags are underweight in the selected sample.
c) For the continuous weight model, determine the value of the constant k and calculate the mean weight of a coffee bag.
d) If a bag is rejected for being less than 1.5 kg, help Namono determine the percentage of bags that would be rejected based on the continuous model.

ITEM 4

In the hilly district of Kapchorwa, two neighboring farmers, Chelangat and Kiptum, share a boundary that has historically been marked by a straight line between two ancient Mutuba trees. On a local land registry map, the first tree is located at coordinates $A(2, 5)$ and the second tree is at $B(10, 11)$, where each unit represents 100 meters.

The farmers intend to construct a joint irrigation reservoir. They initially plan to place it at the exact midpoint of their shared boundary to ensure both farms have equal access to water. However, a local surveyor suggests that a more stable location for the reservoir would be at a point P that divides the boundary AB in the ratio 3:1 from A towards B.

Tasks:

a) Determine the coordinates of the midpoint of the boundary AB where the farmers initially intended to place the reservoir.
b) Help the farmers calculate the coordinates of point P suggested by the surveyor to see if it significantly shifts the location from the center.
c) If the main water source for the reservoir is located at point $S(6, 15)$, advise the farmers on which location (the midpoint or point P) is closer to the source to minimize the cost of piping, providing a mathematical reason for your choice.
d) Formulate the Cartesian equation of the straight line representing the boundary AB to help the land registry formally document the border for future generations.

ITEM 5

A medical drone is programmed to deliver emergency vaccines from a hospital in Entebbe to a health center in the Kalangala islands. The position of the hospital is represented by the point $A(2,5,1)$ and the health center by $B(8,-1,4)$, with coordinates in kilometers. A strong wind blowing from the East is represented by the vector $w = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$. The drone's engine provides a displacement vector $d = \begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix}$ relative to the air.

Tasks:

a) Determine the position vector of the health center relative to the hospital and calculate the direct distance (magnitude) between the two locations.
b) Calculate the resultant velocity vector of the drone and find the angle this resultant makes with the North.
c) Determine the vector equation of the straight path the drone would take if it flies directly from A to B.

ITEM 6

Juma, a driver for a popular bus company, is transporting passengers from Kampala to Mbale. As he navigates the highway, he starts the bus from rest and accelerates uniformly at $1.5\, m/s^2$ for 20 seconds until he reaches a steady cruising speed. He maintains this constant speed for 10 minutes while driving through a flat, open stretch of the road.

Suddenly, Juma notices a herd of cattle crossing the road 150 meters ahead. He immediately applies the brakes, decelerating the bus uniformly to a complete stop just 5 meters before reaching the herd.

Tasks:

a) Calculate the cruising speed Juma was maintaining in km/h and determine the distance covered during the initial acceleration phase.
b) If the legal speed limit on that stretch of the highway is 80 km/h, advise Juma on whether his cruising speed was within the legal limit.
c) Help the bus company manager estimate the total time taken from the moment the bus started in Kampala to the moment it came to a halt at the cattle crossing.

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ITEM 7

An electrical technician in the Katwe industrial area is repairing a solar power inverter. He represents the "impedance" (resistance to AC current) of two components, Z_1 and Z_2, as complex numbers. $Z_1 = 3 + 4i$ ohms and $Z_2 = 1 - i$ ohms. To find the total impedance Z_T when connected in a specific way, he needs to use the formula $Z_T = \frac{Z_1}{Z_1+Z_2}$.

Tasks:

a) Help the technician calculate the total impedance Z_T in the form $a + bi$.
b) Represent Z_1 and Z_2 on an Argand diagram and determine the modulus and argument of Z_1.
c) If a third component Z_3 has a modulus of 5 and an argument of $\frac{\pi}{3}$, express Z_3 in the polar form and then in rectangular form.
d) The technician notices a fault where the impedance Z must satisfy $|Z - (2+i)| = 3$. Sketch the region (locus) representing all possible values of Z and describe the shape of this region to the technician.

ITEM 8

An environmental scientist is studying the rate of spread of an organic fertilizer in a community garden in Mukono. The concentration of the fertilizer over time is modeled by a rational function $C(t) = \frac{7t + 11}{(t-1)(t+3)}$, where $t > 1$ is the time in hours. To integrate this function and find the total accumulation of the fertilizer, the scientist first needs to break the function into simpler parts.

Tasks:

a) Formulate the expression for $C(t)$ into partial fractions.
b) Using your results from (a) above, calculate the exact value of the integral $\int_2^4 C(t)\ dt$ to determine the accumulation between the 2nd and 4th hour.

The Ministry of Water and Environment district engineers are assessing the impact of human activity on the shores of Lake Victoria. To plan for a new community park, they need to estimate the surface area of a specific irregular-shaped swampy bay near Entebbe. A survey team has measured the width of the bay (y) in meters at regular intervals (x) of 25 meters from a fixed starting point. The collected data is recorded in the table below:

Distance x (m)	0	25	50	75	100	125	150
Width y (m)	10	18	24	22	19	14	8

Tasks:

a) Help the engineer to estimate the total surface area of this section of the bay using the trapezium rule.
b) The environmental team suspects that the width measurement at $x = 75$ may have an error of +1.5 meters due to dense vegetation. Show how this specific measurement error would affect the final area estimate for the park.
c) If the cost of planting specialized reeds for water purification is UGX 15,000 per square meter, determine the estimated budget the Ministry needs for this section.

ITEM 9

Musoke, a land surveyor, is measuring a triangular plot of land near the foothills of Mt. Elgon in Kapchorwa. The plot has two sides of length 120 m and 150 m, and the angle between them is θ. He needs to ensure the area of the plot is exactly 6,000 square meters for a planned playground. He also uses a signal mast that is held by two wires inclined at angles α and β to the horizontal, where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{5}{12}$. In order to find the specific alignment for the angles for the playground he must use the equation $3\cos 2x - \sin x = 13$ for $0° \leq x \leq 360°$

Tasks:

a) Help Mr. Musoke to
   i. find the possible values of the angle θ (in degrees) that would satisfy the playground requirements.
   ii. calculate the exact value of $\sin(\alpha + \beta)$ to determine the combined tension angle for the mast supports.
b) Determine the specific alignment angles for the playground equipment.
c) Show that the expression $\frac{\sin 2A}{1+\cos 2A}$ can be simplified to $\tan A$, and explain how this identity could simplify Musoke's calculations when dealing with double-angle slopes.

A market vendor in Wakiso wants to set up a stall. She has a budget of UGX 500,000. The cost of transporting items from point A to her stall is UGX 2,000 per km, and the cost of storage at the stall is UGX 5,000 per day. Her location (x, y) must be within a safe zone defined by the intersection of the line $y = 2x + 1$ and the curve $y = x^2 - 2$. Furthermore, to be near customers, her coordinates must satisfy the inequality $x + y \leq 6$ and $y \geq 2$. She approaches you for advice on how she can set up her stall and technical support.

Tasks:

a) Help the vendor to determine the and visualize the safe zone on a coordinate plane and the possible locations where to setup the stall to be nearer to her customers.
b) Calculate the distance between the two points found in (a) to determine the length of the "safe corridor".

c) If the vendor chooses a spot at $(1, 4)$, calculate the total cost for 5 days of storage and 10 km of transport, and advise her if this fits within her UGX 500,000 budget