OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)
A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2025
ORGANISED ON SATURDAY 04TH OCTOBER 2025

ALGEBRA

1. (a) A group of ten students are to sit around a circular table. Among these students, there are two students who should not sit side by side. Find the possible number of different ways in which the group of students can sit so as to achieve the given condition/ restriction.

(b) Solve the equation $4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0$.

(c) In the equation $ax^2 + bx + c = 0$, one root is the square of the other. Prove that $c(a-b)^3 = a(c-b)^3$

(d) Which is the first term of a G.P 5+10+20+… to exceed 400,000?

2. (a)(i) Find the locus in the complex plane given by $|z - i| = 2$.

(ii) By shading the unwanted region, show in a diagram the region by $|z - i| < 2$

(b) Given that the complex number Z varies such that $|z - 5| = 3$, find the greatest and the least values of $|z + 2 - 4i|$

(c) Show that $z = 1 + i$ is a root of the equation; $z^4 + 3z^2 - 6z + 10 = 0$. Hence, find the remaining roots.

(d) Find the cube roots of $-8i$, and represent them on an Argand diagram.

3. (a) (i) Solve for m; $4^{2m+1} - 2(4^{m+2}) + 48 = 0$

(ii) If $(x-5)^2$ is a factor of $x^3 - 2ax^2 + 3bx - 200$, find the values of a and b

(b) (i) Use binomial theorem to expand $(1 + 4x)^{\frac{1}{4}}$ up to term in x^3.

(ii) State the range of values of x with in which the expansion is valid.

(iii) Using the expansion in (i) above, find $82^{\frac{1}{4}}$ and give your answer to 4 decimal places.

(iv) Find the coefficient of the term x^6 in the expansion of $\left(-3x + \frac{2}{x}\right)^{12}$

4. If x is real and $y = \frac{5x^2+8x+4}{x^2+x}$,

(i) Prove that y cannot lie between -4 *and* 4.

(ii) Find the coordinates of the turning points on the graph of $y = \frac{5x^2+8x+4}{x^2+x}$, and the coordinates of the point where the graph crosses its horizontal asymptotes.

(iii) Sketch the graph, showing all the necessary points, given that it has no intercepts.

ANALYSIS

5. (a) Differentiate $\frac{1}{2x^2-3}$ from first principles, with respect to x.

(b) Given $y = 2e^{2x}cos3x - e^{2x}sin3x$. Show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

(c) The height, h, of a right-angled triangle increases at a constant rate of 0.05m per hour while keeping the base constant at 5 metres. Find the rate of change of the angle opposite the height, θ when $\theta = \frac{\pi}{3}$.

(d) A piece of wire of length L is cut into 2 parts $x \; and \; L - x$. The former is bent into a square and the later into a rectangle, of which the base double the height. Find the sum of the two areas of these figures. Prove that this sum is maximum or minimum when $x = \frac{8L}{17}$ and find which it is.

6. (a) Find the approximate value of the cube root of 732 using small changes

(b) The rate of change of the temperature, $\theta^0 C$ of a mug of coffee is given by $\frac{d\theta}{dt} = \frac{1}{25}t - k$; where t is the elapsed time, in minutes after the coffee is poured into the mug and k is a constant. Initially, the temperature of the coffee is $100^0 C$. 10 minutes later, the temperature has fallen to $82^0\ C$. Express θ interms of t.

(c) The population of a certain town follows a mathematical model $\frac{dx}{dt} = \frac{x}{100} - \frac{x^2}{10^8}$, where t is recorded in years. Given that the population of the town was 100,000 in 1990.

(i) Determine the population of the town as a function of time t. (i.e. t interms of x)

(ii) In what year will the population of the town double that of 1990?

7. (a)(i) Express $\frac{x^4-7x^3+20x^2-15x-50}{(x-1)(x-2)(x-4)}$ in partial fractions.

(ii) Hence evaluate $\int_5^7 \frac{x^4-7x^3+20x^2-15x-50}{(x-1)(x-2)(x-4)} dx$ correct to 3dps

(b) Find the area enclosed by the curve $y = cosec\ \left(\frac{x}{2}\right)$, the $x - axis$, and the lines $x - \pi = 0$ and $3x - 4\pi = 0$.

(c) Find the volume generated when the area between the line $y = x$ and the curve $y^2 = x$ is rotated through 360^0 about the $x - axis$.

TRIGONOMETRY

8. (a) Solve the equation $2sin^2x + sin^2 2x = 2$ for $-180^0 \le x \le 180^0$.

(b) An arc AB subtends an angle of $\frac{2\pi}{3}$, radians at the centre of the circle of radius, R. Show that $\frac{Length\ of\ arc\ AB}{Length\ of\ chord\ AB} = \frac{2\pi}{3\sqrt{3}}$

(c) Given that ABC is a triangle, prove that $\frac{b^2-a^2}{2c^2} = \frac{1-tanAcotB}{2(1+tanAcotB)}$.

9. (a) If $cos\alpha - cos\beta = \frac{2}{5}$ and $sin\alpha - sin\beta = \frac{5}{6}$. Find the value of;

(i) $\text{Sin}\frac{1}{2}(\alpha+\beta)$

(ii) $\cos(\alpha+\beta)$

(b) In any triangle ABC, Prove that; $sin^2A + sin^2B + sin^2C = 2 + 2cosAcosBcosC$

(c) Show that $cos4x = 8cos^4x - 8cos^2x + 1$

10. (a) Factorise the expression $6sin\theta cos\theta + 3cos\theta + 4sin\theta + 2$, and hence solve the equation $6sin\theta cos\theta + 3cos\theta + 4sin\theta + 2 = 0$ for $-180^0 \le \theta \le 180^0$.

(b) Prove that $sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + cot^{-1}(x) = \frac{\pi}{2}$

(c) Given the triangle PQR, show that $tan\left(\frac{Q-R}{2}\right) = \frac{q-r}{q+r}cot\frac{P}{2}$, hence solve the triangle with two sides 5cm and 7cm, and the included angle is 45^0

VECTORS

11.(a) Show that the equation of the line through the points (1,2,1) and $(4,-2,2)$ is given as $\frac{x-1}{3} = \frac{y-2}{-4} = z - 1$

(b) A line L drawn from the point $A(-1,3,-2)$ and is parallel to the line $\frac{x+3}{2} = \frac{y-1}{5} = \frac{z}{1}$ meets the plane $\pi;\ x + y + z = 8$ at point B. Find the;

(i) Coordinates of B

(ii) Angle between L and π

(iii) Distance between L and the line $\frac{x+3}{2} = \frac{y-1}{5} = \frac{z}{1}$.

12. (a) Determine the vector perpendicular to both vectors $\boldsymbol{a} = 4\boldsymbol{i} = 5\boldsymbol{j} - \boldsymbol{k}$ and $b = 3\boldsymbol{k} - 5\boldsymbol{i} + 2\boldsymbol{j}$**.**

(b) Show that the points $A(1,2,3)$, $B(3,8,1)$ *and* $C(7,20,-3)$ are collinear.

(c) In a triangle $\boldsymbol{OAB}, \boldsymbol{OA} = \boldsymbol{a}$ *and* $\boldsymbol{OB} = \boldsymbol{b}$**,** Point **P** lies on $\boldsymbol{OB}$ such that $\boldsymbol{OP}:\boldsymbol{OB} = \mathbf{2}:\mathbf{5}$ and Point $\boldsymbol{Q}$ lies on $\boldsymbol{AB}$ such that $\boldsymbol{AQ}:\boldsymbol{AB} = \mathbf{3}:\mathbf{5}$**.** The lines **OQ** and **PA** intersect at point **R** such that $\mathbf{3}\boldsymbol{PR} = \mathbf{8}\boldsymbol{RA}$. Find interms of **a** and **b** the vectors.

(i) $\boldsymbol{OP}$ (ii) $\boldsymbol{AQ}$ (iii) $\boldsymbol{OQ}$ (iv) $\boldsymbol{PA}$ (v) $\boldsymbol{PR}$

13. (a) Find the cartesian equation of the line of intersection of two planes whose equations are $3x - y + z = 2$ *and* $x + 5y + 2z = 6$.

(b) In a triangle **OAB** where $\boldsymbol{OA} = \boldsymbol{a}$ and $\boldsymbol{OB} = \boldsymbol{b}$, show that area of the triangle OAB is given by $\frac{1}{2}\sqrt{|\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}.\boldsymbol{b})^2}$

(c) Show that the points **A, B** and **C** with position vectors $\mathbf{2}\boldsymbol{i} + \mathbf{3}\boldsymbol{j}$, $\mathbf{4}\boldsymbol{i} + \mathbf{5}\boldsymbol{j}$, $\mathbf{6}\boldsymbol{i} + \mathbf{9}\boldsymbol{j}$**,** respectively are the vertices of a triangle. Find the area of the triangle.

COORDINATE GEOMETRY

14. (a) Show that the equation $y^2 - 4y = 4x$ represents a parabola and hence determine the coordinates of its vertex and focus.

(b) A chord to the parabola $4x - 3y^2 = 0$ is parallel to the line $2x - y = 4$ and passes through the point (1,2), find;

(i) The equation of the chord.

(ii) The coordinates of the points of intersection of the chord with the parabola

(iv) The acute angle between the chord and the directrix of the parabola.

15. (a)(i) Show that the acute angle between two straight lines of gradients m_1 and m_2 is given by $tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

(ii) Find the angle between two lines $y = 2x + 5$ and $3x + y = 7$

(b) Show that $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$

(c) The normal to the curve $xy = 4$ at the point $\left(2p, \frac{2}{p}\right)$ meets the curve again at point Q. Find the coordinates of Q.

(d) calculate the distance between the parallel lines $3x + 4y + 10 = 0$ and $6x + 8y - 5 = 0$

16. (a) Find the equation of a circle which touches the line $3x - 4y = 3$ at the point (5,3) and passes through the point (-2,4)

(b) Find the values of m for which the line $y = mx$ is a tangent to the circle $x^2 + y^2 + fy + c = 0$.

(c) Find the points where the lines $2y - x + 5 = 0$ meets the circle $x^2 + y^2 - 4x + 3y - 5 = 0$. Obtain the equation of the tangents and normals to the circle at these points.

17. (a) The equation of the ellipse is given by $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find the value of the;

(i) Eccentricity

(ii) Coordinates of the foci

(iii) The equations of the directrices

(iv) The length of the latus rectum

(b) The equation of an ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b > 0$. A tangent drawn to the upper part of the ellipse at point (m, n) cuts the $x - axis$ at point $(c, 0)$. Show that; $\frac{a^2}{b^2} = \frac{mc-m^2}{n^2}$

18. (a) Prove that the line $y = mx \pm a\sqrt{m^2 - 1}$ are tangents to the hyperbola $x^2 - y^2 = a^2$ for all values of m

(b) The tangent at any point $P\left(ct, \frac{c}{t}\right)$ on the hyperbola $xy = c^2$ meets x and y axes at A and B respectively. O is the origin.

(i) Prove that AP=PB

(ii) Prove that the area of the triangle AOB is constant.

(iii) if the hyperbola is rotated through an angle of -45^0 about O, find the new equation of the curve.

END