REFRACTION OF LIGHT IN LENSES

A lens is a piece of transparent glass material bounded by one or two surfaces.

Types of lenses.

1. **Convex lens**: This is a lens whose surfaces curve outwards. It is thicker in the middle and thinner at the edges. The convex lens is as shown below;

The convex lens is sub divided into two namely; plano convex lens and convex meniscus lens

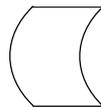Convex meniscus lens   Plano convex lens

2. Concave lens: This is a lens whose surfaces curve outward. It is thinner in the middle and thicker at the edges.

The concave lens is also subdivided into two namely; Plano concave lens and the concave meniscus lens.

Plano convex   Concave meniscus

Wide beam of light incident on a lens.

When a beam of light, parallel and close to the principal axis incident on a convex lens, light is refracted and brought to same focus at the principal focus.

When a beam of light, parallel and close to the principal axis is incident on a concave lens, light is refracted such that it appears to be diverged from the principal focus.

Convex lens

Principal axis   F

Concave lens

Principal axis   F
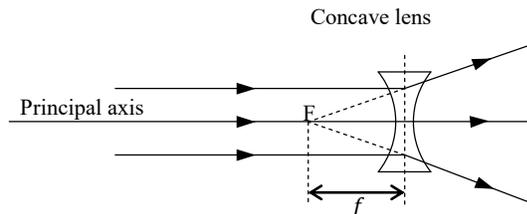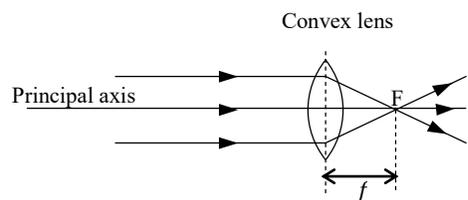
The convex lens converges light and therefore called a converging lens while a concave lens diverges light and therefore called a diverging lens.

Terms used in lenses

1. **Centres of curvature of a lens:** These are centres of the spheres of which the lens surfaces form part.
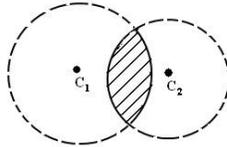
   Points C_1 and C_2 are the centers of curvature of the lens surfaces.

2. **Radii of curvature of a lens:** These are distances from the centres to the surfaces of the spheres of which the lens surfaces form part to the lens lens surfaces.

3. **Radius of curvature:** This is the radius of the sphere of which the lens surface forms part.

4. **Principal axis of a lens:** This is the line joining the centers of curvature of the two surfaces of the lens.

5. **Optical centre of the lens:** This is the mid-point of the lens surface through which rays incident on the lens pass un deviated.

6. **Paraxial rays:** These are rays close to the principal axis and make small angles with the lens optical axis.

7. **Principal focus of a convex lens(F):** This is the point on the principal axis where rays originally parallel and close to the principal axis converge or appear to diverge after refraction by the lens.
   (i) Principal focus of a convex lens(F): This is a point on the principal axis where rays originally parallel and close to the principal axis converge after refraction by the lens.
   (ii) Principal focus of a concave lens(F): This the point on the principal axis where rays originally parallel and close to the principal axis appear to diverge after refraction by the lens.

8. **Focal length of a convex lens(f):** This is the distance from the optical centre of the lens to the focal point.

Rules of construction of ray diagrams in lenses
1. A ray parallel to the principal axis is refracted through the focal point.
2. A ray through the focal point is refracted parallel to the principal axis.
3. A ray through the optical centre passes through un deviated.

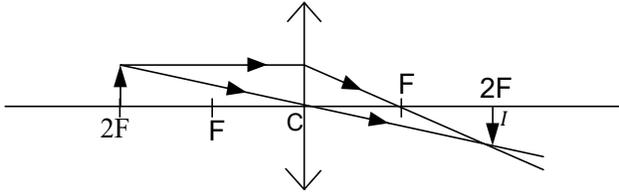## Formation of images by convex lenses
1. Object beyond C
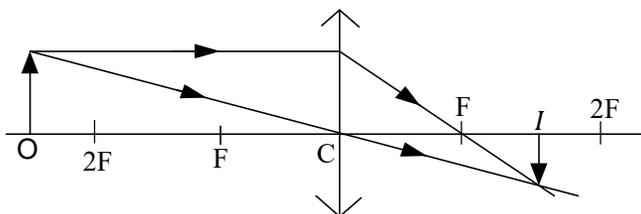
Nature of the image

The image is;

- ✓ Real
- ✓ Inverted
- ✓ Diminished
- ✓ Formed between F and 2F
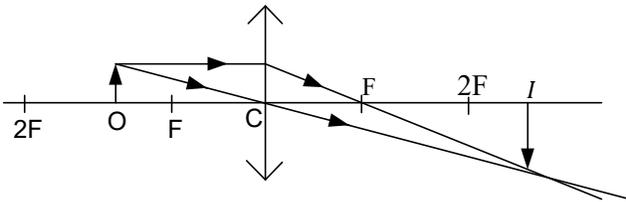
2. Object at 2F

Nature of the image

The image is;

- ✓ Real
- ✓ Inverted
- ✓ Has the same size as the object
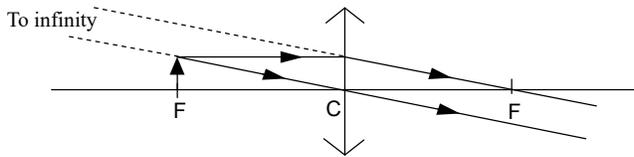- ✓ Formed at 2F

3. Object between F and 2F

Nature of the image

The image is;

- ✓ Real
- ✓ Inverted
- ✓ magnified
- ✓ Formed at beyond 2F

4. Object at F

Nature of the image

The image is;
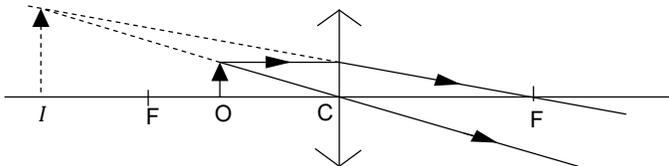
- ✓ Virtual
- ✓ Upright
- ✓ magnified
- ✓ Formed at infinity
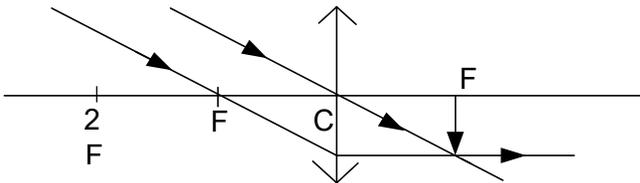
5. Object between F and C

Nature of the image

The image is;

- ✓ Virtual
- ✓ Upright
- ✓ magnified

.

When the object is between the focal point and the optical centre, the lens acts as a magnifying glass.
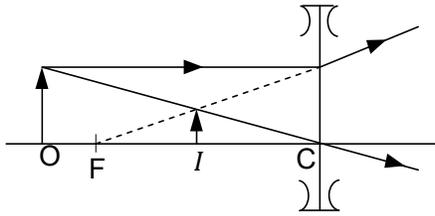
6. Object at infinity

Nature of the image

The image is;

- ✓ Real
- ✓ inverted
- ✓ diminished
- ✓ formed at F

Image formation by a concave lens

Nature of the image

The image is;

- ✓ Virtual
- ✓ Upright
- ✓ diminished
- ✓ formed between F and C

Sign convention

A convex lens has a real principal focus and centre of curvature. Therefore, the focal length and radii of curvature of a convex lens are positive.

The concave lens has a virtual principal focus and centre of curvature. Therefore the focal length and radii of curvature of a concave lens are negative.

The thin lens formula

The thin lens formula gives the relationship between the object distance(u), image distance(v) and the focal length(f)

(a) Using a convex lens and a point object

Consider light from a distant object incident on a convex lens at a height, h from the principal axis.
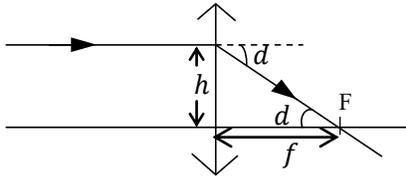
$$\tan d = \frac{h}{f}$$

Consider light from a point object O incident on the lens at a height h above the principal axis as shown below.

$\tan \alpha = \frac{h}{u}$ and $\tan \beta = \frac{h}{v}$

From the ray diagram; $\alpha + \beta = d$ ....(1)

But α, β and d are small angle in radians

Therefore; $\tan d \approx d = \frac{h}{f}$, $\tan \alpha \approx \alpha = \frac{h}{u}$ and and $\tan \beta \approx \beta = \frac{h}{v}$

Equation one becomes;

$$\frac{h}{u} + \frac{h}{v} = \frac{h}{f}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

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(b) Using a convex lens and a finite object
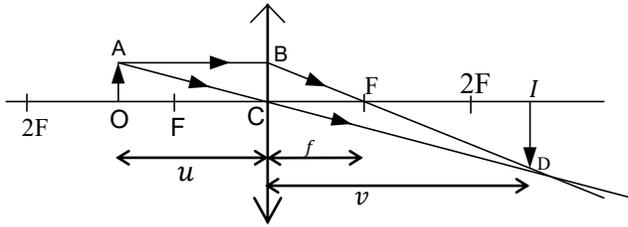
Triangles OAC and CID are similar.

$\dfrac{OC}{IC} = \dfrac{OA}{ID}$…………..(1)

Triangles CBF and FID are also similar.

$\dfrac{CF}{IF} = \dfrac{CB}{ID}$…………………..(2)

But $OA = CB$

Equations (1) becomes.

$\dfrac{OC}{IC} = \dfrac{CB}{ID}$……………..(3)

Equating (2) and (3)

$$\frac{CF}{IF} = \frac{OC}{IC}$$

$$OC = u, \qquad CF = f, \qquad IF = v - f\,, IC = v$$

$$\frac{f}{v-f} = \frac{u}{v}$$

$$vf = uv - uf$$

$$vf + uf = uv$$

Dividing through by uvf.

$$\frac{vf}{uvf} + \frac{uf}{uvf} = \frac{uv}{uvf}$$

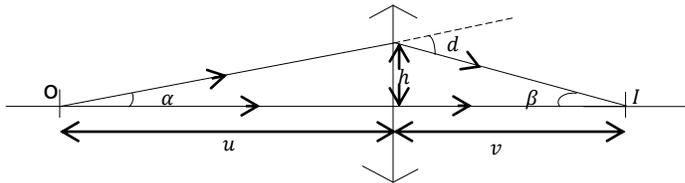$$\Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

(c) Using a concave lens and a point object

Consider a ray of light from a point object incident on a concave lens at a height, h above the principal axis.
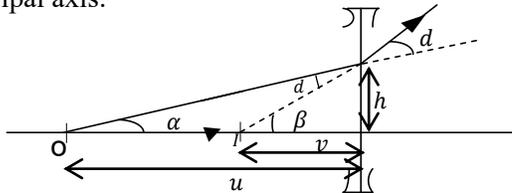
$$\tan \alpha = \frac{h}{u} \text{ and } \tan \beta = \frac{h}{-v}$$

From the ray diagram; $\alpha + d = \beta$………(1)

Consider a ray of light from a distant object incident on the concave lens at a height, h above the principal axis.

$$\tan d = \frac{h}{-f}$$

But α, β and d are small angle in radians.

Therefore; $\tan d \approx d = \frac{h}{-f}$, $\tan \alpha \approx \alpha = \frac{h}{u}$ and and $\tan \beta \approx \beta = \frac{h}{-v}$

Equation (1) becomes;
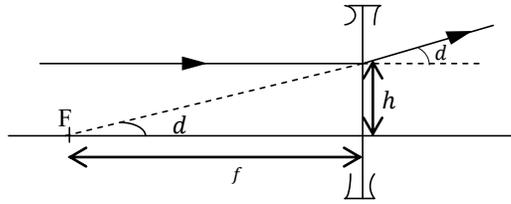
$$\frac{h}{u} + \frac{h}{-f} = \frac{h}{-v}$$
$$\frac{1}{u} - \frac{1}{f} = \frac{-1}{v}$$
$$\boldsymbol{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

Linear magnification produced by a lens

Linear magnification is ratio od image size to object size OR Linear magnification is ratio of image distance to object distance.

Relationships between u, v, f and m

From $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ ....(*)

Multiplying through in (*) by v

$$\frac{v}{u} + \frac{v}{v} = \frac{v}{f}$$

But $m = \frac{v}{u}$

$$m + 1 = \frac{v}{f}$$
$$\boldsymbol{v = (m + 1)f}$$

Multiplying through in (*) by u

$$\frac{u}{u} + \frac{u}{v} = \frac{u}{f}$$

But $\frac{u}{v} = \frac{1}{m}$

$$1 + \frac{1}{m} = \frac{u}{f}$$
$$\boldsymbol{u = \left(\frac{1}{m} + 1\right)f}$$

Examples

1. A real image in a converging lens of focal length 15cm is twice the size of the object. Find the object distance and the image distance.

   Solution

   $$f = 15cm, \ \ h_I = 2h_o \Rightarrow \frac{h_I}{h_o} = 2$$
   $$m = 2$$

   Object distance; $u = \left(\frac{1}{m} + 1\right)f$

$$u = \left(\frac{1}{2} + 1\right) \times 15 = 22.5cm$$

Image distance; $m = \frac{v}{u} \Rightarrow 2 = \frac{v}{22.5}$

$$v = 45cm$$

Alternatively; $v = (m + 1)f$

$$v = (2 + 1) \times 15 = 45cm$$

2. An object is placed 15cm infront of a concave lens of radius of curvature 20cm. find the position of the image and its magnification.
    Solution

$$u = 15cm, f = \frac{-20}{2} = -10cm$$
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
$$\frac{1}{15} + \frac{1}{v} = \frac{1}{-10}$$
$$v = -6cm$$

The image formed 6cm infront of the lens.
Magnification; $m = \frac{v}{u}$

$$m = \frac{-6}{15} = -0.4$$

The magnification of the image is 0.4 (the image is diminished and virtual)

3. A convex lens forms an image whose size is $\frac{3}{4}$ the size of the object. The object is moved away from the lens until the image size is half the object size. If the shift in the image position is 2.5cm, find the;
    (i)   Focal length of the convex lens.
    (ii)   New position of the object.
            Solution

(i)   For case I; $m_1 = \frac{3}{4}$

$$v_1 = (m_1 + 1)f$$

$v_1 = \left(\frac{3}{4} + 1\right)f = 1.75f$.........................(1)

For case II; $m_2 = \frac{1}{2}$

$$v_2 = (m_2 + 1)f$$

$v_2 = \left(\frac{1}{2} + 1\right)f = 1.5f$..............(2)

Shift in the image position $= v_1 - v_2 = 2.5$
$$1.75f - 1.5f = 7.5 = 2.5$$
$$.25f = 2.5$$
$$f = 10cm$$

(iii) $u_1 = \left(\frac{1}{\frac{1}{2}} + 1\right)f$

$$u_1 = (2 + 1)f = 3f$$
$$u_1 = 3 \times 10 = 30cm$$

Exercise
1. A real image in a converging lens of focal length 12cm is twice as long as the object. Find the image distance from the lens.

2. The magnification of an object in a converging lens is **m**. when the lens is moved a distance **d** towards the object, the magnification becomes **m¹**. Show that the focal length **f** of the lens is given by; $f = \frac{dmm\prime}{m\prime - m}$

3. A convex lens forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 20cm, determine the;
   - (i) focal length of the lens.
   - (ii) shift of the object.

4. Draw a ray diagram to show how a concave lens forms an image of a finite object and use it to derive the formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

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Least distance between the object and the image in a convex lens

Consider an object O placed infront of a convex lens of focal length, f to form an image on the screen placed a distance x from the lens. Suppose the object and the screen are a distance, d apart.
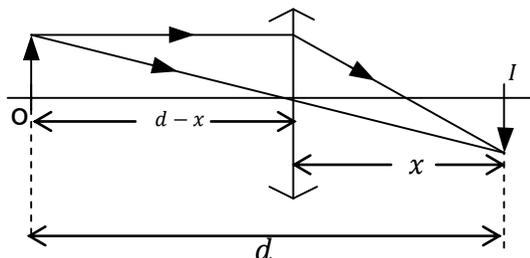
Using the lens formula; $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$u = d - x, v = x$$
$$\frac{1}{d-x} + \frac{1}{x} = \frac{1}{f}$$
$$\frac{x + d - x}{x(d-x)} = \frac{1}{f}$$
$$fd = dx - x^2$$
$$x^2 - dx + fd = 0$$
$$x = \frac{-(-d) \pm \sqrt{(-d)^2 - 4(1)(fd)}}{2(1)}$$
$$x = \frac{d \pm \sqrt{d^2 - 4fd}}{2}$$

For a real image; $d^2 - 4fd \geq 0$

$$d(d - 4f) \geq 0$$
$$(d - 4f) \geq 0$$
$$d \geq 4f$$
$$\boldsymbol{d_{min} = 4f}$$

Hence the least minimum distance between the object and the real image formed by a convex lens is $4f$.

   Conjugate points

These are points on the principal axis such that when the object is placed at one point the image is formed at the other point.

Suppose a convex lens of focal length, f forms an image of the object at I. If the object is placed at I, the lens would form the image of the object at O. Then points O and I are called conjugate points.
Let points O and I be at distances x and y from the focal point, F respectively.
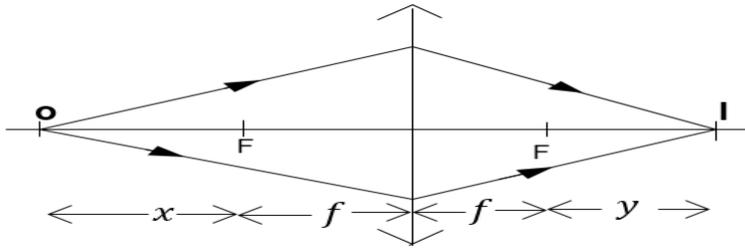
Using the lens formula; $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$u = f + x, v = f + y$$
$$\frac{1}{f+x} + \frac{1}{f+y} = \frac{1}{f}$$
$$\frac{f+y+f+x}{(f+x)(f+y)} = \frac{1}{f}$$
$$\frac{2f+y+x}{(f+x)(f+y)} = \frac{1}{f}$$
$$2f^2 + fx + fy = f^2 + fy + fx + xy$$

$f^2 = xy$. This is called the Newton's equation.

Displacement of a lens keeping the object and screen fixed.
There are two positions where a convex lens can be placed to form real image on the screen.
Consider a convex lens of focal length, f positioned at A infront an Object at O to form the image on the screen at I. Suppose the Object and the screen are fixed and the lens is displaced through a distance, d to position B to form another real image on the screen.
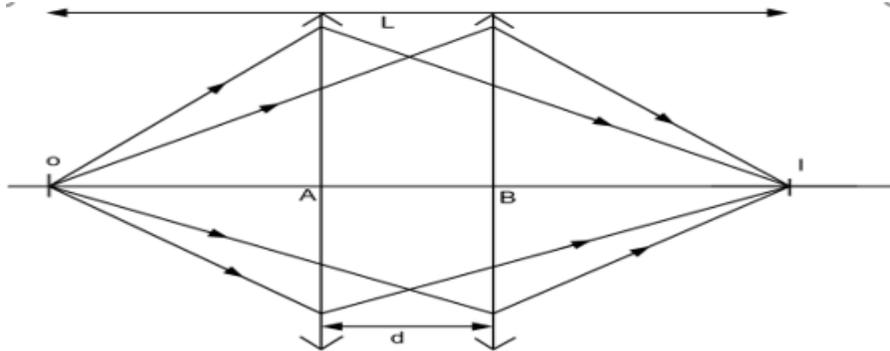
When the lens is at A, $u = OA, v = AI$
Since O and I are conjugate points; $OA = BI$ and $AI = OB$.
$$OA + AB + BI = l$$
$$u + d + u = l$$
$$2u + d = l \Rightarrow u = \frac{l-d}{2}$$
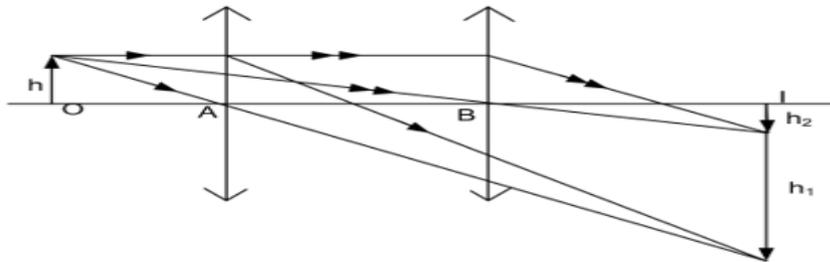$$v = AI = AB + BI$$
$$v = d + \frac{l-d}{2}$$

$$v = \frac{2d + l - d}{2} = \frac{l + d}{2}$$

Using the lens formula; $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{\frac{l-d}{2}} + \frac{1}{\frac{l+d}{2}} = \frac{1}{f}$$

$$\frac{2}{l-d} + \frac{2}{l+d} = \frac{1}{f}$$

$$\frac{2l + 2d + 2l - 2d}{(l-d)(l+d)} = \frac{1}{f}$$

$$\frac{4l}{l^2 - d^2} = \frac{1}{f}$$

$$4lf = l^2 - d^2$$

$$f = \frac{l^2 - d^2}{4l}$$

Relationship between the height of the object and height of the two images

Magnification of the image formed when the lens is at position A; $m_A = \frac{AI}{OA} = \frac{h_1}{h}$

Magnification of the image formed when the lens is at position B; $m_B = \frac{BI}{OB} = \frac{h_2}{h}$

$$m_A \times m_B = \frac{AI}{OA} \times \frac{BI}{OB}$$

But $OA = BI$ and $AI = OB$.

$$\frac{h_1}{h} \times \frac{h_2}{h} = \frac{AI}{OA} \times \frac{OA}{BI}$$

$$\frac{h_1 h_2}{h^2} = 1$$

$$h^2 = h_1 h_2$$

$$h = \sqrt{h_1 h_2}$$

Example.

A luminous object and the screen are placed on an optical bench and a converging lens is placed between them to show a sharp image of the object on the screen. The linear magnification of the image is found to be 3. The lens is now moved 20cm near the screen and a sharp image is again formed on the screen. Calculate the focal length of the lens.

solution

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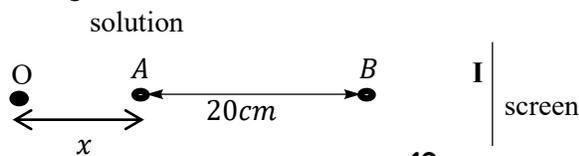
$$OA = BI = x$$
$$m = \frac{v}{u} = \frac{AI}{OA} = \frac{AB + BI}{OA}$$
$$3 = \frac{20 + x}{x}$$
$$3x = 20 + x$$
$$x = 10cm$$
$$u = 10cm, v = 20 + 10 = 30cm$$
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
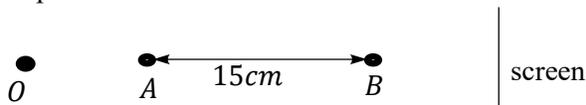$$\frac{1}{10} + \frac{1}{30} = \frac{1}{f}$$
$$f = 7.5cm$$

Alternatively; Using; $f = \frac{l^2 - d^2}{4l}$

$$d = 20cm, \qquad l = 10 + 20 + 10 = 40cm$$
$$f = \frac{40^2 - 20^2}{4 \times 40} = 7.5cm$$

Questions

1. In the diagram below the image of the object at O is formed on the screen when a convex lens is placed either at A or B.


If A and B are 15cm apart and the distance between the object and screen is 45cm, find the;
      (i)   focal length of the lens.
      (ii)   magnification of the image when the lens is at B.

2. A thin convex lens is placed between an object and a screen that are kept fixed at a distance 64cm apart. When the position of the lens is adjusted, a clear focused image is obtained on the screen for two lens positions that are 16cm distant apart.
      (i) Draw a ray diagram to show the formation of the images in the two lens positions.
      (ii) Find the focal length of the lens
      (iii) Find the magnification produced in each lens position.

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Experiments for determining the focal length of a convex lens.
**(a) Using an illuminated object and a plane mirror.**
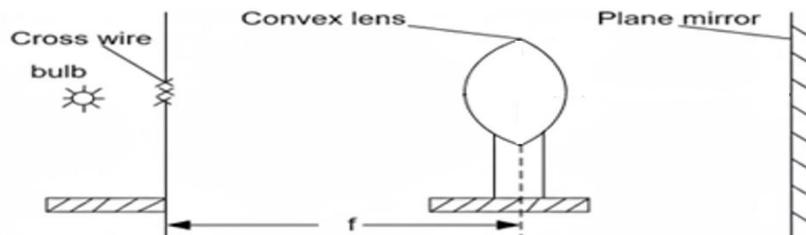
The apparatus are arranged as shown above.

The cross wires are illuminated with the light of bulb. The Position of the convex lens is adjusted between the screen with cross wires and the plane mirror until a sharp image of the cross wires is formed on the screen.
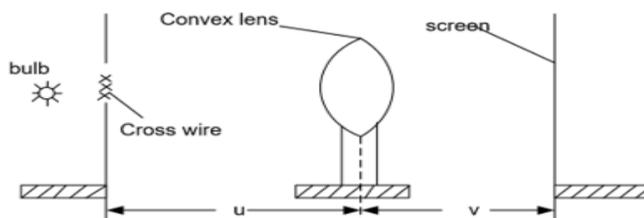
The distance. f between the convex lens and the screen is measured. This gives the focal length of the lens.

(b) Using a plane mirror and the no parallax method.
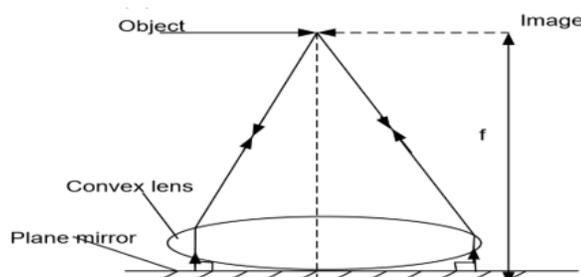
A plane mirror is placed on a horizontal surface with its reflecting surface facing upwards. The convex lens whose focal length is to be determined is placed on the plane mirror as shown.

A pin is clamped horizontally on the retort with its tip lying along the principal axis of the lens. While observing from above, the position of the lens is adjusted vertically until the pin coincides with its image and there is no parallax between the object and the image.

The distance, f between the pin and the lens is measured. This gives the focal length of the convex lens.

(c) Using the illuminated object and the screen (use of the lens formula involving graphical analysis)

The screen with cross wires, the convex lens and the white screen are arranged as shown above. The bulb placed behind the cross wires is lit.

Starting with a measured distance, u, the position of the screen is adjusted until a sharp image of the cross wires is formed. Distance, v between the convex lens and the screen is measured and recorded.

The procedures are repeated with different values of u and the corresponding values are recorded.

Results are tabulated including values of uv and $u + v.$
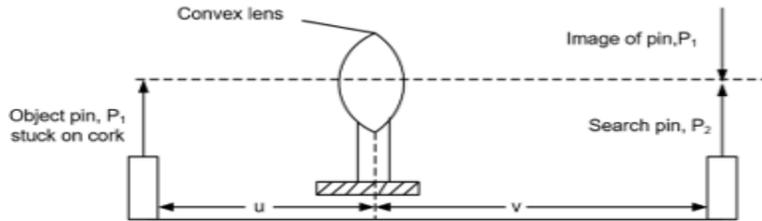
A graph of uv against $(u + v)$ is plotted and the slope of graph is calculated.

The focal length of the lens is equal to the slope.

Alternatively:

A graph of $\frac{1}{u}$ against $\frac{1}{V}$ can be plotted. The intercepts C_1 and C_2 on the axes are read and noted. The focal length of the mirror is obtained from $f = \frac{1}{2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$

(d)   **Using two optical pins and the no parallax method.**

The object pin **P₁** is placed infront of a convex lens so that its tip lies along the principal axis of the lens.

The search pin **P₂** placed behind the lens is adjusted until it coincides with the image of pin P_1 and there is no-parallax method.
The distance **v** of pin **P₂** from the lens is measured.
The procedure is repeated using different values of **u** and the results are tabulated including values of **uv** and **$u + v$.**
A graph of **uv** against **$u + v$** is plotted and the slope of the graph is equal to the focal length **f** of the convex lens.

(e) Using the displacement method.
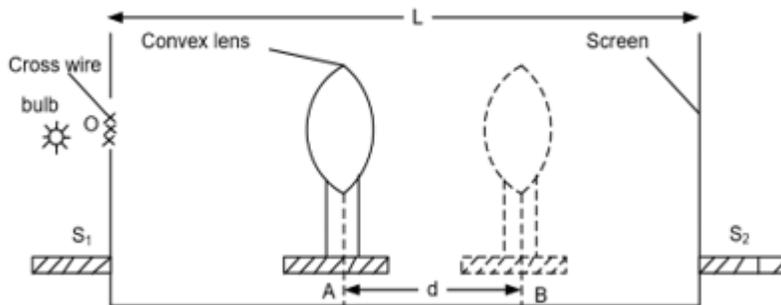
The screen(S_1) with cross wires, the convex lens and the white screen(S_2) are arranged as shown above.
The cross wires are illuminated by the bulb and the position of the lens is adjusted until a sharp magnified image of the cross wires is formed on screen(S_2). Distance, l between the screens is measured and position A of the lens is noted.
Keeping the position of the two screens fixed, the lens is moved towards screen(S_2) until a sharp diminished image is formed on screen(S_2). The new position B of the lens is noted.
Distances OA and OB are measured and distance, d is determined from $d = OB - OA$.
The procedure is repeated with different lengths, l and the corresponding values of d are recorded.
Results are tabulated including values of $(l^2 - d^2)$.
A graph of $(l^2 - d^2)$ against l is plotted and its slope is determined as S.
The focal length of the lens is obtained from $\frac{S}{4}$.

Note: This method is used in determining the focal length of a convex lens whose surfaces are inaccessible (like the lens fixed in a telescope or eye piece tube).
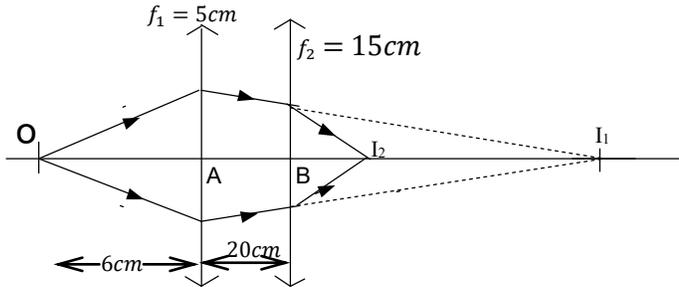
Cases involving more than one lens.

1. Two thin convex lenses A and B of focal lengths 5cm and 15cm respectively are placed coaxially 20cm apart. If an object is placed 6cm from A on the side remote from B.
   - (i)   Sketch a ray diagram to show the formation of the final image.
   - (ii) Find the position, magnification and nature of the final image.

   Solution

   (i)

   (ii)   Considering action of the lens A (Lens A forms the image of object O at I_1)

   $$f_1 = 5cm, \qquad u_1 = 6cm, \qquad v_1 = ??$$

   $$\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1}$$

   $$\frac{1}{5} = \frac{1}{6} + \frac{1}{v_1}$$

   $$v_1 = 30cm$$

   Considering action of the lens B (Image I_1 acts as the object for lens B to form the final image at I_2). I_1 acts as a virtual object for lens B.

   $$f_2 = 15cm, \qquad u_2 = -(30 - 20) = -10cm, \qquad v_2 = ??$$

   $$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$$

   $$\frac{1}{15} = \frac{1}{-10} + \frac{1}{v_1}$$

   $$v_2 = 6cm$$

   The final image is formed 6cm behind lens B.

   Magnification; $m = m_1 \times m_2$

   $$m = \frac{v_1}{u_1} \times \frac{v_2}{u_2}$$

   $$m = \frac{30}{6} \times \frac{6}{10} = 3$$

   The magnification of the final image is 3.
   Nature: The final image formed is real and magnified.

2. A thin convex lens **X** of focal length **10cm** and a thin concave lens **Y** of focal length **15cm** are placed coaxially **50cm** apart. If an object is placed **12cm** from **X** on the side remote from **Y.**
   - (i)   Sketch a ray diagram to show the formation of the final image.
   - (ii) Find the position, nature and magnification of the final image.

   Solution

(i)

$f_1 = 10cm$

$f_2 = 15cm$

O   X   Y   I₁   I₂

u_2

v_2

12cm   50cm

v_1
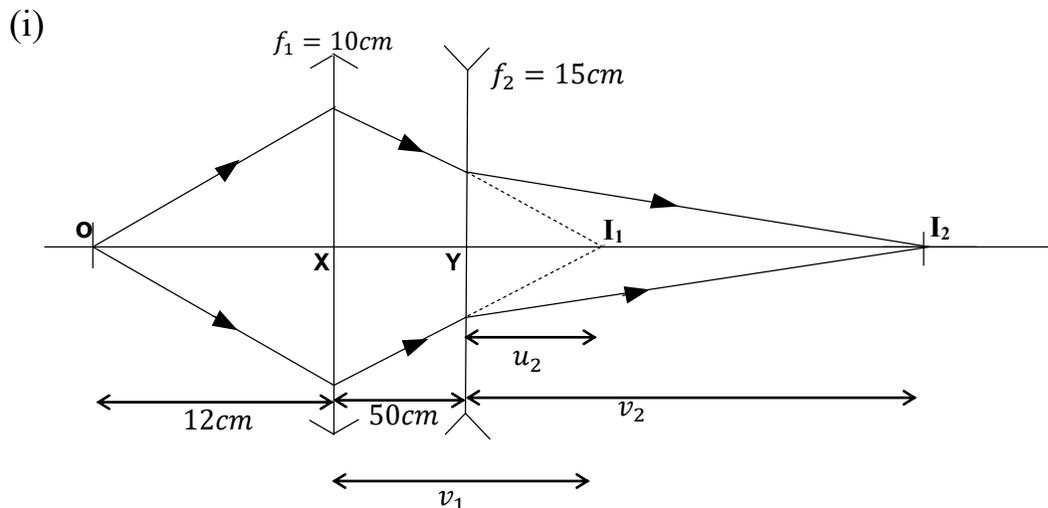
(ii)   Considering action of the convex lens X (Lens X forms the image of object O at I₁)

$$f_1 = 10cm, \qquad u_1 = 12cm, \qquad v_1 =??$$

$$\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1}$$

$$\frac{1}{10} = \frac{1}{12} + \frac{1}{v_1}$$

$$v_1 = 60cm$$

Considering action of the concave lens Y (Image I₁ acts as a virtual object for lens Y to form the final real image at I₂).

$$f_2 = 15cm, \qquad u_2 = -(60 - 50) = -10cm, \qquad v_2 =??$$

$$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$$

$$\frac{1}{-15} = \frac{1}{-10} + \frac{1}{v_1}$$

$$v_2 = 30cm$$

The final image is formed 30cm from lens Y.

Magnification; $m = m_1 \times m_2$

$$m = \frac{v_1}{u_1} \times \frac{v_2}{u_2}$$

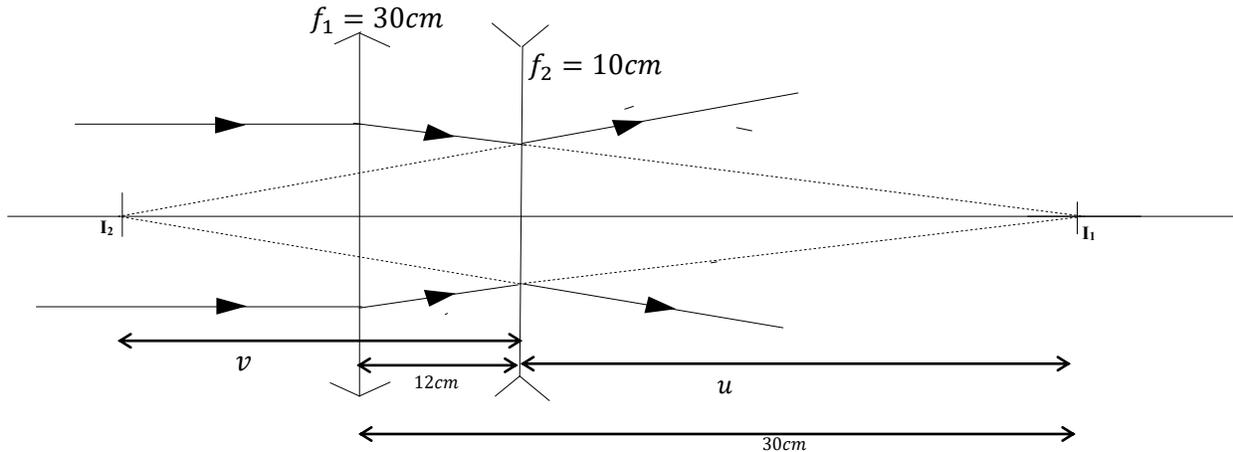$$m = \frac{60}{12} \times \frac{30}{10} = 15$$

The magnification of the final image is 15
Nature: The final image formed is real and magnified.

3. Light from a distant object is incident on a converging lens of focal length 30cm placed 12cm infront of a diverging lens of focal length 10cm. Determine the position and nature of the final image.

    Solution

(i)

(ii)   Considering action of the converging lens: The image of the distant object is formed at I_1 the focal point of the lens.

Considering action of the diverging lens: Image I_1 acts as a virtual object for the concave lens to form the final image at I_2.

$$f_2 = 10cm, \qquad u = -(30 - 12) = -18cm, \qquad v = ??$$

$$\frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$$
$$\frac{1}{-10} = \frac{1}{-18} + \frac{1}{v}$$
$$v = -22.5cm$$

The final image formed is virtual and 22.5cm from lens.

Exercise.
1. A thin converging lens A of focal length 6cm and a thin diverging lens B of focal length 15cm are placed coaxially 14cm apart. If an object is placed 8cm from A on the side remote from B. Find the position and magnification produced by the lens B. **(Ans: 30cm from lens B, 9)**
2. A thin converging lens P of focal length 20cm and a thin diverging lens Q of focal length 30cm are placed coaxially 10cm apart. If an object 4cm tall is placed 70cm from Q on the side remote from Q.
       (i)   Find the position final image.
       (ii)   The height of the final image. **(Ans: 60cm, 6cm)**
3. An object is placed 24cm in front of a convex lens P of focal length 6cm. When a concave lens Q of focal length 12cm is placed beyond lens P, the screen has to be 10cm away from lens P so as to locate the real image formed. Draw a ray diagram Find the distance between the two lenses P and Q.
       (Ans: 4cm)
4. A lens L_1 casts a real image of a distant object on a screen placed at a distance 15cm away. When another lens L_2 is placed 5cm beyond lens L_1, the screen has to be shifted by 10cm further away to locate the real image formed. Find the focal length and the type of lens L_2. **(Ans: $f = -20cm$)**
5. Light from a distant object is incident on a diverging lens of focal length 9cm placed 6cm in front of a converging lens of focal length 10cm. Determine the position and nature of the final image.

Experiments to determine the focal of a concave lens.

The concave lens cannot form a real image on the screen. Therefore the concave lens is used together with a converging mirror (concave mirror) or a converging lens (convex lens) inorder to produce a real image on the screen.

(a) Determination of the focal the focal length of a concave lens by using a convex lens.

   (i)   Using a convex lens not in contact with the concave lens.
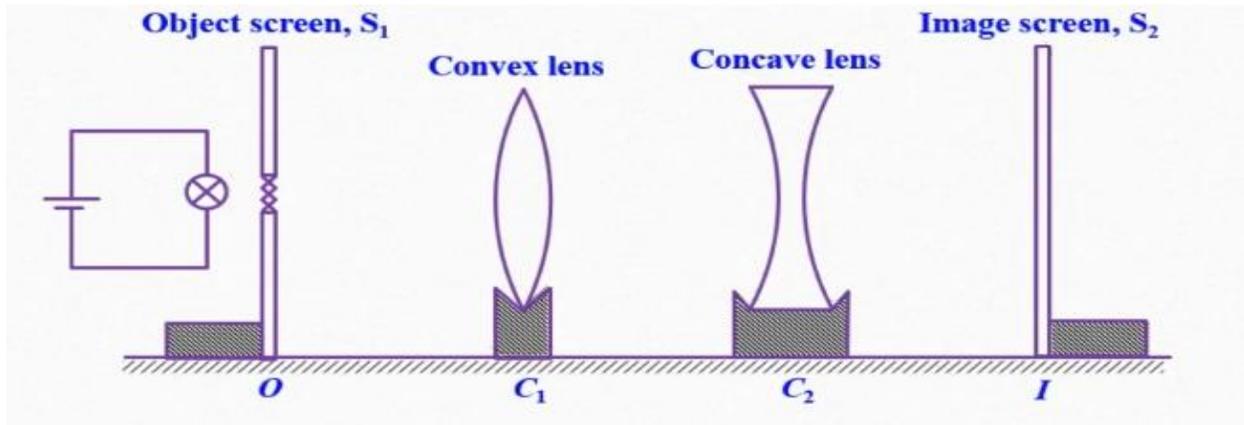
The screen with cross wires, the convex lens and the screen are arranged as shown above.

First without the concave lens, the convex lens is placed such that distance OC_1 is greater than the focal length of focal length of the convex lens.
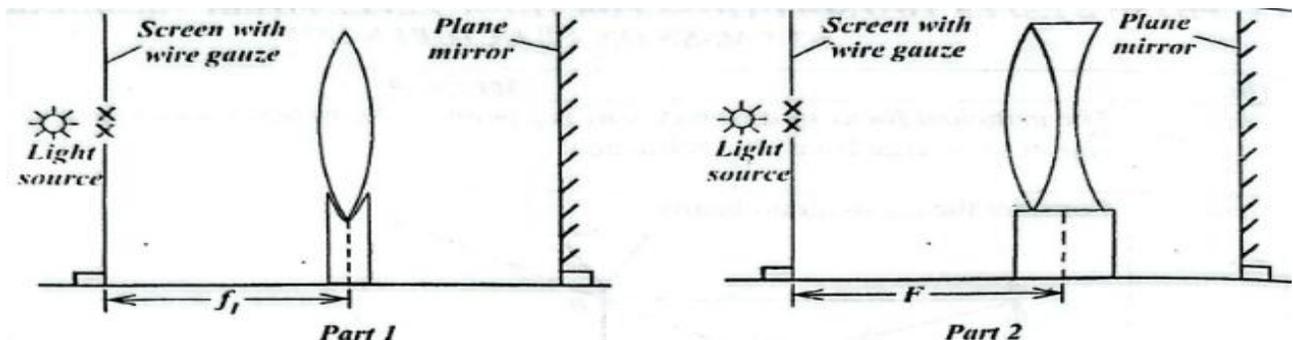
The position of the screen (S_2) is adjusted until a sharp image of the cross wires is formed on it. Distance C_1I is measured and recorded.

The concave lens whose focal length is required is placed between the convex lens and the screen S_2. The screen(S_2) is again adjusted until a sharp image is formed on the screen. Distance C_1C_2 and C_2I is measured.

The focal length of the concave lens is determined by using the formula; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where

$$u = -(C_2I - C_1C_2) \text{ and } v = C_2I.$$

   (ii)   Using a convex lens in contact with the concave lens and a plane mirror

The screen with cross wires, the convex lens and the plane mirror are arranged as shown above in part 1.
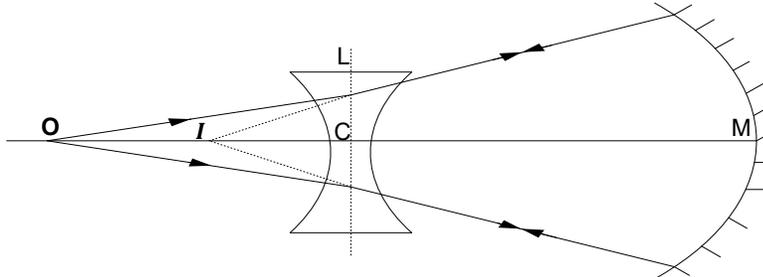
First without the concave lens, the position of the convex is adjusted until a sharp image of cross wires is formed on the screen. The distance between the lens and the screen is measured and is the focal length of the convex lens recorded as (f_1).

The concave lens whose focal length is required is placed in contact with the convex lens as shown in part 2. The combined lenses are adjusted until a sharp image is again formed on the screen. The distance

between the combined lenses and the screen is measured and is the focal length of the combined lenses recorded as (f).

The focal length of the concave lens is determined by using the formula; $\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$.

**(b) Determination of the focal the focal length of a concave lens by using a concave mirror.**

The screen with cross wires at O, the concave lens(L) and a concave mirror(M) of known radius of curvature, r are arranged coaxially as shown.
The position of the concave mirror is adjusted until the sharp image is formed on the object screen. Distances OC and CM are measured.

The focal length of the concave lens is determined by using the formula; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where

$v = -(r - CM)$ and $v = OC$.

Theory of the experiment
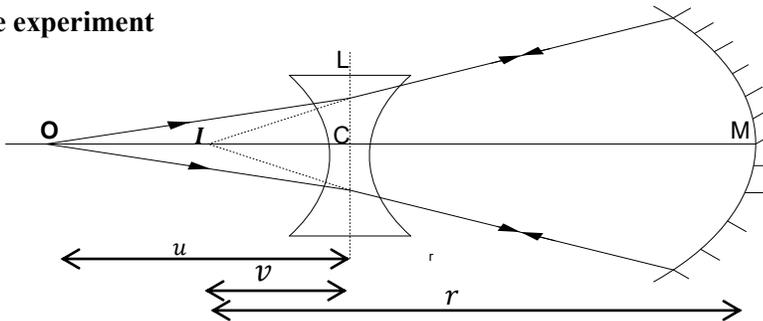
Light rays from the object at O incident on the concave lens is diverged by lens onto the concave mirror. When the object and the image are coincident at O, light rays refracted by the lens must be incident normally on mirror M, thus the reflected rays trace back their original path and pass through the centre of curvature of the mirror. Therefore Point I the position of the virtual image is also the centre of curvature of the concave mirror. The image distance, $v = IC = IM - CM = r - CM$ and the object distance $u = OC$. Image I is virtual, thus a negative is imposed on the numerical value of v by sign convention.
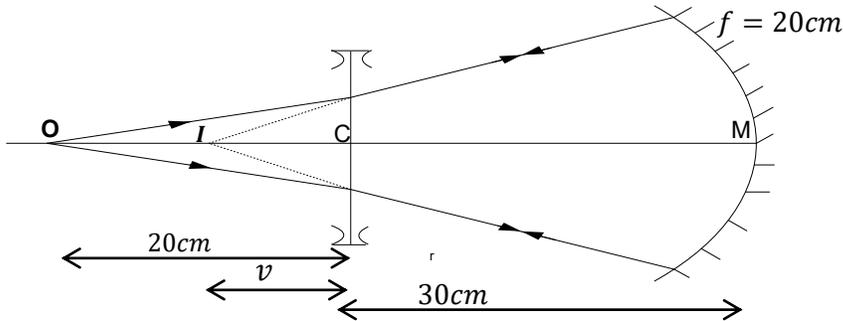**Note**: If the radius of curvature of the concave mirror is not known, it can be determined using the same arrangement but without the concave lens.

Examples
1. An object is placed 20cm in front of a diverging lens that is coaxially with a concave mirror of focal length 20cm. When the concave mirror is 30cm from the lens, the final image coincides with the object.
    (i)   draw a ray diagram to show how the image is formed.
    (ii)   determine the focal length of the diverging lens.

Solution

(i)

$f = 20cm$

O   I   C   M

20cm

v

30cm

(ii)   Considering action of the concave mirror

$IM = r = 2f = 40cm$ (radius of curvature of the concave mirror)

Considering action of the concave mirror

$$u = 20cm, \qquad v = -(r - CM) = -(40 - 30) = -10cm$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
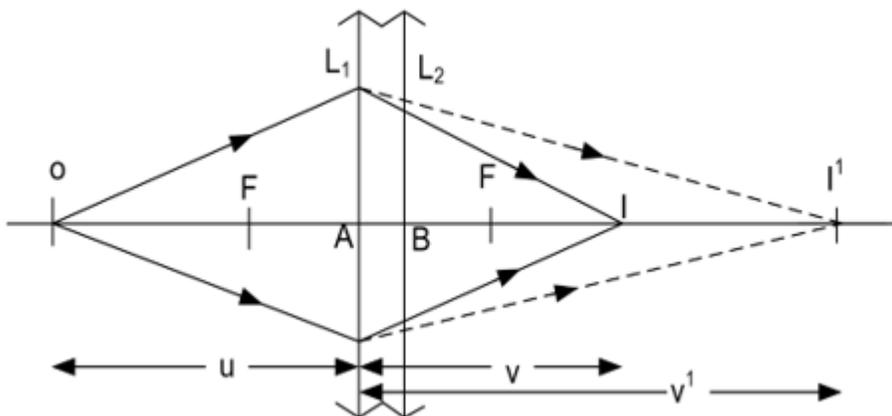
$$\frac{1}{f} = \frac{1}{20} + \frac{1}{-10}$$

$$\frac{1}{f} = \frac{-1}{20} \Rightarrow f = -20cm$$

The focal length of the concave lens is 20cm.

Combined focal length of two thin lenses in contact.

Consider two lenses A and B of focal length f_1 and f_2 respectively placed in contact.

Considering action of lens A

$$\frac{1}{u} + \frac{1}{v^1} = \frac{1}{f_1} \quad \ldots\ldots(1)$$

Considering action of lens B: (Image I^1 acts as a virtual object for lens B to form the image I)

$$\frac{-1}{v^1} + \frac{1}{v} = \frac{1}{f_2} \quad \ldots\ldots\ldots(2)$$

Adding equations (1) and (2)

$$\left(\frac{1}{u} + \frac{1}{v^1}\right) + \left(\frac{-1}{v^1} + \frac{1}{v}\right) = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$; where f is the focal length of the two combined lenses.

Note:
  (i)   This formula for applies for any two lenses in contact such as two diverging lenses or a converging and diverging lens.
  (ii)   When the formula is used, the signs of the focal length must be considered (the negative is imposed on the focal length of the concave)

Examples.
  1. A convex lens of focal lens of focal 18cm is placed in contact with a concave lens of focal length 12cm. Find the combined focal length of the lenses.
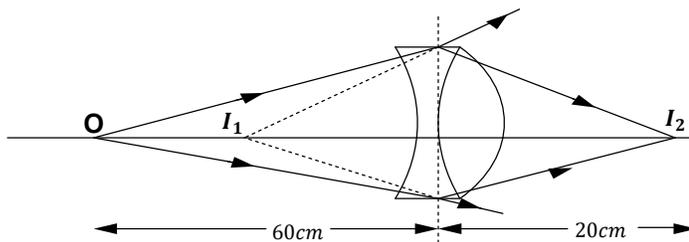   Solution

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{18} + \frac{1}{12} \quad \Rightarrow \quad \frac{1}{f} = \frac{5}{36}$$

$$f = 7.2cm$$

  2. A small object is placed at a distance of 60cm infront of a diverging lens of focal length 30cm. A converging lens is then placed in contact with the diverging lens on the side face the screen. If a real image is formed on the screen placed 20 cm from the combined lenses, find the focal length of the converging lens.

Solution

Considering the action of the concave lens: $u = 60cm$, $f_1 = 60cm$

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-30} = \frac{1}{60} + \frac{1}{v} \quad \Rightarrow \quad \frac{1}{v} = \frac{1}{20}$$
$$v = -20cm$$

Considering the action of the convex lens: The virtual image I_1 acts as a real object for the converging lens to form image I_2.

$u = 20cm, \ v = 20cm$

$$\frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f_2} = \frac{1}{20} + \frac{1}{20} \quad \Rightarrow \quad \frac{1}{f_2} = \frac{1}{10}$$
$$f_2 = 10cm$$

Alternatively: Considering the combination of the lenses.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{60} + \frac{1}{20}$$

$f = 15cm$ (this is the combined focal length of the lenses)

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

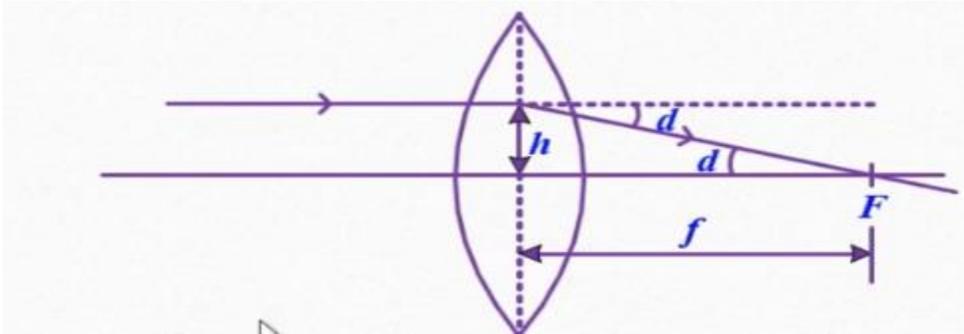$$\frac{1}{15} = \frac{1}{-30} + \frac{1}{f_2}$$

$$f_2 = 15cm$$

The focal length of the convex lens is 15cm.

Thin lens formula

The thin lens formula gives the relationship between focal length of a lens, its refractive index and the radii of curvature of the lens surfaces.
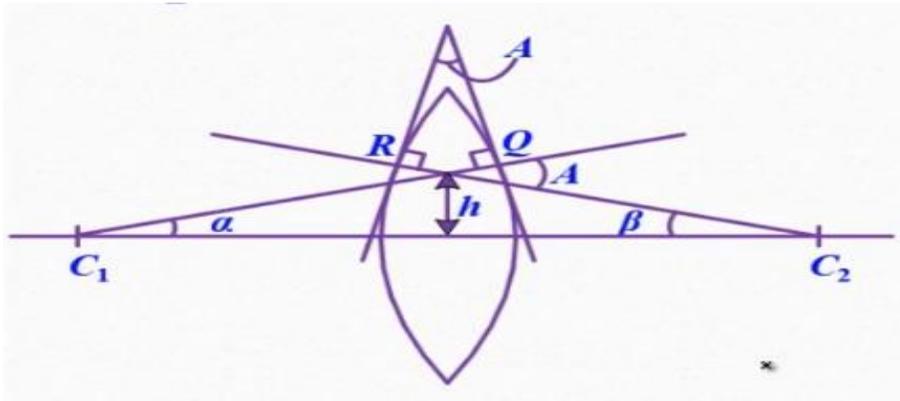
Consider light from a distant incident on the convex lens of focal length f at a height h above the principal axis.

$$\tan d = \frac{h}{f}$$

d is a small angle in radians, thus $\tan d \approx d = \frac{h}{f}$.

Consider normals Q and R to the lens surfaces that pass through centres of curvature C_1 and C_2 respectively.

$$A = \alpha + \beta$$

$$\tan \alpha = \frac{h}{r_1} \quad \text{and} \quad \tan \beta = \frac{h}{r_2}$$

α and β are small angles in radians, thus $\tan \alpha \approx \alpha = \frac{h}{r_1}$ and $\tan \beta \approx \beta = \frac{h}{r_2}$

$$A = \frac{h}{r_1} + \frac{h}{r_2}$$
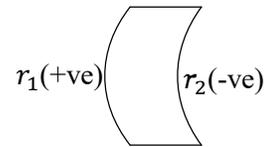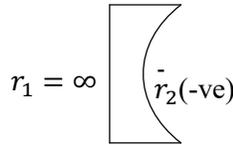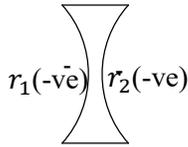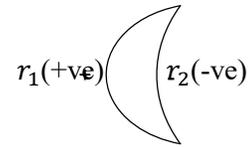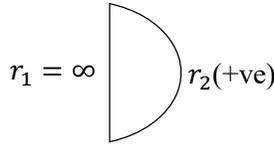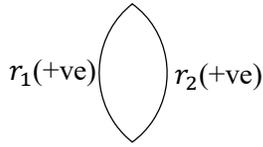
For a small angle prism, $d = (n-1)A$

$$\frac{h}{f} = (n-1)\left(\frac{h}{r_1} + \frac{h}{r_2}\right)$$
$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

While using the thin lens formula, sign convention is put into consideration on the focal length f and the radii of curvature r_1 and r_2 as shown below.

In numerical work, the focal length of the biconvex lens, plano convex and the converging meniscus are considered positive while the focal length of the biconve lens, plano concave lens and the diverging meniscus are considered negative.
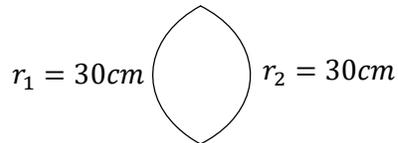
Examples

1. A thin lens with surfaces of radii of curvature 30cm is to be made from the glass with refractive index 1.6. what will be the focal length of the lens if it is;
    (i) Biconvex.
    (ii) Biconcave.

    Solution

(i)

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = (1.6-1)\left(\frac{1}{30} + \frac{1}{30}\right)$$
$$f = 25cm$$

The focal length of the biconvex lens is 25cm.

(ii)

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = (1.6-1)\left(\frac{-1}{30} + \frac{-1}{30}\right)$$
$$f = -25cm$$

The focal length of the biconcave lens is 25cm.

2. A converging meniscus with radii of curvature 15cm and 6cm is made of glass material of refractive index 1.5. Calculate its focal length when surrounded by:
   (i)   Air.
   (ii) a liquid of refractive index 1.2.
                      solution

   (i)
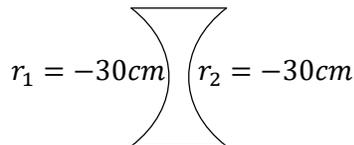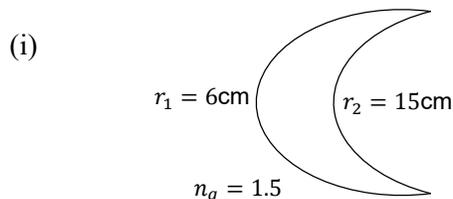

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{6} + \frac{1}{-15}\right)$$
$$f = 20cm$$

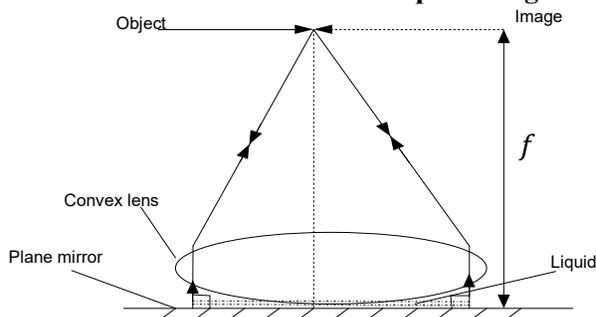   (ii)

$$\frac{1}{f} = \left(\frac{n_g}{n_l} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = \left(\frac{1.5}{1.2} - 1\right)\left(\frac{1}{6} + \frac{1}{-15}\right)$$
$$f = 40cm$$

Experiment to determine the refractive index of a liquid using a convex lens and a plane mirror.

A plane mirror is placed on a horizontal surface with its reflecting surface facing upwards and a convex lens is placed above it.
A pin is clamped horizontally on a retort stand with its tip along the principal axis of the lens.
The pin is adjusted vertically to locate the position where the pin coincides with its image and there is no parallax between the object and the image.
The distance of the pin from the lens is measured recorded as f_1.
The lens is removed from the mirror and a small quantity of the test liquid is poured on the mirror.
The lens is placed back on the mirror and the position of the pin is again adjusted vertically until the pin coincides with its image again. The distance of the pin from the lens is measured and recorded as f.

The focal length of the liquid is got from; $\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$.

The refractive index of the liquid is got from; $n = 1 - \frac{r}{f_2}$

Examples

1. A converging lens is placed on top of a liquid of refractive index 1.4 poured on a plane mirror. By adjusting the position of pin O, the position is found where O coincides with it image. If both surfaces of the lens have radii of curvature of 15cm and refractive index of the lens 1.5. Determine the position of coincidence

Solution

For the liquid lens (When the convex lens is placed on top of the liquid, it forms a diverging meniscus lens with the liquid as the material).
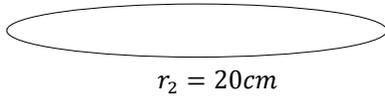
$r_1 = -20cm$

Liquid

$r_2 = \infty$
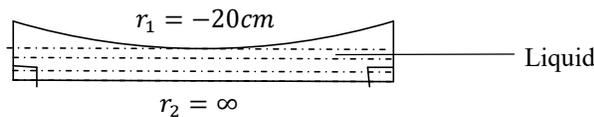
$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{-15} + \frac{1}{\infty}\right)$$

$$\frac{1}{f_l} = \frac{-2}{75}$$

For the bi convex lens

$r_1 = 20cm$

$r_2 = 20cm$

$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_g} = (1.5 - 1)\left(\frac{1}{15} + \frac{1}{15}\right)$$

$$\frac{1}{f_g} = \frac{1}{15}$$

Considering the combination of the liquid and glass lens (the point of coincidence is located at the focal point of the liquid- convex lens combination).

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$$

$$\frac{1}{f} = \frac{-2}{75} + \frac{1}{15} = \frac{1}{25}$$

$$f = 25cm$$

Point of coincidence is 25cm from the convex lens.

2. An equi-convex lens of refractive index 1.5 is placed on a horizontal plane mirror. A clamped pin coincides with its own image when it is 60cm above the lens. When the space between the mirror and the lens is filled with a liquid, the pin has to be raised by 20cm for coincidence to occur again. What is the refractive index of the liquid?

Solution

Considering the equiconvex lens.

$r_1 = r$

$r_2 = r$
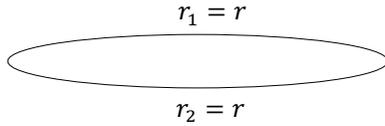
$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{60} = (1.5 - 1)\left(\frac{1}{r} + \frac{1}{r}\right)$$

$$\frac{1}{30} = \frac{2}{r}$$

$$r = 60cm$$
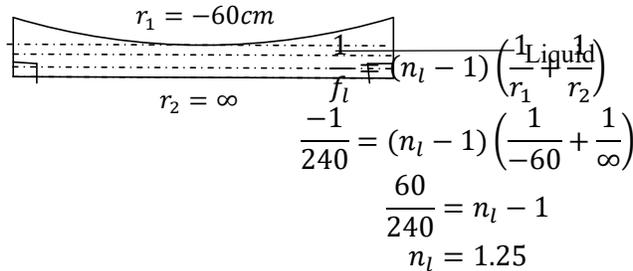
Considering the combination of the liquid and glass lens.

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$$

$$f = 60 + 20 = 80cm$$

$$\frac{1}{80} = \frac{1}{f_l} + \frac{1}{60}$$

$$\frac{1}{f_l} = \frac{-1}{240}$$

Considering the liquid lens

$r_1 = -60cm$

$r_2 = \infty$

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{-1}{240} = (n_l - 1)\left(\frac{1}{-60} + \frac{1}{\infty}\right)$$

$$\frac{60}{240} = n_l - 1$$

$$n_l = 1.25$$

The refractive index of the liquid is 1.25.

Exercise

1. An object is placed 12cm in front of a diverging lens placed coaxially with a concave mirror of focal length 14cm. When the concave mirror is 20cm from the lens, the final image coincides with the object. Determine the focal length of the diverging lens. **(Ans: 24cm)**

2. A concave lens of focal length 15cm is placed 26cm infront of concave mirror of focal length 18cm. Draw a ray diagram to show how the final image is formed and calculate the distance from the lens at the object will coincide with its image. **(Ans: 30cm)**

3. A thin converging lens of focal length 8cm is placed in contact with a diverging lens of focal length 10cm. what is the focal length of the combined lenses. **(Ans: 40cm)**

4. A thin converging lens of focal length 6cm is placed in contact with a diverging lens of focal length 10cm.
   (i)   What is the combined focal length of the two lenses.
   (ii) Find the position of the final image if the object is placed 18cm from the lenses.

5. A thin concave lens is placed in contact with a convex lens made of glass of refractive index 1.5 and its surfaces have radii of curvature 10cm and 15cm. If an object placed 75cm in front of the lens combination gives rise to an image on a screen at a distance 50cm from the combination, calculate the focal length of the concave lens.
6. A diverging meniscus lens has surfaces with radii 25cm and 20cm. If the refractive index of the material of the lens is 1.5, calculate the focal length of lens.
7. A thin equiconvex lens of refractive index 1.5 whose surfaces have radius of curvature 24cm is placed on a horizontal plane mirror. When the space between the mirror and the lens is filled with a liquid, a pin held 40cm vertically above the mirror is found to coincide with its own image. What is the refractive index of the liquid **(Ans: 1.4)**
8. A small quantity of a liquid of refractive index 1.4 is poured on a horizontal plane mirror and a biconvex lens of focal length 30cm and refractive index 1.5 is then placed on top of the liquid. The pin is moved along the axis of the lens until no parallax between it and its image find the distance between the pin and the lens. **(Ans: 50cm)**
9. The curved face of a Plano convex lens of refractive index 1.5 is placed in contact with a plane mirror. A pin placed at a distance 20cm coincides with its image. A film of a liquid is now introduced between the lens and the plane mirror. Then the coincidence of the pin and its image is found to be at a distance 100cm. Calculate the refractive index of the liquid **(Ans: 1.4)**
10. Two equiconvex lenses of focal length 20cm and made of glass of refractive index 1.6 are placed in contact and the space between them is filled with a liquid of refractive index 1.4. Find the focal length of the lens combination. **(Ans: 15cm)**

(skip 5 pages)

Defects in images (Aberrations)

This is the distortion of images formed by either spherical mirrors or lenses.

Mirrors and lenses form images that differ in shape and color from that of the object. Such defects are known as aberrations in images. These aberrations are of two kinds namely: Spherical aberration and chromatic aberration.

Spherical aberration

This is the defect of images in lenses or mirrors which occurs when a wide beam of light is incident on a mirror or a lens of wide aperture resulting into formation of blurred/distorted images.
Explanation
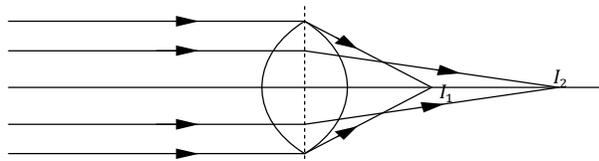
When a wide beam of light is incident on a convex lens of wide aperture, the marginal rays converged to a focus close to the lens while central rays are brought to a focus further away from the lens. This results into formation of images that are blurred or distorted at different foci along the principal axis of the lens.

Correction of spherical aberration

(i) In lenses, Spherical aberration can be minimized using an opaque disc with a central hole to cut off marginal rays. Since only marginal rays are incident on the lens, a single image is formed at a single focus.
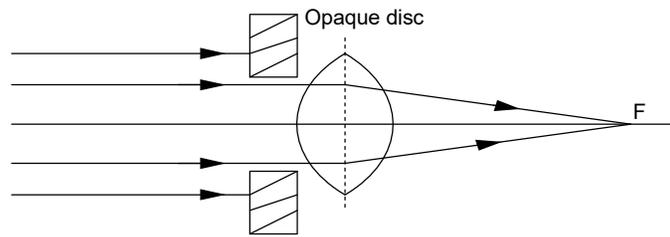
The use of the opaque disc has a disadvantage that the brightness of the image is reduced since the intensity of the incident light cut down.

(ii)   In mirrors, spherical aberration can be minimized by using a parabolic mirror. The parabolic mirror converges a wide parallel beam of light incident onto its surface to a single focus thus forming a single clear image.

Chromatic aberration

This is the defect of images in lenses which occurs when white light is incident on a lens resulting into formation of images with coloured edges.

Explanation
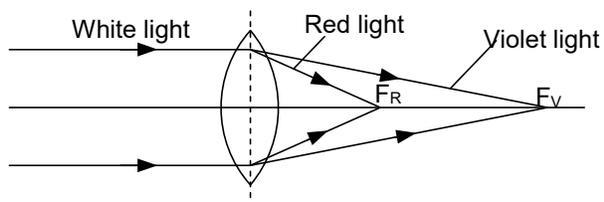
When white light is incident on a lens, the different colour components are refracted by different amounts because the lens has different refractive indices for each colour thus images corresponding to the different colour components are formed at different foci along the principal axis. This results into formation of an image with coloured images with red light being the least deviated and violet light the most deviated.

Correction of chromatic aberration

Chromatic aberration can be reduced by using an achromatic doublet. The achromatic doublet consists of a convex lens combined with a concave lens made from different glass materials. The convex lens deviates the rays while the concave lens nullifies the deviation.

Conditions for lenses that form an achromatic doublet
  ✓ The two lenses should have opposite dispersive powers (one lens should be converging and the other lens should be diverging)

  ✓ The two lenses should be made from materials of different refractive indices.

  ✓ The ratio of the focal lengths of the two lenses should be numerically equal to ratio of the dispersive powers of the lens materials.

  ✓ One surface of the lens should have the same radius of curvature as the other to enable easy cementing.