PASS MATHEMATICS

ITEM 1

MTN Uganda has been tracking its mobile money subscribers over the past 5 years. The number of active users has been growing exponentially. The company started with 850,000 active users in 2020 and has experienced a constant annual growth rate.

You are a data analyst at MTN Uganda tasked with analyzing this growth to help the company plan for future infrastructure needs. In 2023, the number of active users was recorded as 1,309,500.

Tasks:

a) Utilizing the knowledge of logarithms, determine the annual growth rate of MTN mobile money subscribers. Express your answer as a percentage to 1 decimal place.

b) Predict the number of active subscribers MTN Uganda will have in 2025.

c) How many years would it take for the number of subscribers to reach 5 million users?

d) how can exponential growth models can be used by telecommunications companies for planning resource allocation.

ITEM 2

Kiira Motors Corporation, Uganda's vehicle manufacturer, is designing components for their new electric vehicle model. The engineers need to calculate exact dimensions for specific components to ensure proper fitment.

You are an engineering intern at Kiira Motors working on the battery bracket design. The chief engineer has given you measurements for a triangular mounting bracket that involves surds. The diagonal support brace of the bracket needs to be exactly √50 cm long. Two sides of the triangular bracket are (5 + 3√2) cm and (7 − 2√2) cm long.

Tasks:

a) Express the length the bracket diagonal support in the simplified form $a\sqrt{b}$, where a and b are integers and b is not divisible by a perfect square.

b) Establish the exact length of material needed for both sides combined.

c) If the perimeter of the entire triangular bracket must be exactly 25 cm, calculate the length of the third side.

ITEM 3

A farmer in Mbarara has 800 square meters of land for growing beans and maize. Based on local market prices, she makes a profit of UGX 5,000 per square meter of beans and UGX 3,000 per square meter of maize. Due to crop rotation requirements, she must plant at least 200 square meters of beans and at least 300 square meters of maize. The farmer has approached you he needs help of how he can maximize her profits while meeting all the constraints.

Tasks:

a) Help the farmer to represent all the constraints as inequalities including the total profit

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b) Determine the maximum possible profit and the corresponding areas of land to be allocated for each crop.

c) If the profit per square meter for maize increases to UGX 4,000, how would this change your recommendation? Show your workings.

ITEM 4

The National Water and Sewerage Corporation (NWSC) is designing a water distribution system for three neighboring communities in Kampala. Each community has different water requirements and infrastructure constraints. A consultant engineer working on this project wants to determine the optimal flow rates for each community.

The water distribution system is modeled by the following equations

$X+2Y+Z=2400$ **(Total available water supply in liters per minute)**

$2X+Y+3Z=3900$ **(Pressure balancing equation)**

$3X+4Y+2Z=5100$ **(Flow optimization equation)**

Where X, Y and Z represent the flow rate in liters per minute in communities A, B and C respectively

The polynomial equation $P(X)=X3-7X2+14X-8$ **models the operational efficiency of the pumping systems**

Tasks

a) Help the engineer to determine the optimal flow for each community

b) If the community A's water requirements increase by 200 liters per minute. What adjustments should be made to other communities' supply to maintain the system balance?

c) Determine all the possible value X **where the efficiency of the pumping system is zero**

ITEM 5

A small-scale manufacturer in Jinja produces handcrafted furniture. Their workshop makes tables and chairs from locally sourced wood. Each table requires 5 hours of labor and 8 square meters of wood, while each chair requires 3 hours of labor and 2 square meters of wood. The workshop has 60 hours of labor and 72 square meters of wood available per week.

Tasks

a) Obtain all the constraints as inequalities that can help the production manager to determine the optimal production strategy.

b) The profit is UGX 70,000 per table and UGX 25,000 per chair. Model the total weekly profit as a function of the number of table and the number of chairs

c) Illustrate the feasible region defined by the constraints and use if to find the optimal strategy that maximizes the profit.

PASS MATHEMATICS

ITEM 6

The Entebbe Municipal Council is planning to develop a new neighborhood with roads, residential areas, and a community center. The planning department uses a coordinate system where each unit represents 100 meters. An urban planner, needs to design the road network and determine optimal locations for key facilities. Road A is planned to connect points A(2,3) and B(8,15) and another road B will connect points C(4,5) and D(12,9).

Tasks

a) Help the urban planner to determine

i) The length of the road A in kilometers.

ii) The gradient of the road A and the equation that models the path of the road.

b) Establish whether the two roads A and B intersect, and if so, at what point and the angle between the two roads to the nearest degree.

c) If a community center needs to be located at a point that is equidistant from points A, B, and C. Find the coordinates of this point

ITEM 7

The Uganda National Roads Authority (UNRA) is studying traffic patterns on the Kampala-Jinja highway. They've developed a mathematical model to predict traffic density at different times of the day.

The traffic density function (vehicles per kilometer) at time t hours after 6:00 AM is given by;

$D(t)$ =20t2 + 30t + 15(t2 + 1)(t + 3)

Tasks

As a traffic engineer, you need to analyze this model to optimize traffic flow and plan future road expansions.

a) Representing the function as a sum of partial fractions.

b) By partial fraction decomposition determine the time of day when traffic density is at its minimum

c) If the average speed $v(t)$ of vehicles is related to traffic density by the equation :$v(t)$=80$D(t)$. Determine the time of day when the average speed is maximized

ITEM 8

A renewable energy company is installing solar panels on buildings in Tororo district. To maximize energy collection, they need to determine optimal installation angles based on the building's orientation and the sun's position throughout the year. The company engineer needs to know the ideal mounting angles for a solar project at a school. He finds out that the amount of solar energy collected by the panels is modeled by the function: $E(\theta)$=800sinθ − 200sin2θ+ 100 where E is the energy collected in watt-hours per square meter and θ is the angle of the panels from the horizontal in

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radians and the shadow length L of a vertical pole of height h at time t hours after noon is given by: $L = h\tan(\pi 12t + \pi 4)$

Tasks

a) If the school building has a roof with a pitch of 35° facing south. If the optimal angle for solar panels is 15° relative to the roof surface, determine the absolute angle of the panels from the horizontal.

b) Establish the value of θ that maximizes energy collection.

c) At what time will the shadow be equal to the height of the pole?

ITEM 1

Phresh School Management System is a digital platform used by schools to streamline administrative tasks, track student performance, and manage school activities efficiently. Over the past five years, the number of schools adopting Phresh has been growing exponentially. The system initially had 850,000 registered users in 2020, with a steady annual growth rate.

As a data analyst for Phresh, you are tasked with evaluating this growth to support future infrastructure planning. By 2023, the number of active users had reached 1,309,500.

Tasks:

a) Utilizing the knowledge of logarithms, determine the annual growth rate of Phresh School Management System users. Express your answer as a percentage to one decimal place.

b) Predict the number of active users Phresh will have in 2025.

c) How many years would it take for the number of users to reach 5 million?

d) How can exponential growth models be used by school management systems for planning resource allocation?

ITEM 2

A small-scale bakery in Kampala produces handcrafted cakes and pastries. Their kitchen prepares cakes and muffins using locally sourced ingredients. Each cake requires 5 hours of baking and preparation time along with 8 kilograms of flour, while each muffin requires 3 hours and 2 kilograms of flour. The bakery has a total of 60 hours of available labor and 72 kilograms of flour per week.

Tasks:

a) Obtain all the constraints as inequalities that can help the bakery manager determine the optimal production strategy.

b) The profit is UGX 70,000 per cake and UGX 25,000 per muffin. Model the total weekly profit as a function of the number of cakes and muffins.

c) Illustrate the feasible region defined by the constraints and use it to find the optimal strategy that maximizes the profit.

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PASS MATHEMATICS

ITEM 4

The Ministry of Energy and Mineral Development is implementing a rural electrification project in three neighboring districts. Each district has distinct energy demands and infrastructure challenges. An electrical engineer is assigned to design an efficient power distribution system that ensures adequate supply across all areas. The system is modeled using three equations representing total energy availability, voltage regulation, and transmission efficiency. Additionally, a polynomial equation determines the reliability of transformers used in the project.

Mathematical Model:

The power distribution system is modeled by the following equations:

A+2B+C=2400 (Total available energy supply in kilowatts per hour)

2A+B+3C=3900 (Voltage regulation equation) © 2025 PRE REGISTRATION ASSESSMENT AITEL CONTACT US ON: 0780298043 | 0740739087 | 0758489600 | 0789756728 | 0777139918 All payments are made from Equity Bank | Account Name: Ask Integrated Teachers Examination Bureau (AITEL) LIMITED | Account Number: 1042203234600

3A+4B+2C=5100 (Transmission efficiency equation)

Where A, B, and C represent the energy supply in kilowatts per hour for districts X, Y, and Z respectively.

The polynomial equation $T(A)=A3-7A2+14A-8$ models the reliability of the transformers in the system.

Tasks:

a) Assist the engineer in determining the optimal energy distribution for each district.

b) If district X's energy demand increases by 200 kilowatts per hour, what adjustments should be made to maintain system balance?

c) Determine all possible values of A where the reliability of the transformers is zero.

ITEM 5

The Kampala City Authority is redesigning a section of the central business district to improve traffic flow and pedestrian accessibility. The urban planning team is using a coordinate system where each unit represents 50 meters. A transport engineer needs to plan new roads, determine optimal crossing points, and ensure safe and efficient movement. The proposed Road X will connect points P(5,10) and Q(15,30), while another Road Y will connect points R(8,12) and S(20,24).

Tasks:

a) Assist the transport engineer to determine:

i. The length of Road X in kilometers.

ii. The gradient of Road X and its equation modeling the path of the road.

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b) Check whether Roads X and Y intersect, and if they do, determine the coordinates of the intersection and the angle between the two roads to the nearest degree.

c) If a pedestrian plaza needs to be located at a point equidistant from points P, Q and R, find the coordinates of this point.

MEASUREMENT AND DIMENSIONS OF PHYSICAL QUANTITIES

In a rural Ugandan village, a group of engineers is working on a project to construct a small footbridge over a river to connect two communities. The bridge will be made of locally sourced materials, including wooden beams and steel cables. The engineers propose a formula to calculate the maximum load W the bridge can support: $W=k\sqrt{E}\cdot A\cdot\Delta LL$, , where E is the Young's modulus of the steel cables, A is the cross-sectional area of the cables, ΔL is the elongation under load, L is the original length of the cables, and k is a dimensionless constant. However, the village elders are concerned about the safety of the bridge and want to ensure the formula is scientifically valid before construction begins.

Tasks:

a) Determine whether the proposed formula is dimensionally consistent.

b) Explain to the village elders how the formula ensures the bridge's safety, using the given values:E=2×1011Pa,A=0.01m2,ΔL=0.03m and L=20m. Calculate W when k=2

c) Describe two potential sources of error in measuring ΔL and suggest practical ways to minimize these errors in the field.

STATICS

In a bustling market in Kampala, a vendor is setting up a makeshift stall using a 5-meter-long wooden plank leaning against a wall. The plank has a mass of 15kg and forms an angle of 600 with the ground. The vendor places a 50kg sack of maize 3m from the base of the plank. The ground is uneven, and the coefficient of static friction between the plank and the ground is 0.4. The vendor is worried that the plank might slip, causing the sack to fall and injure someone.

Tasks:

a) Determine the normal reaction force from the wall and the frictional force at the base of the plank.

b) Explain whether the plank will slip under the current conditions. If it will, suggest a practical solution to prevent slipping.

c) Contrast the forces acting on the plank if the angle is reduced to 450. Would this make the stall more stable?

LINEAR MOTION

A taxi is traveling along a straight road at a constant speed of 20$m\backslash s$. Suddenly, a goat runs onto the road 50m ahead. The driver applies the brakes, causing the taxi to decelerate uniformly at 4ms−2. At

the same time, a boda-boda (motorcycle) traveling in the opposite direction at 15m/s is 100m away from the taxi.

Tasks:

a) Determine whether the taxi stops before hitting the goat.

b) Explain whether the taxi and the boda-boda will collide if both continue moving at their current speeds.

c) Describe how the driver's reaction time (assume 0.5s) affects the stopping distance of the taxi.

MOTION UNDER GRAVITY

In a rural Ugandan community, a farmer is using a slingshot to scare away birds from his crops. He launches a small stone at an angle of 300 to the horizontal with an initial velocity of 15ms−1. The stone must clear a 2m tall fence located 20m away. The farmer wants to ensure the stone does not harm any birds beyond the fence.

Tasks:

a) Determine whether the stone clears the fence.

b) Explain how the farmer could adjust the launch angle or velocity to ensure the stone lands safely beyond the fence.

c) Describe the effect of air resistance on the stone's trajectory and how it might affect the farmer's aim.

WORK, ENERGY AND POWER

A small hydroelectric power plant is being constructed near a waterfall in western Uganda. Water falls from a height of 30m at a flow rate of 400kgs−1. The turbine converts 65% of the gravitational potential energy into electrical energy. The plant will supply electricity to a nearby village, but the engineers are concerned about the efficiency of the system.

Tasks:

a) Determine the power generated by the turbine.

b) Explain how the engineers could increase the power output to 100KW by modifying the flow rate or height of the waterfall.

c) Describe the impact of energy losses due to friction in the pipes and suggest ways to minimize these losses.

SOLID FRICTION

In a Ugandan brick-making factory, workers are pushing heavy carts loaded with bricks across a rough concrete floor. Each cart has a mass of 200kg, and the workers apply a horizontal force of 800N to

start the cart moving. Once moving, the cart requires a force of 600N to keep it moving at a constant speed. The factory manager wants to reduce the effort required to move the carts to improve efficiency.

Tasks:

a) Determine the coefficients of static and kinetic friction between the cart and the floor.

b) Explain how the workers could reduce the force required to move the carts by modifying the floor or the cart's wheels.

c) Contrast the frictional forces acting on the cart if the mass of the bricks is reduced by half. Would this significantly reduce the effort required?

Item1

A city is planning to construct a new suspension bridge. The design team uses the formula for the sag of a cable, which involves exponential functions and surds:

- The sag (in meters) of the cable is modeled by $S_t = 50 \times 2^{-0.05t}$, where (t) is time in years.

- The length of the cable (in meters) is given by $L = \sqrt{(200^2 + (4S_t)^2)}$, where S_t is the sag at time (t).

Tasks:

a) - Determine after how many years the sag of the cable will reduce to 10 meters. Use logarithms to solve for (t).

- Justify why logarithmic transformations are necessary for solving this problem.

b) - Calculate the exact length of the cable (in surd form) when the sag is 10 meters.

- Rationalize your final answer and explain how surds ensure precision in engineering calculations.

Item2

A telecommunications company is designing a satellite antenna. The signal strength decays exponentially over distance, and the antenna's optimal length involves surds for precise alignment.

The signal strength P(d) in watts at distance d in km is modeled by:

$P(d) = 1000 \times 3^{-0.1d}$

The antenna's length L in meters is calculated using:

$L = (75/\sqrt{12}) + (50/\sqrt{27})$

Task:

Help the company to;

a)Determine the distance d at which the signal strength drops to 50 watts.

b) Simplify L to its exact form.

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c)Explain why rationalization is critical in antenna design.

Item 3

An airline offers three ticket classes (Economy, Premium and Business) on a flight. The revenue from ticket sales must cover fixed costs while maximizing profit.

On a given flight,The sum of all tickets sold is 200 passengers.

when Economy tickets cost is $300, Premium $600, and Business $900.

Total revenue that was collected was exactly $120,300.

Support:

the number of Economy tickets is equal to twice the number of Premium tickets minus a 4 times the number of the Business ticket plus 6.

Task

Using knowledge of solving simultaneous equations using reduction to row echelon form,help the manager know the number of tickets in each class.

Item 4

Aerospace engineers are analyzing the stability of a satellite's elliptical orbit. The orbital path is modeled by the quadratic equation:

$f(t) = t^2 - (2e + 4)t + (e^2 + 4e + 3) = 0$

where:

- Roots (α, β) represent critical stability points in the orbit

- Parameter e represents orbital eccentricity ($0 < e < 1$)

For stability calculations, engineers need:

a)Sum of cubes $\alpha^3 + \beta^3$

b)Sum of reciprocals squared $(1/\alpha^2) + (1/\beta^2)$

Task:

Use knowledge of mathematics to Derive expressions for a) and b) in terms of e.

TOPIC 1: NUMERICAL CONCEPTS

Item 1

Nakato is a mobile money agent in her village, Mukono. When she started in January 2024, she had 50 regular customers. She observed that her customer base seemed to grow exponentially each month. By the end of March 2024 (after 3 months), she had 135 regular customers. She wants to predict her customer growth to plan for liquidity (cash and e-float) and potentially hire an assistant. Assume the

growth follows the model $N = N0 \times Kt$, where N is the number of customers after t months, $N0$ is the initial number of customers, and K is the monthly growth factor.

Tasks:

a) Help Nakato in determining her monthly growth factor,K. Express your answer to 3 significant figures.

b) Determine the number of customers Nakato can expect by the end of December 2024 (after 12 months) if this growth rate continues.

c) Nakato estimates she needs UGX 10,000 in float per regular customer per month. Using the predicted number of customers for December 2024, calculate the total float she would need. Express this amount using index notation in terms of powers of 10.

Item 2

Mr. Okello is weaving a traditional Ugandan mat (ekikeeka) with intricate geometric patterns. One key element involves fitting square tiles made of dyed reeds into a rectangular border. The side length of each square tile needs to be exactly (√5−√2)cm for the pattern to align perfectly. The rectangular border has a length of (10√5 + 5√2)cm and a width of (8√5− 4√2) cm.

Tasks:

a) Calculate the exact area of one square tile. Express your answer in the simplest surd form $a + b\sqrt{c}$.

b) Determine the exact area of the rectangular border. Express your answer in the simplest surd form.

c) If Mr. Okello wants to fit as many *whole* square tiles as possible within the border without overlapping, estimate the maximum number of tiles he can fit. Justify your answer.

Item 3

A local environmental group in Jinja is studying the population growth of a specific fish species in a protected section of the Nile River. Their initial estimate in 2020 was 1,200 fish. They believe the population P after t years can be modelled by $P(t) = 1200 \times (1.15)t$. However, another model proposed is based on logarithms: $\log 10P = \log 101200 + t\log 101.15$.

Tasks:

a) Using the index model $P(t) = 1200 \times (1.15)t$, calculate the predicted fish population in the year 2025.

b) Using the logarithmic model, show that it is equivalent to the index model.

c)

d) The group wants to know when the fish population is predicted to reach 5,000. Using logarithms and the model $P(t) = 1200 \times (1.15)t$, determine the approximate number of years (t) it will take.

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TOPIC 2: EQUATIONS AND INEQUALITIES

Item 4

Mrs. Nabukalu, a farmer in Masaka, finds that the yield of her maize crop (in bags per acre), Y, depends on the amount of a specific fertilizer x used (in kg per acre). The relationship is modelled by the quadratic equation: $Y(x) = -0.5x2 + 20x + 50$. She wants to maximize her yield but also knows the fertilizer costs money.

Tasks:

a) Help Mrs. Nabukalu to determine the amount of fertilizer (x) that will give her the maximum maize yield.

b) Calculate the maximum possible yield in bags per acre.

c) If the cost of the fertilizer is UGX 1,500 per kg. Mrs. Nabukalu wants the yield to be at least 200 bags per acre. Formulate a quadratic inequality to represent this situation.

d) By solving the inequality in part (c) above determine the range of fertilizer amounts (in kg per acre) she can use to achieve a yield of at least 200 bags per acre.

Item 5

A community group in Gulu is managing the costs for drilling three boreholes (A, B, and C). The total cost was UGX 25,000,000. The cost of Borehole B was UGX 1,000,000 less than Borehole A. The combined cost of Boreholes A and C was three times the cost of Borehole B. Let the costs of drilling boreholes A, B, and C be a,b, and c (in UGX) respectively.

Tasks:

a) Formulate a system of three linear simultaneous equations representing the information given.

b) By Using Row reduction, calculate the individual cost of drilling each borehole (a, b, and c).

c) If the cost per meter drilled was UGX 250,000 for all boreholes, determine the depth of Borehole A.

Item 6

A school in Mbarara wants to create a rectangular vegetable garden. They have 80 meters of fencing available. They want the area of the garden to be greater than 300 square meters to grow enough vegetables for the school lunch program. Let the length of the garden be L meters and the width be W meters.

Tasks:

a) Express the perimeter of the garden in terms of L and W and form an equation using the available fencing.

b) Using your equation in a) above formulate the area A of the garden purely in terms of L.

c) Formulate a quadratic inequality representing the condition that the area must be greater than 300 square meters and solve it to determine the possible range of values for the length (L) of the garden that satisfies both the fencing constraint and the area requirement.

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TOPIC 4 COORDINATE GEOMETRY 1

Item 7

Mr. Kato owns a rectangular farm near Fort Portal. On a map grid, the corners of his main plot are at $A(1,2)$,$B(9,2)$,$C(9,8)$, and $D(1,8)$. He plans to install two straight irrigation pipes. *Pipe 1* will run from corner A to corner C. *Pipe 2* will run from the midpoint of side AB to the midpoint of side CD. A water source is located at point W (5, 5).

Tasks:

a) Determine the coordinates of the midpoints of sides AB and CD.

b) Formulate the equation of the line representing Pipe 1 (line AC) and the equation of the line representing Pipe 2.

c) Determine the shortest distance from the water source W (5, 5) to Pipe 1 (line AC). Will Pipe 1 pass directly through the water source? Justify your answer.

Item 8

A new road (Road 1) is being constructed in Kampala, represented by the equation $y = 2x + 3$. It will intersect an existing road (Road 2), represented by the equation $3x + 2y = 12$. A traffic light needs to be installed at the intersection point. Another planned road (Road 3) needs to be parallel to Road 1 and pass through the point $P(4,1)$. A fourth road (Road 4) must be perpendicular to Road 2 and pass through the same point $P(4,1)$.

Tasks:

a) Calculate the coordinates of the intersection point of Road 1 and Road 2 where the traffic light will be placed.

b) Determine the equation of the line representing the planned Road 3 and Road 4.

c) Calculate the acute angle between Road 1 and Road 2 at their intersection point. Give your answer in degrees.

Item 9

Three villages, A, B, and C, are located on a map grid at coordinates A(2, 1), B(8, 3), and C(4, 7). A new health centre needs to be built such that it is equidistant from villages A and B. It must also lie on the line that passes directly between village C and the midpoint of the line segment connecting A and B.

Tasks:

a) Determine the coordinates of the midpoint M of the line segment connecting villages A and B.

b) Formulate the equation of the perpendicular bisector of the line segment AB. (This line represents all points equidistant from A and B).

c) Find the equation of the line passing through village C (4, 7) and the midpoint M calculated in Task 1.

d) Calculate the coordinates where the two lines found in b) and c) intersect. This point represents the ideal location for the health centre. Justify why this location satisfies both conditions.

TOPIC 5: PARTIAL FRACTIONS

Item 10

An Engineer in a chemical engineering plant in Namanve, want to use a chemical with concentration $C(t)$ of a product over time t, which is modelled by complex rational functions. Suppose the rate of change of concentration involves the expression: $f(t)$ =5t + 3(t+1)(t+2). To analyse the long-term behaviour but doesn't know the appropriate techniques to use.

Tasks:

a) Help the engineer to identify the type of factors in the denominator of $f(t)$.

b) Express $f(t)$ as the sum of its partial fractions.

Item 11

An electrical engineering student at Makerere University is analysing a signal whose behaviour over time x is related to the function $g(x)$ =2x2+x−1 x(x−1)2. This expression needs to be broken down for further analysis.

Tasks:

a) Set up the appropriate form for the partial fraction decomposition of $g(x)$.

b) Determine the values of the unknown constants in the partial fraction decomposition.

c) Write the final partial fraction decomposition of $g(x)$.

Item 12

An economist is studying the relationship between investment I and national income Y. The relationship involves a complex function where a particular term is given by $h(Y)$ = Y3+ 2Y2− Y + 5 Y2+ Y − 2 . Before proceeding with the economic analysis, the economist needs to simplify this expression.

Tasks:

Help the economist to;

a) Identify h(Y) as a proper or improper rational function. Justify your answer.

b) express h(Y) as the sum of a polynomial and a proper rational fraction.

c) Take the proper rational fraction part obtained in b) and decompose it into its partial fractions.

d) Combine the results from b) and c) to write the complete simplified expression for h(Y).

TOPIC 5: TRIGONOMETRY

Item 13

A surveyor is mapping a triangular piece of land in the hilly region of Kabale. The vertices are marked as P, Q, and R. The distance PQ is measured as 120 meters, and the distance PR is 150 meters. The angle at P, $\angle QPR$, is measured as 75°. The surveyor needs to find the length of the third side QR and the area of the land.

Tasks:

a) Determine the length of the side QR to the nearest meter and angle $\angle PQR$ to 1 decimal place.

b) Calculate the area of the triangular piece of land PQR.

c) If $\angle QPR$ was actually m easured as (45°+ 30°), apply an appropriate formula find the exact value of cos75°.

Item 14

An architect is designing a symmetrical roof truss for a community hall in Lira. The truss is shaped like an isosceles triangle ABC, with AB = AC. The base BC has a length of 16 meters. The angle at the apex A, $\angle BAC$, needs to be determined such that the height (altitude from A to BC) is exactly 6 meters. Let M be the midpoint of BC.

Tasks:

a) Consider the right-angled triangle AMB. Calculate the length of the side AB

b) In triangle AMB, determine the value of tan($\angle ABM$) and hence find $\angle ABM$ in degrees.

c) determine the measure of \angle BAM hence calculate the angle at the apex, $\angle BAC$.

Item 15

A fishing boat leaves Kasenyi landing site (Point K) and travels 15 km on a bearing of 060° to reach Point A. From Point A, it then travels 20 km on a bearing of 135° to reach Point B. The boat captain now wants to know the direct distance and bearing from Kasenyi (K) back to Point B.

Tasks:

a) Help the captain to map the journey on a diagram, showing the points K, A, B, and their bearings.

b) By applying cosine rule calculate the direct distance KB, correct to one decimal place.

c) By applying sine rule calculate the angle $\angle AKB$. Hence, determine the bearing of Kasenyi (K) from Point B.

Item 18

Two market vendors, Aisha and Ben, operating in Owino Market, Kampala, recorded their daily sales (in thousands of UGX) over a period of 30 days. The data is summarized below:

Aisha: Mean Sale = 150, Standard Deviation = 25

Ben: Mean Sale = 160, Standard Deviation = 40

Tasks:

a) Determine which vendor has higher average daily sales.

b) Calculate the coefficient of variation for both Aisha and Ben.

c) Using the coefficient of variation, determine whose sales are relatively more consistent. Justify your answer.

TOPIC 8: DYNAMICS 1

Item 22

Two farm workers, Okello and Lanyero, are pulling a heavy sack of maize (mass 80 kg) across level ground in a Kireka warehouse. Okello pulls with a force of 300 N at an angle of 20° above the horizontal. Lanyero pulls with a force of 250 N at an angle of 15° above the horizontal, in the same direction as Okello. The coefficient of kinetic friction between the sack and the ground is 0.3. (Assume g = 9.8 m/s2).

Tasks:

a) Represent all the forces acting on the sack on a diagram.

b) Resolve the forces applied by Okello and Lanyero into horizontal and vertical components.

c) Calculate the total upward vertical component from the workers' pulls and hence determine the Normal Reaction force exerted by the ground on the sack.

d) Calculate the maximum possible frictional force and the total horizontal component of the pulling forces hence determine the net horizontal force acting on the sack.

Item 23

In a mechanics lab, a block A of mass 5 kg rests on a rough inclined plane angled at 30° to the horizontal. The coefficient of kinetic friction between block A and the plane is 0.2. Block A is connected by a light inextensible string passing over a smooth pulley at the top of the incline to a block B of mass 3 kg, which hangs freely. The system is released from rest. (Assume g = 9.8m/s2).

Tasks:

a) Illustrate the forces acting on block A and block B on separate diagrams.

b) For block A, resolve its weight into components parallel and perpendicular to the inclined plane. Calculate the normal reaction force on block A.

c) Determine the frictional force acting on block A as it slides (assume it slides up the plane initially, if unsure, calculate net force without friction first to determine direction).

d) Apply Newton's Second Law to both block A and block B to formulate two simultaneous equations involving the acceleration (a) of the system and the tension (T) in the string. Solve these equations to find the values of a and T.

Item 24

A lorry of mass 5000 kg is parked on a road in Kisoro inclined at an angle θ to the horizontal, where sinθ= 0.1. The coefficient of static friction between the lorry's tyres and the road is 0.4. The driver has applied the handbrake. We want to determine if the lorry will remain stationary. (Assume g = 9.8 m/s2).

Tasks:

a) Come up with a diagram showing the forces acting on the lorry assuming it is about to slide down the slope.

b) Resolve the weight of the lorry into components parallel and perpendicular to the road surface, hence find the Normal Reaction force acting on it.

c) Determine the maximum possible static frictional force that can be exerted by the road on the tyres (Fmax =μsN). Compare this maximum friction with the component of the lorry's weight acting down the slope. Establish wether lorry will remain stationary or slide down. Justify your conclusion.

TOPIC 9: PROBABILITY THEORY

Item 25

In a certain region of Uganda, it is estimated that 2% of the population has a particular disease. A medical test is developed to detect the disease. The test is not perfect:

If a person has the disease, the test correctly gives a positive result 95% of the time (Sensitivity).

If a person does not have the disease, the test correctly gives a negative result 90% of the time (Specificity). A person from the region is selected at random and tested.

Tasks:

a) Construct a tree diagram and use it to calculate the overall probability that a randomly selected person tests positive.

b) Using Bayes' Theorem, determine the probability that a person actually has the disease given that they tested positive.

c) Interpret your result from b) above. What does this tell you about the reliability of a positive test result in this scenario?

Item 26

A factory in Jinja produces light bulbs using three machines: Machine A, Machine B, and Machine C.

Machine A produces 40% of the total output, and 5% of its bulbs are defective.

Machine B produces 35% of the total output, and 3% of its bulbs are defective.

Machine C produces 25% of the total output, and 2% of its bulbs are defective. A bulb is selected at random from the factory's output.

Tasks:

a) Determine the probability that the selected bulb was produced by Machine A AND is defective. Similarly, calculate the probabilities for Machine B being defective and Machine C being defective.

b) Using the results from a), determine the overall probability that a randomly selected bulb from the factory's output is defective.

c) Given that the selected bulb is found to be defective, calculate the probability that it was produced by Machine B.

Item 27

In a class of 60 students at a Kampala school, 40 own an Android phone (A), 25 own an iPhone (I), and 15 own both types.

Tasks:

a) Represent this information on a Venn diagram.

b) Determine the number of students who own:

i) Only an Android phone.

ii) Only an iPhone.

iii) Neither type of phone.

c) A student is selected at random from the class. Calculate the probability that the student owns:

i) An Android phone or an iPhone.

ii) Exactly one type of phone.

d) Given that a selected student owns an Android phone, calculate the probability that they also own an iPhone.

TOPIC 10: DIFFERENTIATION 1

Item 28

A farmer in Mukono wants to create a rectangular enclosure for chickens next to a long, straight existing wall. He has 100 meters of fencing wire available for the other three sides of the rectangle. He wants to maximize the area enclosed for his chickens. Let the side parallel to the wall have length x meters, and the other two sides perpendicular to the wall have length y meters each.

Tasks:

a) Help the farmer to express the total length of the fencing used in terms of x and y, and formulate an equation based on the available wire.

b) Express the area A of the enclosure ($A = xy$) as a function of only one variable x. Hence, find the value of x that maximizes the area.

c) Determine the maximum possible area of the enclosure and confirm it is a maximum.

Item 29

The displacement s (in meters) of a particle moving along a straight line from a fixed point O, at time t (in seconds), is given by the equation $s(t) = t3 - 6t2 + 9t + 5$, for $t \geq 0$.

Tasks:

a) Determine the expressions for the velocity $v(t)$ and acceleration $a(t)$ of the particle at time t by differentiating the displacement function.

b) Calculate the initial velocity and initial acceleration of the particle $at\ t$=0.

c) Find the time(s) when the particle is momentarily at rest $v(t) = 0$.

d) Determine the acceleration of the particle at the time(s) when it is at rest. Describe the motion of the particle during the first 4 seconds.

Item 30

A scientist has a spherical balloon is which being inflated. Its radius r is increasing at a constant rate of 0.1 cm per second. The volume of a sphere is given by $V = 43\pi r3$. He wants to find the rate at which the volume is increasing when the radius is 5 cm. He also wants to estimate the approximate increase in volume as the radius increases from 5 cm to 5.1 cm.

Tasks:

a) Help the scientist to determine the rate at which the volume of the bowl is changing with respect to the radius.

b) Determine the rate at which the volume is increasing when the radius r = 5 cm.

c) Estimate the approximate increase in volume (δV) as the radius increases from r=5 cm to r=5.1 cm.

TOPIC 11: INTEGRATION 1

Item 31

Water flows into a storage tank in Mbale at a rate given by $R(t)$ = 10 + 0.5t liters per minute, where t is the time in minutes from the start (t=0). The tank was initially empty.

Tasks:

a) Obtain expression for the volume V(t) of water in the tank as an indefinite integral.

b) Using the initial condition, determine the value of the constant of integration C.

c) Calculate the volume of water in the tank after 60 minutes

d) Determine the average rate of flow into the tank during the first 60 minutes using the mean value function.

Item 32

A piece of land is bounded by a river whose shape can be modelled by the curve y =√x, the straight line x=9, and the x-axis (representing a straight fence). The coordinates are measured in meters. The owner wants to calculate the area of this piece of land.

Tasks:

a) Help the owner to sketch the region bounded by the land.

b) Set up the definite integral that represents the area of this region hence use it to calculate the exact area of the piece of land.

c) If this area were revolved around the x-axis, it would form a solid shape. Set up the definite integral representing the volume of this solid of revolution hence Calculate this volume.

Item 33

A boda-boda rider accelerates away from a traffic light in Fort Portal. His velocity v (in m/s) after time t (in seconds) is given by $v(t)$ = 6t − t2 for 0≤ t≤ 6.

Tasks:

a) Obtain an expression for his displacement $s(t)$ (assume $s(t$=0)=0).

b) Determine the displacement of the boda-boda from the traffic light after 3 seconds.

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c) Calculate the total distance travelled by the boda-boda in the first 6 seconds.

d) Determine the time t at which the boda-boda reaches its maximum velocity within the interval $0 \leq t \leq 6$. Hence Calculate its maximum velocity.

TOPIC 12: PERMUTATIONS AND COMBINATIONS

Item 34

A student at Ntare School has 4 distinct Mathematics books, 3 distinct Physics books, and 2 distinct Chemistry books. He wants to arrange them on a single shelf.

Tasks:

Help the student to know

a) how many different ways he can arrange 9 books be on the shelf if there are no restrictions?

b) how many ways he can arrange the books if all the Mathematics books must be kept together, all the Physics books must be kept together, and all the Chemistry books must be kept together?

c) In how many ways he can arrange the books if only the Mathematics books must be kept together?

Item 35

Exodus College School needs to form a student committee of 5 members. There are 8 eligible students from Senior Five and 6 eligible students from Senior Six.

Tasks:

a) In how many ways can the committee of 5 be formed if there are no restrictions on the class level?

b) In how many ways can the committee be formed if it must consist of exactly 3 students from Senior Five and 2 students from Senior Six?

c) In how many ways can the committee be formed if it must include at least 4 students from Senior Five?

d) Suppose two specific Senior Six students, Mary and Jane, refuse to be on the committee together. In how many ways can the committee be formed if it must have exactly 3 Senior Five students and 2 Senior Six students, considering this restriction?

Item 36

A mobile banking App requires users to create a 4-digit PIN using the digits 0 to 9.

Tasks:

a) How many different 4-digit PINs can be created if digits can be repeated?

b) How many different 4-digit PINs can be created if digits cannot be repeated?

c) How many different 4-digit PINs can be created if digits cannot be repeated and the PIN must be an even number?

d) How many different 4-digit PINs can be created if digits can be repeated, but the PIN cannot start with 0?

1. Amina Exports (Kampala) ships roasted Arabica coffee in containers.

Her weekly profit, $P(x)$ (in UGX hundred thousands) when she ships x containers, in a week, is modeled by the quadratic equation;

$$P(x) = -2x2 + 36x - 160.$$

Amina wants to determine the number of containers to ship that guarantees profits, and the number of containers shipped that will make her break even in a week but she lacks knowledge on how to undertake it.

Task:

Help Amina.

2. John is a bodaboda rider around Kampala. John and his friend plan to purchase motorcycle jointly, and thus they set their joint savings plan.

They want to save for the first thirty one consecutive days.

On first day, they saved UGX 42, 000. They increased their daily savings by a fixed amount, and on 10th day, they saved UGX 150,000.

At the start of the year, they are to open an account with a co – operative savings scheme which offers a 12.5% compound interest per annum. The scheme doesn't allow any withdrawal until a period of 5 years. They are to deposit their money they saved for thirty one days, on the account. At the beginning of each year henceforth, they are required to deposit a third of the amount of money they opened the account with.

Tasks:

Help John and his friend to determine;

a) Amount of money they had jointly saved in the first thirty one days

b) How much they will have in saving scheme at the end of 5th year, given that they have to purchase their motorcycle at the end of 5th year.

ITEM ONE:

John, a Ugandan city tycoon, has a rectangular piece of land around Nakivubo center.

He wants to rent out this land to road – side traders in a way that each trader rents 2 square metres of this land at UGX 840,000 per month.

The length, x meters, and the width, y meters, of his land are defined by the following simultaneous logarithmic equations;

$logyx=5$ and $log2x=2+log2y$

He wants to determine the number of $2\ m2$ plots he can have out of his full rectangular piece of land, for renting.

Tasks:

a) Calculate the number of plots He can divide out of His rectangular piece of land.

b) How much is He likely to earn in a month from His land if all the plots are rent out?

Mugisha, a builder in Kampala, is constructing the roof for a new community library.

He uses wooden beams to support the roof structure and needs to calculate the number of cement bags required for the base of the beam support system.

He wants to ensure stability while minimizing unnecessary expenditure.

The strength of the beam support system is modeled by the polynomial function;
$S(n)=-n4+26n3-239n2+910n$

Where; $S(n)$ is the structural strength score and n is the number of beams that support the roof structure.

Each beam requires a base made with cement. The number of cement bags C needed for maximum number of beams is modeled by the inequality; $C2-7C+(n+2)\leq 0$

The builder wants to stay within the recommended range of cement usage to avoid wastage. The cost of a bag of cement is UGX 35,000, and each wooden beam costs UGX 42,000.

Tasks:

Help Mugisha calculate the;

a) Maximum possible number of wooden beams required given that the structure is only considered stable if the strength score is at least 1200.

b) Maximum number of bags required.

c) Total cost is likely to incur.

ITEM FOUR:

Mzee Okello from a village near Gulu, Uganda, needs to lay a new water pipe to irrigate his crops.

All the water system and the main channel have been mapped on a 2D grid system as follows;

The main water channel runs along a straight path; $2x-y+5=0$. His farm is located at $P(6,8)$. To save pumping costs, Okello plans to use short – cut pipe, l, from the main channel at point Q to his farm.

However, Mzee Okello's farm engineer suggests a pipe, $l1$, that meets the main channel at, R, at an angle of; 450, then to the Mzee Okello's farm.

His Neighbor, Ms. Tinah, wants to lay a pipe,$l0$, that runs parallel to the main channel, passes through Mzee Okello's farm up to Her home's tap.

Each unit represents 100 metres.

Tasks:

a) Calculate the length of the short – cut pipe, l, needed. Correct your answer to 1 decimal place.

b) Find the possible equation(s) of the Engineer's suggested pipe, $l1$.

c) Obtain the equation of Ms. Tinah's pipe, lo

d) Calculate the coordinates of; Q and R.

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