LOGARITHMS, INDICES & SURDS

1. . Without using tables, find

$(243)^{\frac{3}{5}} \times (81)^{\frac{-3}{2}}$ ***(4 marks)***

2. Find $\frac{y}{x}$ if $9^x = 27^y$ ***(4 marks)***

3. Given that $\log_{10}a = 1.621$ and $\log_{10}b = 1.152$, evaluate $\log_{10}a + \log_{10}b^{1/2}$. ***(4 marks)***

4. Without using tables or calculator, evaluate
$3\ell og_{10}2 + \ell og_{10}20 - \ell og_{10}1.6$

5. Simplify $\frac{3^x \times 9^{x+1}}{27^{x-1}}$

6. $\frac{6}{3\sqrt{2}-2\sqrt{3}} = a\sqrt{3} + b\sqrt{3}$, find the values of a and b

7. . Given that $81^x = \left(1/3\right)^{x-5}$ find the value of x

8. Given $\frac{5}{\sqrt{5}} + \sqrt{20} = a\sqrt{5}$ determine the value of a

9. . Simplify $\frac{2^x \times 8^{x-1}}{6^{x-1}}$

10. Without using tables or calculator simplify
$\frac{1}{2}\ell og_{10}16 - 2\ell og_{10}\left(\frac{a}{5}\right) + \ell og10a^2$

11. Express $\frac{9}{\sqrt{5}-\sqrt{2}}$ in the form $a(\sqrt{b}+\sqrt{c})$, where a, b and c are integers.

12. Show that $\sqrt{18}+\sqrt{50}-\sqrt{72} = 2\sqrt{2}$

13. Simplify $\frac{2^{-2}\times 3^{-3}}{2^{-4}\times 3^{-6}\times 18}$

14. Simplify $\frac{\sqrt{63}+\sqrt{28}}{\sqrt{175}-\sqrt{63}}$ as far as possible.

15. With out using table or calculator, evaluate
$\left(\frac{1}{16}\right)^{\frac{-1}{2}} x \left(\frac{1}{64}\right)^{\frac{-1}{3}}$

16. Simplify $\frac{3^3 \times 9^2 \times 125^{1/3}}{9^3}$

17. If $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} = a+\sqrt{b}$,
Find the values of a and b

18. With out using tables or calculator

Simplify $\frac{\sqrt{30}}{\sqrt{6}}+\frac{\sqrt{35}}{\sqrt{7}}$

19. Simplify $\frac{(12)^{3/2}\,x(16)^{\frac{1}{8}}}{(27)^{1/6}\,x(18)^{1/2}}$

20. Solve for x in the equation $(x + 2)(x-4) - x^2 < -6$

21. Express $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ in the form $a + b\sqrt{3}$.

Hence evaluate $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ correct to 3 significant figures $\sqrt{3} = 1.732$

22. A trader made a 35 % profit after selling a goat at h 45,900. How much profits did the trader get?

23. Simplify $\log 75 + 2\log 2 - \log 3$.

24. Simplify $\frac{(12)^{3/2}\,x(16)^{\frac{1}{8}}}{(27)^{1/6}\,x(18)^{1/2}}$

25. Express $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ in the form $a + b\sqrt{3}$.

Hence evaluate $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ correct to 3 significant figures $\sqrt{3} = 1.732$

25. Simplify $\log 75 + 2\log 2 - \log 3$.

26. Express $\frac{1}{\sqrt{5}-\sqrt{2}}$ with a rational denominator

27.Simplify $\left(\frac{8}{27}\right)^{\frac{-2}{3}}$

28. . Solve: $\log_{10}(7x + 2) - \log_{10}(x - 1) = 1$

29. Given that $\frac{a+b\sqrt{2}}{c} = \frac{4+\sqrt{2}}{4-\sqrt{2}c}$, find the values of a, b and c

30.. Express $\frac{3}{1-\sqrt{2}}$ in the form $a + b\sqrt{2}$ ***(04 mks)***

31. Simplify

$$\frac{\left(3^2\right)^{\frac{3}{2}}\times\left(3^{-\frac{1}{2}}\right)}{3^{\frac{1}{2}}}$$

32. Evaluate

$$\ell\log_a\left(\frac{5}{7}\right)+2\ell\log_a\left(\frac{7}{6}\right)-\ \ell\log_a\left(\frac{5}{6}\right)$$

33. . Without using tables, evaluate

$$\frac{12^{3/2}\times16^{1/8}}{27^{1/6}\times18^{1/2}}$$

34. Evaluate $\dfrac{9^{1/2}\times3^{5/2}}{3^{2/3}\times3^{-1/6}}$

35. Find the value of x

$2^{(x-2)} = 32$

36. Simplify $\dfrac{3^{n+1}\times9^{n}}{27^{(2/3)n}}$

$$\log_a 36+\frac{1}{2}\log_a 256-2\log_a 48=-\log_a 4\,.$$

37.

38.

Simplify each of the following expressions, giving the final answer as an integer.

a) $\log_2 3-\log_2 24\,.$

b) $\log_a a^2-4\log_a\left(\dfrac{1}{a}\right),\ a>0,\ a\neq1\,.$

Full workings, justifying every step, must support each answer.

39.

Given that $y=\log_2 x$, write each of the following expressions in terms of y.

a) $\log_2 x^2$

b) $\log_2\left(8x^2\right)$

Solve each of the following logarithmic equations.

a) $\log_x 16=\log_x 9+2\,.$

b) $\log_y 27=3+\log_y 8\,.$

40.

41. Simplify the following expression.

$$9\log_{24} 2+\log_{24} 27 .$$

Show detailed workings in this simplification.

42. Write each of the following surd expressions as simple as possible.

a) $\left(\sqrt{5}+2\right)\left(3-\sqrt{5}\right)$.

b) $\dfrac{14}{\sqrt{2}}-\sqrt{18}-\left(\sqrt{2}\right)^3$.

43. Write each of the following surd expressions as simple as possible.

a) $\sqrt{98}-\sqrt{50}$.

b) $3\sqrt{8}\times 2\sqrt{32}-\dfrac{10}{\sqrt{2}}$.

44. Write each of the following surd expressions as simple as possible.

a) $\dfrac{\left(2+\sqrt{2}\right)\left(1+\sqrt{2}\right)}{\sqrt{2}}$.

b) $\dfrac{5\sqrt{5}-\sqrt{45}}{\sqrt{20}}$.

END