TOPIC 5: FURTHER DIFFERENTIATION

This is a continuation of what you covered in S.5

It will also cover differentiation involving logarithms and exponential

Example

Differentiate $\dfrac{e^{x^2}\sqrt{\sin x}}{(2x+1)^3}$

Let $y=\dfrac{e^{x^2}(\sin x)^{1/2}}{(2x+1)^3}$

$$Iny = Ine^{x^2} + In(\sin x)^{1/2} - In(2x+1)^3$$

$$Iny = x^2 Ine \ + 1/2\, In(\sin x) \ -3In(2x+1)$$

$$Iny = x^2 + 1/2\, In(\sin x) \ -3In(2x+1)$$

Note that apply the law of logarithms properly before differentiating remember we only differentiate or integrate in log base e

$$\frac{1}{y}\frac{dy}{dx} = x^2 + 1/2\frac{Cosx}{\sin x} - 3\frac{(2)}{2x+1}$$

$$\frac{1}{y}\frac{dy}{dx} = 2x + 1/2\cot x - \frac{6}{2x+1}$$

$$\frac{dy}{dx} = \left(2x + 1/2\cot x - \frac{6}{2x+1}\right)\frac{e^{x^2}(\sin x)^{1/2}}{(2x+1)^3}$$

Please simplify your answer as much as it can be done, a lot of marks lies in that area

Example

Differentiate $\sqrt{\dfrac{(2x+3)^3}{1-2x}}$

Let = $\sqrt{\dfrac{(2x+3)^3}{1-2x}} = \dfrac{(2x+3)^{3/2}}{(1-2x)^{1/2}}$

$$Iny = In(2x+3)^{3/2} - In(1-2x)^{1/2}$$

$$Iny = \tfrac{3}{2} In(2x+3) - \tfrac{1}{2} In(1-2x)$$

$$\frac{1}{y}\frac{dy}{dx} = \tfrac{3}{2}\left(\frac{2}{2x+3}\right) - \tfrac{1}{2}\left(\frac{-2}{1-2x}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2x+3} + \frac{1}{1-2x}$$

$$= \frac{3(1-2x)+2x+3}{(2x+3)(1-2x)}$$

$$= \frac{3-6x+2x+3}{(2x+3)(1-2x)}$$

$$= \frac{6-4x}{(2x+3)(1-2x)}$$

$$\frac{dy}{dx} = \frac{2(3-2x)}{(2x+3)(1-2x)} \bullet \frac{(2x+3)^{3/2}}{(1-2x)^{1/2}}$$

$$= \frac{2(3-2x)\sqrt{(2x+3)}}{(1-2x)^{3/2}}$$

Example

Differentiate with respect to x the expression

$$\sin^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right)$$

Let $y = \sin^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right)$

$$\sin y = \frac{3+5\cos x}{5+3\cos x}$$

$$Cosy\frac{dy}{dx} = \frac{(5+3\cos x)(-5Sinx) - (3+5Cosx)(-3\sin x)}{(5+3\cos x)^2}$$

$$= \frac{(-25Sinx - 15Sinx\cos x) + (9Sinx + 15SinxCosx)}{(5+3\cos x)^2}$$

$$\cos y\frac{dy}{dx} = \frac{-16Sinx}{(5+3\cos x)^2}$$

$$\sin y = \frac{3+5\cos x}{5+3\cos x}$$

$(5+3\cos x)^2-(3+5\cos x)^2=b^2$

b= 4sinx

$$Cosy = \frac{4Sinx}{5+3\cos x}$$

$$\therefore \frac{dy}{dx} = \frac{(-16Sinx)(5+Cosx)}{(5+3\cos x)^2 4\sin x}$$

$$= \frac{-4}{5+3\cos x}$$

Example

Differentiate $Cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Let $y = Cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$\cos y = \frac{1-x^2}{1+x^2}$$

$$-Siny\frac{dy}{dx} = \frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$$

$$= \frac{-2x(1+x^2+1-x^2)}{(1+x^2)^2}$$

$$+Siny\frac{dy}{dx} = \frac{+4x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{4x}{(1+x^2)Siny}$$

$$\cos y = \frac{1-x^2}{1+x^2}$$

$$h^2 = \left(1+x^2\right)^2 - \left(1-x^2\right)^2$$

$= 1+2x^2+x^4-(1-2x^2+x^4)$

$=4x^2$

$h \quad =2x$

$$\frac{dy}{dx} = \frac{4x}{\left(1+x^2\right)^2} = \frac{2}{1+x^2}$$

Example

Given $y = \sqrt[3]{\dfrac{x-1}{(x^2-1)^2}} = \dfrac{(x-1)^{1/3}}{(x^2-1)^{2/3}}$

$$Iny = \tfrac{1}{3} In(x-1) - \tfrac{2}{3} In(x^2-1)$$

$$\frac{1}{y}\frac{dy}{dx} = \tfrac{1}{3}\left(\frac{1}{x-1}\right) - \tfrac{2}{3}\left(\frac{1}{x^2-1}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \tfrac{1}{3}\left(\frac{1}{x-1}\right) - \tfrac{2}{3}\left(\frac{1}{x^2-1}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \tfrac{1}{3}\left(\frac{(x^2-1)-4x(x-1)}{(x^2-1)(x-1)}\right)$$

$$= \tfrac{1}{3}\frac{(x^2-1-4x^2+4x)}{(x^2-1)(x-1)}$$

$$= \frac{-3x^2+4x-1)}{3(x^2-1)(x-1)}$$

$$= \frac{(1-3x)(x-1)}{3(x^2-1)(x-1)}$$

$$\frac{dy}{dx} = \frac{(1-3x)}{3(x^2-1)} \bullet \frac{(x-1)^{1/.3}}{(x^2-1)^{5/.3}}$$

$$\frac{dy}{dx} = \frac{(1-3x)(x-1)^{1/3}}{3(x-1)(x-1)(x+1)^{2/3}(x-1)^{2/3}}$$

$$\frac{dy}{dx} = \frac{(1-3x)^{1/3}}{3(x+1)^{5/3}(x-1)^{4/3}}$$