SENIOR FIVE TERM 1

TOPIC 1/6: measurement and dimensions of physical quantities

Competency: The learner uses a variety of instruments to accurately measure physical quantities and applies the concept of dimensions to establish the relation between these quantities.

Measurements

Measurement involves assigning numerical values to physical properties like length, mass, time, and temperature. Measurements of different quantities are expressed in different unit. The **International System of Units (SI)** is the globally accepted standard for measuring physical quantities. It is a modern form of the metric system and is used in science, industry, and everyday commerce worldwide

Importance of measurements in our daily life

Measurements play a crucial role in everyday life, helping us make accurate decisions and perform tasks efficiently. They are essential in the following ways.

- **Health & Medicine**: Doctors measure body temperature, blood pressure, and medication doses to ensure proper treatment.
- **Cooking & Baking**: Precise measurements of ingredients help achieve the right taste and texture in food.
- **Tailoring**: Precise measurements are vital for **trendy** outfits.

example of a trendy outfits

- **Time Management**: Clocks and schedules help us plan our day effectively.
- **Construction & Engineering**: Builders and engineers rely on measurements for designing safe and functional structures.
- **Sports & Fitness**: Athletes track speed, distance, and weight to improve performance.
- **Shopping & Finance**: Measuring weight, volume, and currency ensures fair transactions.
- **Environmental Monitoring**: Scientists measure air quality, temperature, and pollution levels to protect ecosystems.

Measuring length

The standard unit of length is metres (m).

Other metric system units include

Millmeter (mm) = 0.001m or $\frac{1}{1000}m \text{ or } 10^{-3}m$

Centimetres (cm) = 0.01m or $\frac{1}{100}m \text{ or } 10^{-2}m$

Kilometer (km) = 1000m $\text{or } 10^{3}m$

Common measuring instruments for length

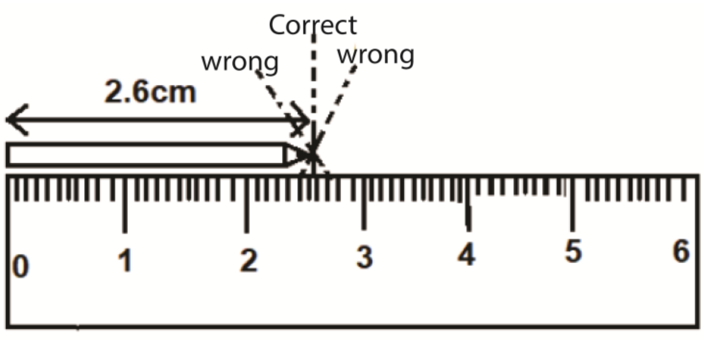
(a) **Meter rule**

How to read a metre rule

(i) Metre rule reads distances in centimetres to 1 dip e.g., 0.0cm, 10.1 cm. 94.5cm

(ii) A metre rule is suitable for distances from 0. 0cm to 100cm such as dimensions of books, small boxes, tables, chairs and so on.

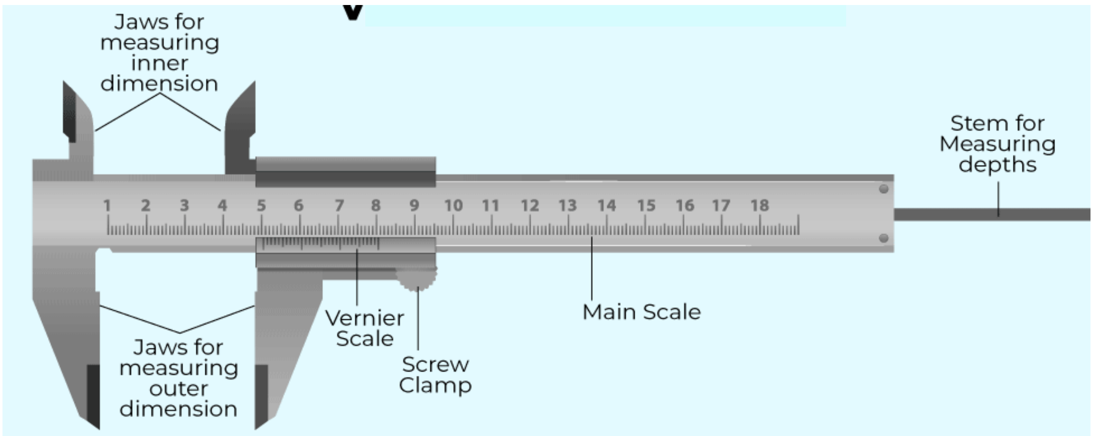
(iii) To accurately take a reading, the eye must be right over the mark on the scale as show below



(iv) If required to measure distances in metres; first measure the distances in cm and then change the readings to metres by dividing by 100. For instance,

10.1 cm = $\frac{10.1}{100}$ = 0.101m

(b) Vernier calipers



How to read a Vernier Calipers

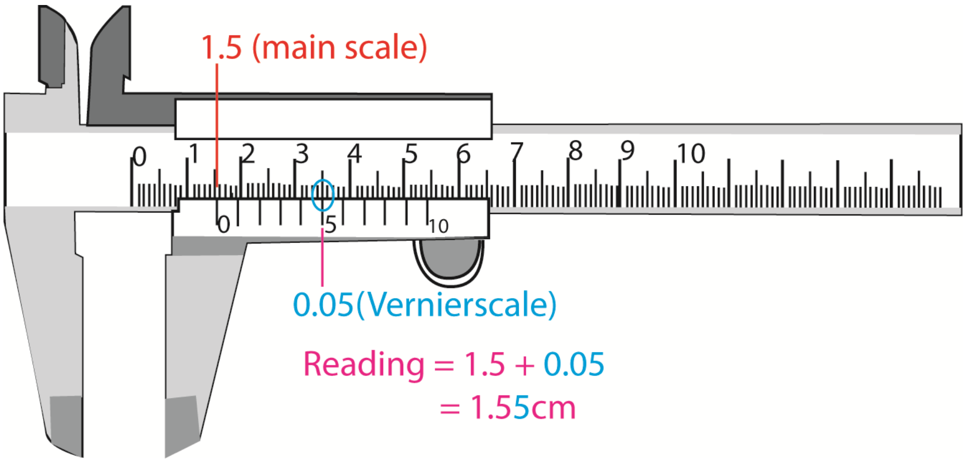
Reading a **Vernier caliper** involves understanding both the **main scale** and the **Vernier scale** to obtain precise measurements. Here's a simple step-by-step guide:

1. **Read the Main Scale** – Identify the nearest whole number or decimal value on the main scale where the zero of the Vernier scale aligns.
2. **Read the Vernier Scale** – Find the first mark on the Vernier scale that perfectly aligns with a mark on the main scale.
3. **Add the Values** – Combine the main scale reading with the Vernier scale reading to get the final measurement.
4. Vernier calipers in physics laboratory often measure from 0.00cm to 15cm or 20.00cm

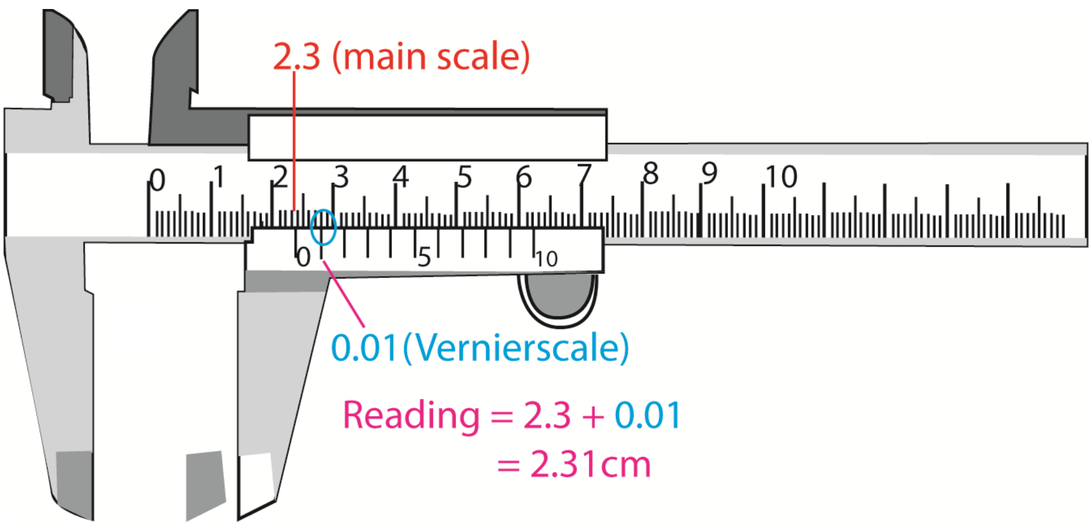
Example 1

Reading a Vernier caliper

(i)

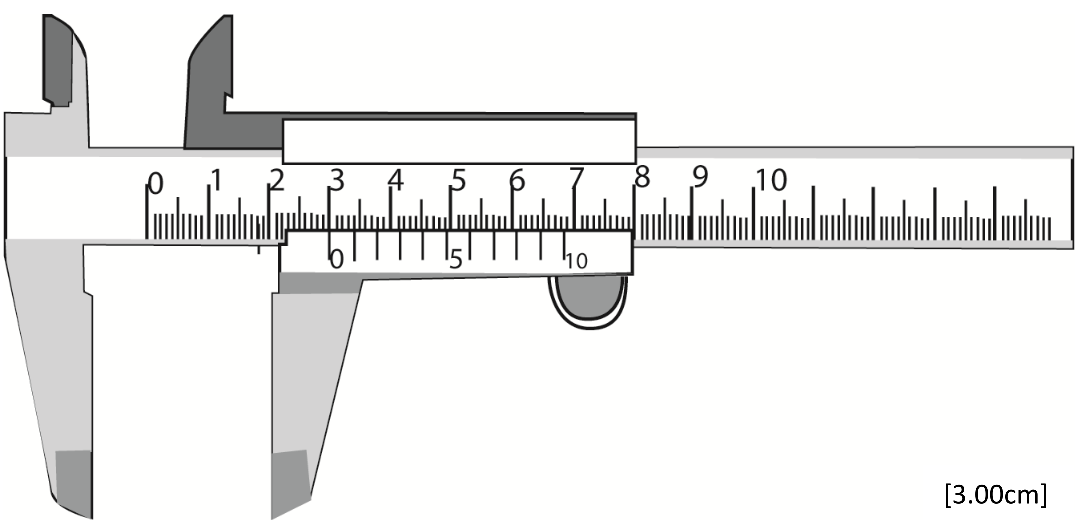


(ii)



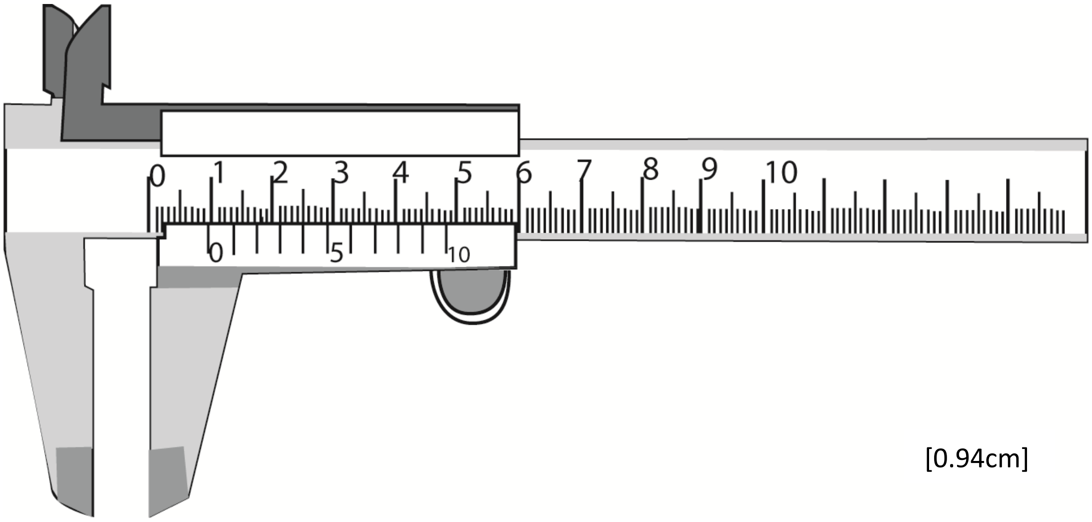
Trial 1: Find the readings on the following Vernier calipers

(i)



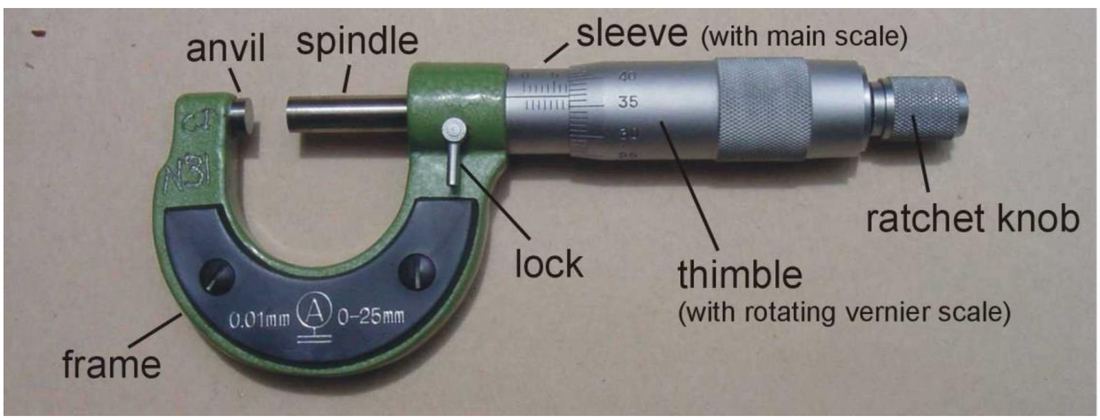
[3.00cm]

(ii)



[0.94cm]

(c) Micrometer screw gauge



How to read a micrometer screw gauge

(a) Key Parts

- **Sleeve/Barrel**: Contains the **main scale** (usually in millimeters).
- **Thimble**: Contains the **rotating scale**.
- **Ratchet Stop**: Helps apply the correct force for measurement.

(b) Read the Main Scale (Sleeve)

- Using the adjust the anvil to set the micrometer screw gauge to 0.00mm before making any readings
- Hold the object between the anvil and spindle and rotate the thimble until the object is secured and a clicking sound is heard.
- Read the visible marking on the sleeve just before the rim of thimble. Each small division on the upper side of the horizontal middle line represents **1 mm** while lower division subdivides the upper division in the middle.

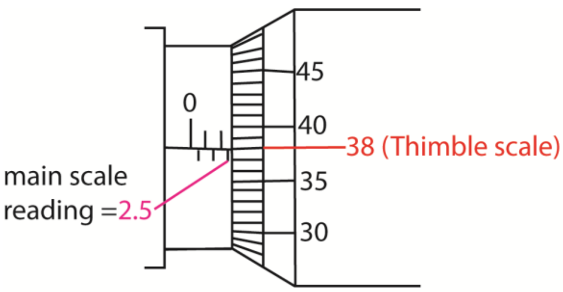
(c) Read the Thimble Scale

- Read the thimble until mark that coincides with the horizontal line of the main scale.
- Each division on the thimble represents **0.01 mm**.
- Micrometer screw gauge measures distances from 0.01mm to 25mm

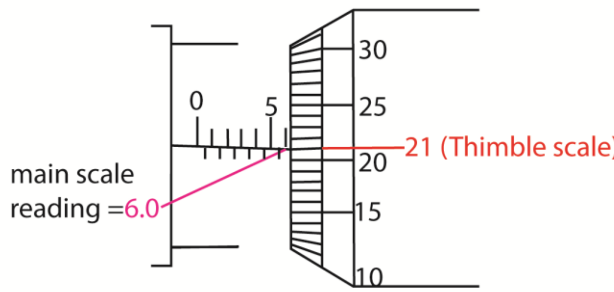
(d) Find the Total Measurement

- Add the sleeve reading to the thimble reading.
- Example: If the sleeve shows **5.0 mm** and the thimble shows **0.35 mm**, the total measurement is: **5.0 mm + 0.35 mm = 5.35 mm**

Example 2



Total reading = 2.5 + 38 x 0.01 = 2.88mm

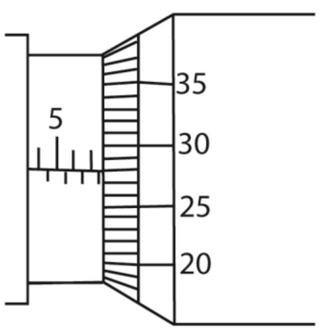


Total reading = 6.0 + 21 x 0.01 = 6.21mm

Trial 2:

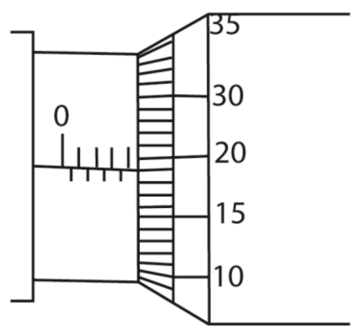
Find the readings on the following micrometer screw gauges

(i)



Total reading = 7.5 + 28 x 0.01= 7.78mm

(ii)



Total reading = 4.0 + 19 x 0.01= 4.19mm

Trail 3:

1. Convert the following to metres

   (a) 10mm
   (b) 5cm

(c) 4km
(d) 2000cm

2. Convert the following as instructed

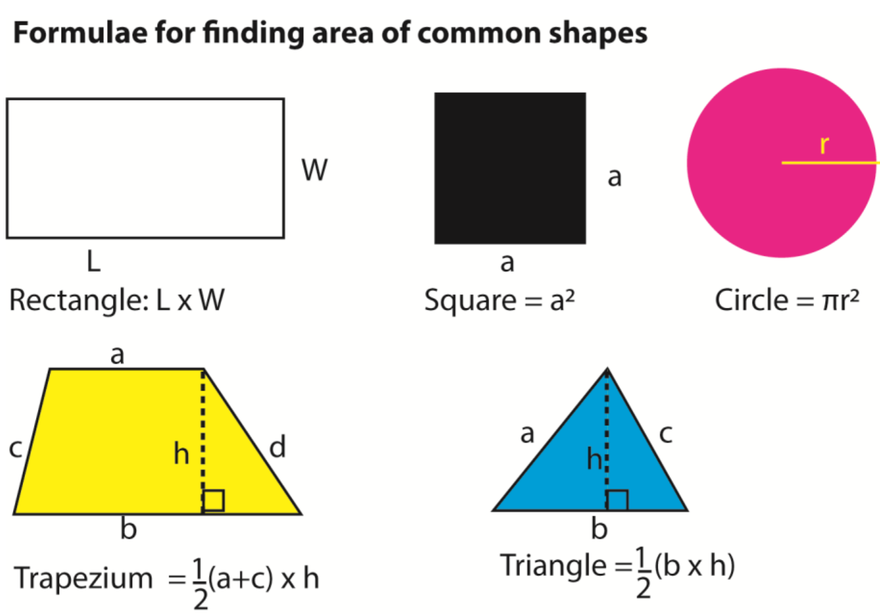
   (a) 2m to cm
   (b) 30cm to mm
   (c) 0.5km to mm
   (d) 500mm to km
   (e) 20cm to m

3. Name the instrument you would use to accurately measure the following giving reasons for your answer

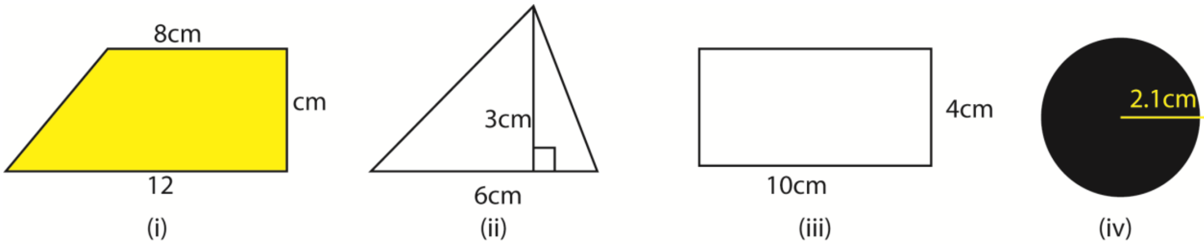
   (a) Thickness of pin
   (b) Length and width of classroom
   (c) Diameter of beaker
   (d) Thickness of paper
   (e) Length of table
   (f) Thickness of paper

Area

This is the amount of space an object occupies.



Trial 4: Find the area of each of the following:

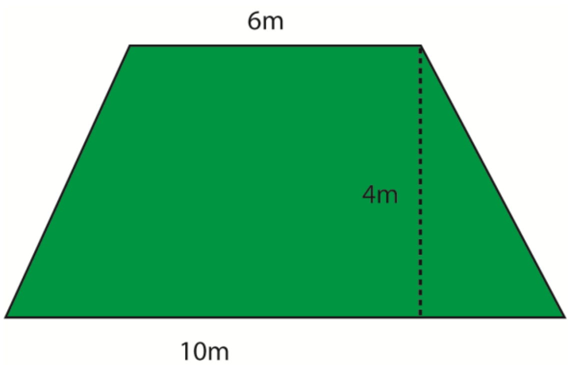


Converting units of area

To convert the units of area one may change the units of the dimensions to the preferred units and then multiply the results; or obtain the area and then convert the units directly.

Example 3

The dimensions of a maize field are given in diagram below



Find the area of the field in

(i) cm^2

Method I1m = 100cm

∴ 6m = 600cm; 10m = 1000cm and 4m = 400m

$A = \frac{1}{2}(a + b)x\,h$

$= \frac{1}{2}(600 + 1000)x\,400$

$= 320{,}000cm^2$

Method II

Area in metres = $\frac{1}{2}(\mathbf{6}+\mathbf{10})\boldsymbol{x}\ \mathbf{4}$ = 32m^2

But 1m^2 = 100cm x 100cm = 10,000cm^2

∴ 32m^2 = (32 x 10,000)cm^2

= 320,000cm^2

(ii) km^2

Method I

1m = $\frac{1}{1000}$cm

∴ 6m = $\frac{6}{1000} = 0.006$km; 10m = $\frac{10}{1000} = 0.010$km and 4m = $\frac{4}{1000} = 0.004$km

$A = \frac{1}{2}(a+b)x\ h$

$= \frac{1}{2}(0.006+0.010)x\ 0.004$

$= 0.000032km^2$

Method II

Area in metres = $\frac{1}{2}(6+10)x\ 4$ = 32m^2

But 1m^2 = $\frac{1}{1000}$ km x $\frac{1}{1000}$ km = $\frac{1}{1000000}$ km^2

∴ 32m^2 = (32 x $\frac{1}{1000000}$) km^2

$= 0.000032km^2$

Trial 4:

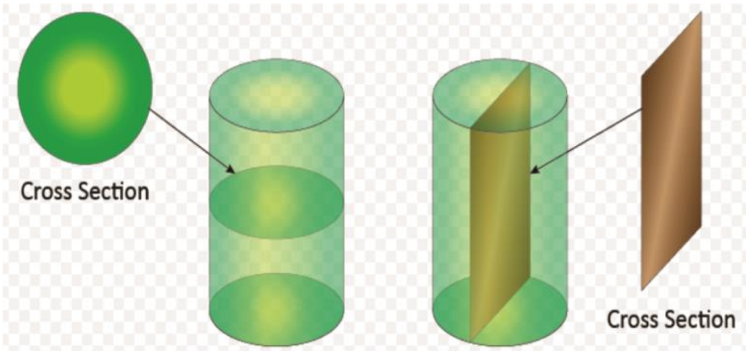
(a) Calculate following units as instructed

(i) 30mm^2 to cm^2
(ii) 100mm^2 to m^2
(iii) 24m^2 to cm^2
(iv) 32km^2 to m^2

(b) A house requires tiles for a hall measuring 30m by 15 m. If a tile measures 20cm by 10cm, how many tiles are requires.

Cross-sectional Area

The **cross-sectional area** of an object refers to the surface area of its slice perpendicular to its length. It's commonly used in engineering, physics, and construction.



Circular cross section area   Rectangular cross section area

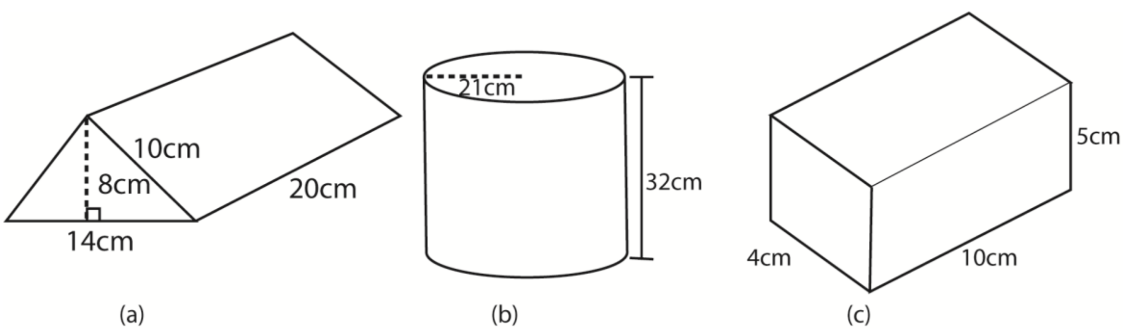
How to Calculate Cross-Sectional Area

The formula depends on the shape of the object:

- **Circle**: A= πr^2 (where r is the radius)
- **Rectangle**: A= width× length
- **Square**: A= $side^2$
- **Triangle**: A=$\frac{1}{2}$ $x\ base\ x\ height$
- **Hollow Pipe**: A=$\pi(R^2-r^2)$ (where R is the outer radius and r is the inner radius)

Trial 5:

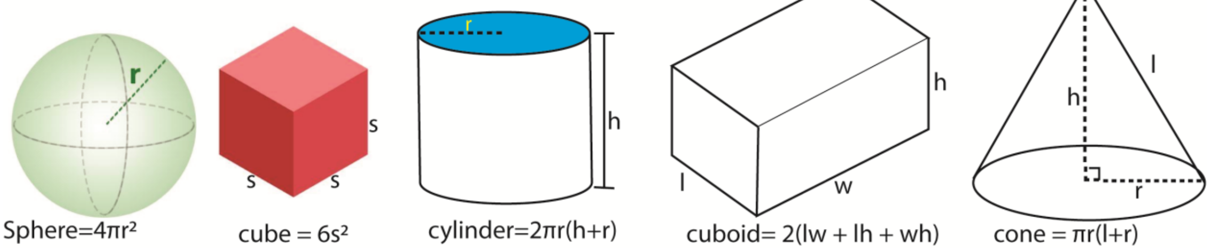
Find the cross sectional areas of the following objects in m^2.



Total surface area

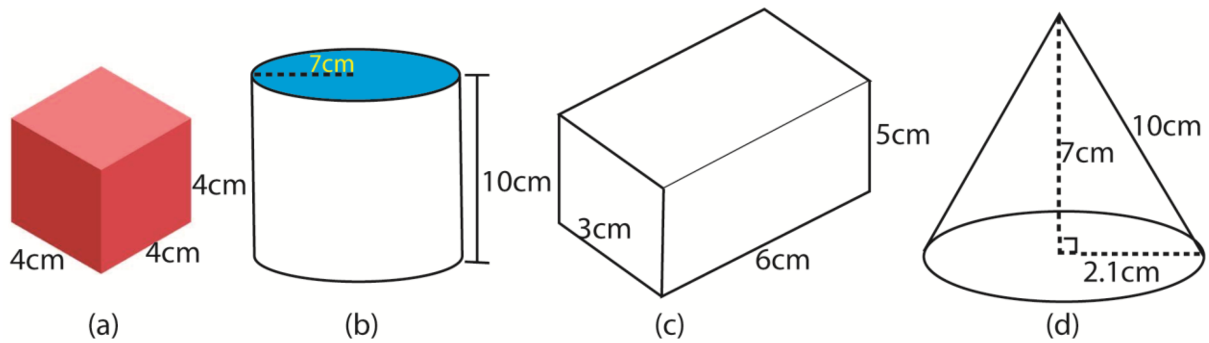
The **total surface area** of an object is the sum of the areas of all its faces or surfaces. The formula depends on the shape:

Formulae for total surface area of common shapes



Trial 6:

Find the total surface areas of the following objects in m^2.



How to find area of an irregular object

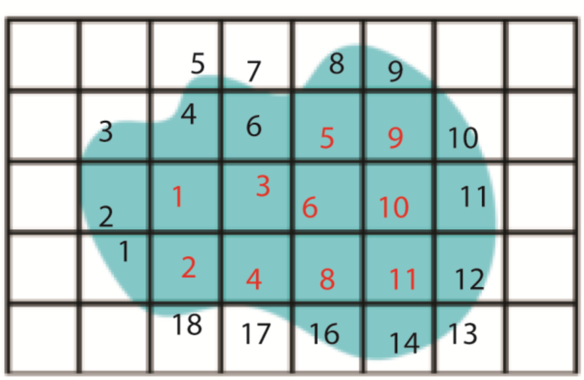
(i) Place the object on a graph paper and trace out its boundary.

(ii) Area of object =

$$\text{area of a square}\left[\text{Number of full squares} + \frac{\text{number of incomplete squares}}{2}\right]$$

Example 5

Find the area of the shape below



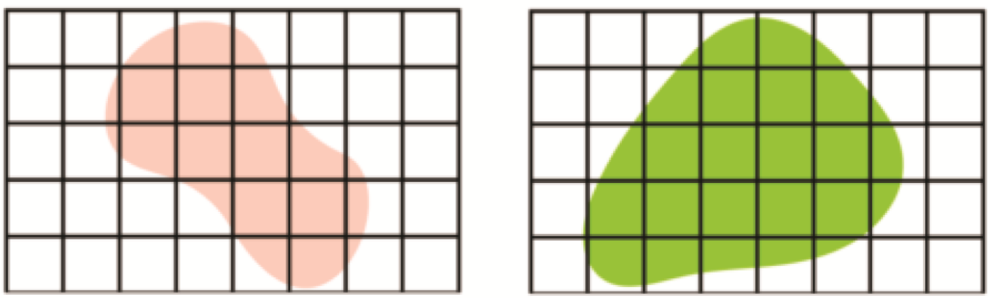
The area of the shape = $11 + \frac{18}{2}$ = 20 squares

If the area of one square is 0.25cm^2

The area of the shape = 20 x 0.25cm^2 = 5cm^2

Trial 7: Estimate the area for the following shapes below.

[the area of each square is 4cm^2]

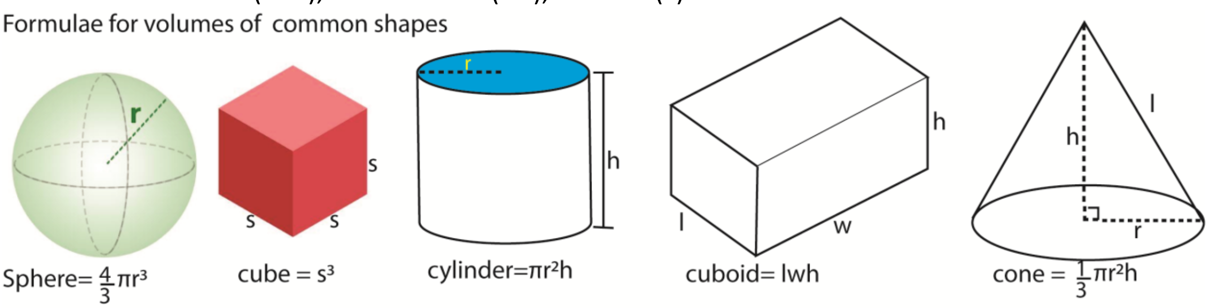


(a) (b)

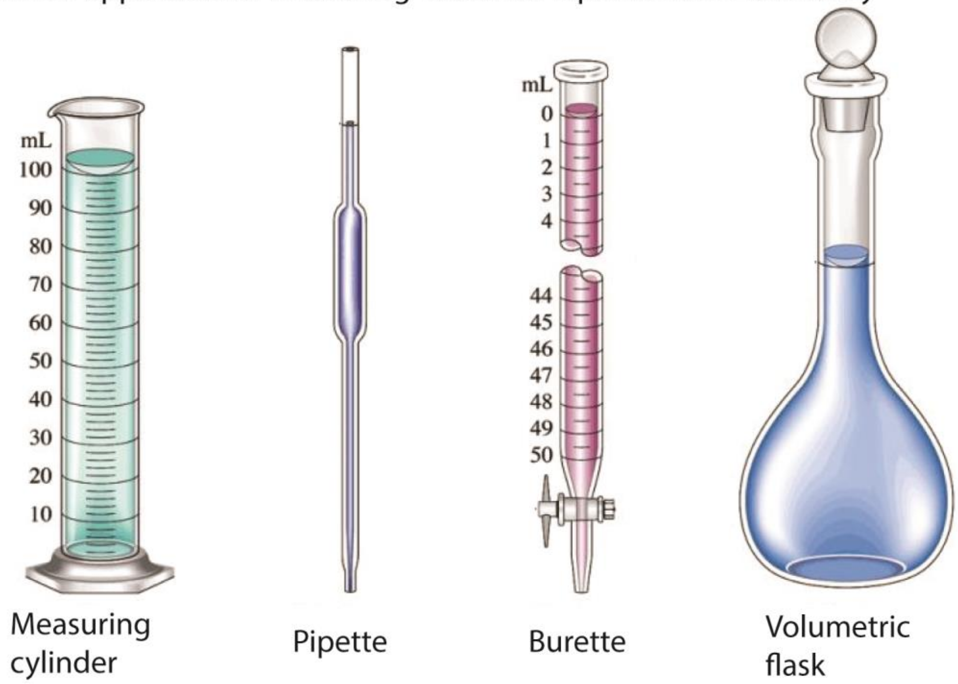
Volume

Volume is the **amount of space** that an object or substance occupies. It is measured in **cubic units** such as cubic centimeters (**cm³**), cubic meters (**m³**), or liters (**L**).

Formulae for volumes of common shapes



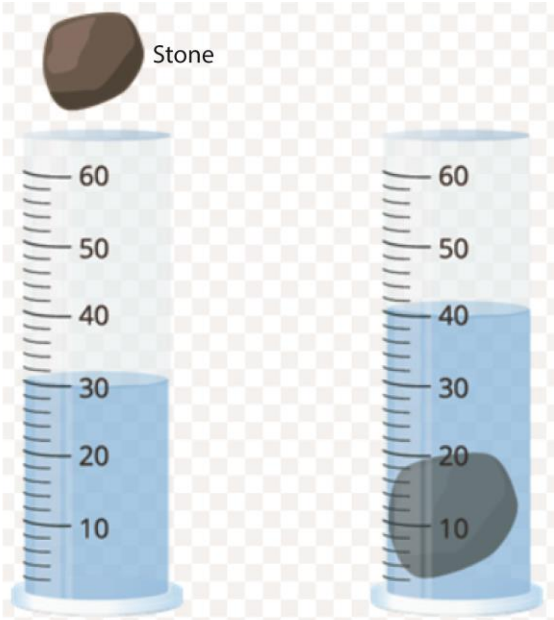
Some apparatus for measuring voume of liquids in the Laboratory



Measuring volume of irregular objects

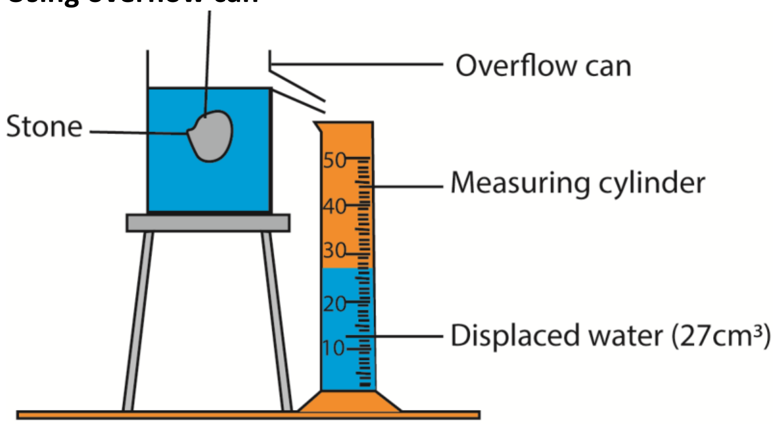
(a) Water Displacement Method (For Solid Objects)

(i) Fill a **graduated cylinder** or container with water and record the initial volume.
(ii) Submerge the **irregular object** completely in the water. (You may use a small string]
(iii) Record the **new water level**.
(iv) **Volume = Final Volume - Initial Volume**.

(v) 

Volume of the stone = 40 - 30 = 10cm³

(b) Using overflow can



Volume of stone = 27cm³

Step-by-Step Method

(i) **Fill the Overflow Can:** Fill the can with water **until it just reaches the spout**. Let any excess water **drain out** to ensure an accurate measurement.

(ii) **Submerge the Irregular Object**: Gently place the object into the water, ensuring it is **fully submerged**. The displaced water will flow out of the spout.

(iii) **Collect the Overflow Water**: Place a **measuring cylinder** or another container under the spout to collect the displaced water. Be precise—avoid splashes or evaporation.

(iv) **Measure the Volume** Read the **graduated scale** on the measuring cylinder.

(v) The volume of the collected water is **equal to the volume of the object**.

Trial 8:

(a) Convert the following units as instructed

(i) $200cm^3$ to m^3
(ii) 20 liters to cm^3
(iii) 4m3 to mm^3
(iv) 6litres to m^3

(b) Bbosa has $40{,}000cm^3$ of milk and sells a litre at UGX 500. How much will he earn from this milk.
(c) After dropping a stone into a measuring cylinder containing $50cm^3$ of water, the volume raised to $80cm^3$-mark. Find the volume of the stone.
(d) Explain how the volumes of an Irish potato can be measured.

Time

Time **is a fundamental concept** that helps us measure the duration between events. Scientifically, time is the ongoing progression of existence, SI unit of time is seconds (s) but it can be measured in minutes, hours, days and years.

Relationships among seconds, minutes, hours and days

1day = 24hours

1hour = 60 minutes

1minute = 60 seconds

Measuring time

Measuring time involves using different tools and units depending on the level of precision needed.

Instrument for measuring time

Stop clock

Stop watch

Trial 9:

(a) Convert the following

- (i) 2.5hour to seconds
- (ii) 48hours to days
- (iii) 1 day to minutes and then seconds
- (iv) 72minutes to hours

(b) How many seconds are in

- (i) 4 milliseconds
- (ii) 30 microseconds

Mass

Mass is the quantity of matter contained in the body. SI unit of mass is kilogram. Other units are grams, and tonnes.

Apparatus for measuring mass in school laboratories

Beam balance

Digital balance

Trial 10:

Convert the following to kg

- (i) 10mg
- (ii) 100g
- (iii) 2 tonnes

Scientific significant figures

Significant figures are the digits in a number that contribute to its precision. They are important in scientific measurements and calculations to ensure accuracy without unnecessary rounding or exaggeration.

Rules for Determining Significant Figures

1. Nonzero digits are always significant (e.g., 123 has 3 significant figures).
2. Leading zeros (zeros before the first nonzero digit) are not significant (e.g., 0.0032 has 2 significant figures).
3. Zeros between nonzero digits are significant (e.g., 103 has 3 significant figures).
4. Trailing zeros in a decimal number are significant (e.g., 2.400 has 4 significant figures).
5. Trailing zeros in a whole number without a decimal point may or may not be significant, depending on notation (e.g., 1500 has 2 significant figures, but 1500. has 4).

Why Significant Figures Matter

- They help indicate **measurement precision**.
- They prevent **overestimating accuracy** in scientific calculations.
- They ensure consistency in rounding when performing mathematical operations.

Rounding off numbers

Rounding off numbers helps simplify values while maintaining reasonable accuracy. The rounding method depends on the place value you're focusing on. Here's how it works:

General Rounding Rules

1. **Identify the place value** to round to (e.g., nearest **ten, hundred, decimal place**).
2. **Check the next digit**:
    - If it's **5 or more**, **increase** the rounding digit by **1**.
    - If it's **4 or less**, **keep** the rounding digit the same.
3. **Replace remaining digits** with zeros (for whole numbers) or remove them (for decimals).

Example 6

- Rounding 376 to the nearest ten → 380 (since 6 is **5 or more**, increase the 7 to 8).
- Rounding 2.347 to the nearest hundredth → 2.35 (since 7 is **5 or more**, increase the 4 to 5).

Multiplication of numbers

(i) When multiplying or dividing numbers with differing significant figures, the resultant takes the lower number of significant figures used in obtaining the result

(ii) For addition or subtraction of numbers, the result takes the lower number of decimal places.

Example 7

(a) Find the difference between 1.456 and 0.9

1.456 – 0.9 = 0.556

= 0.6 [0.9 has least number of d.p (1d.p)]

(b) Divide 3.42 by 1.645

3.42 ÷ 1.645 = 2.079

= 2.08[3.42 has the least number of significant figures (3)]

Trial 11:

(a) Write the following numbers to 3 s.f.

(i) 304678
(ii) 0.62547
(iii) 4.267×10^{-3}
(iv) 9.4593
(v) 5.4754×10^{5}

(b) Perform the following arithmetic

(i) 2.005x 3.07
(ii) 1.068 x 0.42
(iii) 1.25 + 14.235
(iv) 1.067 ÷ 0.25
(v) $\frac{3.68+4.978}{0.0973-3.26\ x\ 20.3}$

(c) Round of the following to 2 significant figures

(i) 9.4678

(ii) 0.63247

(iii) 4.467×10^{-3}

(iv) 5.4744×10^{6}

Scientific notation (exponential or standard form)

Standard form is a way of writing very large or very small numbers concisely using powers of ten. It is commonly used in mathematics and science.

How to Write a Number in Standard Form

The format is: $a \times 10^{n}$

Where:

- a is a number between **1 and 9**
- n is the **power of 10** (positive for large numbers, negative for small numbers)

Example 8

1. **Large number**:
    - 5,600,0005,600,000 → 5.6×10^{6}
    - The decimal point moves **6 places** to the left.
2. **Small number**:
    - 0.00042 → 4.2×10^{-4}
    - The decimal point moves **4 places** to the right.

Trial 12: Write the following number in scientific notation

(i) 0.00009

(ii) 4500000

(iii) 0.049

(iv) 527200 to 2 s.f.

Physical quantities

Physical quantities are divided into two groups

Fundamental quantities

These are physical quantities which cannot be expressed in form of other quantities using any mathematical equations.

They include

Quantity	S.I unit	Symbol of S.I unit
Mass	kilogram	kg
Time	second	s
Length	metres	m
Temperature	Kelvin	K
Current	Ampere	A

Dimensional/derived quantities

These are physical quantities which can be expressed in terms of fundamental quantities. For example velocity, work, volume, density

Dimensions of physical quantities

This is a way in which derived quantities can be expressed in form of fundamental quantities. i.e.

Mass- M

Length- L

Time –T

The square bracket, [ ] is used to show dimensions

For example

(i) [Area] = length x length or length x width

$L \times L = L^2$

(ii) Volume = length x width x height

$= L \times L \times L = L^3$

(iii) Density = $\left[\frac{mass}{volume}\right] = \frac{M}{L^3}$ or ML^{-3}

(iv) Velocity = $\left[\frac{lenght}{time}\right] = \frac{L}{T} = LT^{-1}$

(v) Acceleration $= \frac{velocity}{time} = \frac{LT^{-1}}{T} = LT^{-2}$

(vi) Force = mass x acceleration

$= M \times LT^{-2} = MLT^{-2}$

Application of dimensions

(i) To check the validity of equation (equations that are not dimensionally consistent are obviously wrong expression and should be discarded.

(ii) To derive equations: for correct equation, the units of the left hand side must be similar to the units of the right hand side.

All right equations must be dimensionally consistent but not all dimensionally consistent equations are correct.

Examples 9

(a) The centripetal force F on a body of Mass M moving at constant speed V round a circular path of radius, r, is given by F = $\frac{MV^2}{r}$.

Show that the equation is dimensionally consistent

Solution

LHS = F = Ma = M x LT^{-2} = MLT^{-2}

RHS = $\frac{MV^2}{r}$. = $\frac{M\,x\,(LT^{-1})^2}{L}$ = MLT^{-2}

Since [LHS] =[RHS], the equation is dimensionally consistent.

(b) Show that the second equation of motion is dimensionally correct

s = ut + ½ at^2

[LHS] = [s] = L

[RHS]= [u][t] + ½ [a]$[T]^2$

=LT^{-1} x T + ½ LT^{-2} xT^2

= L + ½ L = $\frac{3}{2}L$

Since $\frac{3}{2}$ is a constant,

[LHS] =[RHS] showing that the equation is dimensionally consistent.

(c) The equation of a transverse wave of a rod of youngers modulus (E) and density, ρ, is given by v = $\sqrt{\frac{E}{\rho}}$. Show that it is dimensionally consistent.

Solution

[LHS] = [v] = LT^{-1}

[RHS]

E = $\frac{stress}{strain}$ = $\left[\frac{MLT^{-2}}{L^2}\right]$ = $ML^{-1}T^{-2}$

$\sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}}$ = $\sqrt{(L^2T^{-2})}$= LT^{-1}

[LHS] =[RHS] , the equation is dimensionally consistent

Derivation of equation

the method of dimensions can be used to derive equations.

Examples 10

The period (T) of a pendulum bob depends on the length of the pendulum (L), mass of the pendulum ball, M, and acceleration due to gravity, g. Determine an expression for the period of a simple pendulum T in terms of the quantities mentioned.

Solution

T $\propto MLg$

T = $kM^xL^yg^z$ where k is dimensionless constant

$[T] = [M]^x[L]^y[g]^z$
$[LHS] = [T] = T$
$[RHS] = M^xL^y[LT^{-2}]^z$

Equate powers of T, M,L

For T; $-2z = 1$
$z = -½$
For L; $y + z = 0$
$y – ½ = 0$
$y = ½$
For M, $x = 0$

Therefore, $T = KL^{-\frac{1}{2}} \text{ x } g^{\frac{1}{2}} = K\sqrt{\frac{L}{g}}$

$T = K\sqrt{\frac{L}{g}}$

Trial 13

1. The sphere of radius, α, moving through a liquid of density, ρ, and velocity, v, experiences a retarding force given by $F = k\alpha^x\rho^y v^z$, where K is a non-dimensional constant. Use dimensions to find the values of x, y and z. [Ans, y =1, z =2 , y = 2]

2. Use dimensional analysis to show how the process of velocity transverse process vibration of a stretched string depends on its length, L, mass, m, and the tension F of the string $V = KL^xM^yF^z$, where k is a non-dimensional constant. Find the values of x, y, z. [Ans, x = ½ , y = - ½ , z = 1/2]
3. A cylindrical vessel of cross section area, A, contains air of volume, V, atmospheric pressure, ρ, trapped by frictionless down and released.

If the piston oscillates with simple harmonic motion, show that the frequency is given by $f = \frac{A}{2\pi}\sqrt{\frac{\rho}{MV}}$ and show that the expression is correction.

4. The equation for volume, V, of a liquid flowing through a pipe in time, t, under steady flow id given by, $\frac{V}{t} = \frac{\pi r^4 \rho}{8\eta l}$
r = radius of the pipe
ρ = pressure difference between the two ends of the pipe
l = length of the pipe
η =coefficient of viscosity of the liquid
If the dimensions of η are $ML^{-1}T^{-1}$ show that the above equation is dimensionally consistent.
5. For streamline flow of a non-viscous incompressible fluid , the pressure, ρ, at a point is related to height, h. and velocity, V, by the equation

(p-a)=pg(h-b) + ½ p(v^2 –d)
Where a, b, and d are constant and p is the density of fluid and g is the acceleration due to gravity. Given that the equation is dimensionally consistent. Find the dimensions of a, b, and d.

Solution

Hint, we add or subtract quantities that have the same dimensions

{ans. [a] has the same units as pressure = $ML^{-1}T^{-2}$, [b] has the same dimension as h = L, [d] has the same dimensions as v^2 = L^2T^{-2}}

Thank you
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