SENIOR FIVE TERM 1

TOPIC 1/6: Numerical Concepts

Contents

1.1 Indices

A number or a variable can have an index. The index tells us how many times to multiply a number its itself. In simple words, an index is a numerical value that indicates how many times a number (the base) is multiplied by itself.

It is written like this: $\mathbf{a^m = a \times a \times a \times \ldots \times a}$ (m times)

Here, **a** is called the **base**, and **m** is the **index**.

Key points:

- **Base:** The number being multiplied.
- **Index (Exponent):** Indicates the number of times the base is used as a factor.

In simple words, the index shows how many times we should multiply the base by itself. It is a short way to write repeated multiplication and makes calculations easier.

- **For example:** $\mathbf{2^3} = 2 \times 2 \times 2 = \mathbf{8}$
- Here, **2** is the base and **3** is the index.

There are five basic rules of indices

(a) $a^p \, x \, a^q = a^{p+q}$

(b) $\frac{a^p}{a^q} = a^{p-q}$

(c) $(a^p)^q = a^{pq}$

(d) $a^{\frac{1}{q}} = \sqrt[q]{a}$

(e) $a^{\frac{p}{q}} = \left(\sqrt[q]{a}\right)^p$

Example 1

Evaluate the following

(a) $2^2 \text{ x } 2^3$

(b) $\frac{4^3}{4^2}$

(c) $(3^2)^3$

(d) $2^{\frac{1}{2}} \, x \, 2^{\frac{1}{2}}$

(e) $\sqrt[3]{27}$

(f) $125^{\frac{2}{3}}$

Solution

(a) $2^2 \text{ x } 2^3 = 2^{2+3} = 2^5 = 32$

(b) $\frac{4^3}{4^2} = 4^{3-2} = 4^1 = 4$

(c) $(3^2)^3 = 3^{2\,x\,3} = 3^6 = 729$

(d) $2^{\frac{1}{2}} \, x \, 2^{\frac{1}{2}} = 2^{\frac{1}{2}+\frac{1}{2}} = 2^1 = 2$

(e) $\sqrt[3]{27} = (3^3)^{\frac{1}{3}} = 3^{3\,x\frac{1}{3}} = 3^1 = 3$

(f) $125^{\frac{2}{3}} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

Example 2

Evaluate the following

(a) $\left(\frac{125}{27}\right)^{\frac{4}{3}}$

(b) $81^{\frac{3}{4}}$

Solution

(a) $\left(\frac{125}{27}\right)^{\frac{4}{3}} = \left(\frac{125^{\frac{4}{3}}}{27^{\frac{4}{3}}}\right) = \left(\frac{\left(\sqrt[3]{125}\right)^4}{\left(\sqrt[3]{27}\right)^4}\right) = \frac{625}{81}$

(b) $81^{\frac{3}{4}} = \left(\sqrt[4]{81}\right)^3 = 27$

The zero index

From $\frac{a^p}{a^p} = a^{p-p} = a^0 = 1$

∴ Any number raised to power zero =1

i.e. $100^0 = 529^0 = 83^0 = 1$

Negative indices

It can be shown that

$\frac{1}{a} = \frac{a^0}{a^1} = a^{0-1} = a^{-1}$

Also

$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

Hence a negative index is the inverse of a given number

Example 3

Evaluate the following

(a) $16^{\frac{-3}{2}}$

(b) $\left(\frac{64}{27}\right)^{-\frac{2}{3}}$

Solution

(a) $16^{\frac{-3}{2}} = \left(\frac{1}{16}\right)^{\frac{3}{2}} = \left(\frac{1}{\sqrt{16}}\right)^{3} = \left(\frac{1}{4}\right)^{3} = \frac{1}{64}$

(b) $\left(\frac{64}{27}\right)^{-\frac{2}{3}}=\left(\frac{27}{62}\right)^{\frac{2}{3}} = \left(\frac{\sqrt[3]{27}}{\sqrt[3]{64}}\right)^{2} = \frac{9}{16}$

Solving equations with unknown indices

It involves making appropriate substation after expressing terms containing powers in simplified form

Example 4

Solve the equation

$2^{2x+1} - 7(2^x) + 6 = 0$

Solution

$2^{2x+1} - 7(2^x) + 6 = 0$

$2^1 . 2^{2x} - 7(2^x) + 6 = 0$

$2(2^x)^2 - 7(2^x) + 6 = 0$

Let p = 2^x

⇨ $2p^2 - 7p + 6 = 0$
$(2p - 3)(p - 2) = 0$

Either 2p -3 = 0

p = $\frac{3}{2}$

or

p – 2 =0

p = 2

when p= $\frac{3}{2}$ => $2^x = \frac{3}{2}$

$\log 2^x = \log\frac{3}{2}$

$x \log 2 = \log\frac{3}{2}$

$x = \frac{\log\frac{3}{2}}{\log s} = 0.585$

When p = 2

$2^x = 2 = 2^1$

x = 1

Hence x = 1 and x = 0.585 (3d.p)

Example 5

Show that

$\frac{3(2^{x+1})-4(2^{x-1})}{2^{x+1}-2^x} = 4$

Solution

$\frac{3(2^{x+1})-4(2^{x-1})}{2^{x+1}-2^x} = 4$

Handling terms on the LHS

$\frac{3(2^{x+1})-4(2^{x-1})}{2^{x+1}-2^x}$

$= \frac{3(2^x\, x 2^1)-4(2^x.2^1)}{2^x.2^1-2^x}$

$= \frac{2^x(3\,.\,2^1-4\,.\,2^1)}{2^x(2^1-1)} = \frac{6-2}{1} = 4$

Example 5

Solve $x^{\frac{4}{3}} = 81$

$x^{\frac{4}{3}} x^{\frac{3}{4}} = 81^{\frac{3}{4}}$

x = $\left(\sqrt[4]{81}\right)^2$ = $3^3 = 27$

Solving equations with squares

Example 6

$\sqrt{2x+5} = x + 1$

Square both sides

$\left(\sqrt{2x+5}\right)^2 = (x+1)^2$

2x + 5 = x^2+ 2x + 1

x^2 = 4

x = ± 2

Testing/checking using -2

$\sqrt{2x+5} = x+1$

$\sqrt{2\,x-2+5} = -2+1$

1 $\neq$ -1

Hence -2 is **not** a solution to the equation

Testing/checking using 2

$\sqrt{2x+5} = x+1$

$\sqrt{2\,x2+5} = 2+1$

$3 = 3$

Hence 2 is the solution to the equation

Example 7

Solve for x: $\sqrt{x+2} = 4$

Square both sides

$\left(\sqrt{x+2}\right)^2 = 4^2$

x + 2 = 16

x = 14

Finding square roots of terms containing rational and irrational quantities

When finding roots of terms expressed in the form a + $\sqrt{b}$, where a is a rational and b is an irrational quantity, we let the root to be in the form of $\pm(\sqrt{x_1}+\sqrt{x_2})$ where x_1 and x_2 are integers.

Example 8

Find the square root of 6 +2$\sqrt{5}$

Let $\pm(\sqrt{x_1}+\sqrt{x_2})$ be square root of 6 +2$\sqrt{5}$

⇨ $\pm(\sqrt{x_1}+\sqrt{x_2}) = \sqrt{6\ +2\sqrt{5}}$

Squaring both sides

$(\sqrt{x_1}+\sqrt{x_2})^2 = \left(\sqrt{6+\sqrt{5}}\right)^2$

$x_1 + x_2 + 2\sqrt{x_1.x_2}$ =6 +2$\sqrt{5}$

Comparing terms on the two sides

$x_1 + x_2 = 6$

$x_1 = 6 - \ x_2$ ....(i)

$x_1.x_2 = 5$.........................(ii)

Substituting eqn. (i) into eqn. (ii)

$(6 - \ x_2)x_2$ = 5

$x_1{}^2 - 6x_2 + 5 = 0$

$x_1{}^2 - x_2 - 5x_2 + 5 = 0$

$x_2(x_2 - 1) - 5(x_2 - 1) = 0$

$(x_2 - 1)((x_2 - 5) = 0$

Either: $x_2 - 1$ = 0 => $x_2 = 1$

Or $x_2 - 5$ = 0 => $x_2 = 5$

When$x_2 = 1$: $x_1 = 6 - 1$ = 5

When$x_2 = 5$: $x_1 = 6 - 5$ = 1

Hence the square root of 6 +2$\sqrt{5}$ is $\pm\left(1+\sqrt{5}\right)$

Example 9

Find the square root of 8 - 2$\sqrt{15}$

et $\pm(\sqrt{x_1}-\sqrt{x_2})$ be square root of 8 -2$\sqrt{15}$

$\pm(\sqrt{x_1}+\sqrt{x_2}) = \sqrt{8-2\sqrt{15}}$

Squaring both sides

$(\sqrt{x_1}-\sqrt{x_2})^2 = \left(\sqrt{8-2\sqrt{15}}\right)^2$

$x_1 + x_2 - 2\sqrt{x_1.x_2} = 8\ - 2\sqrt{15}$

Comparing terms on the two sides

$x_1 + x_2 = 8$

$x_1 = 8 - x_2$ ……………….(i)

$x_1 . x_2 = 15$……………………(ii)

Substituting eqn. (i) into eqn. (ii)

$(8 - x_2)x_2$ = 15

${x_1}^2 - 8x_2 + 15 = 0$

${x_1}^2 - 3x_2 - 5x_2 + 15 = 0$

$x_2(x_2 - 3) - 5(x_2 - 3) = 0$

$(x_2 - 3)((x_2 - 5) = 0$

Either: $x_2 - 5$ = 0 => $x_2 = 5$

Or $x_2 - 3$ = 0 => $x_2 = 3$

When$x_2 = 5$: $x_1 = 8 - 5$ = 3

When$x_2 = 3$: $x_1 = 8 - 3$ = 5

Hence the square root of 8 -2$\sqrt{15}$ is $\pm(\sqrt{5} - \sqrt{3})$

Application of indices

Indices appear frequently in mathematical expressions. Here are some real-life examples:

(i) **Scientific notation:**

e.g. The mass of an electron is 9.11×10^{-31}kg.

(ii) **Algebra:** $x^3 + 2x^2 - x + 5$

(iii) **Geometry:** Area of a square: $A = s^2$

(iv) **Compound interest:**

Formula: $A = P(1 + r)^t$,

E.g. If you invest $1,000 at 5% interest for 3 years:

$A=1000\cdot(1.05)^3=1157.63$

(v) Population Growth

Formula: $P=P_0\cdot r^t$

Example: If a population doubles every year, after 5 years it's $P_0\cdot 2^5=P_0\cdot 32$

(vi) Computing: Data Storage

Example: 1 GB = 2^{30}bytes

These examples illustrate how useful and versatile indices are in maths.

Conclusion

Mastering the concept of index is a key part of mathematics, especially with exponential expressions and algebraic simplifications. From understanding the definition to applying the laws and rules of indices, the use of indices in maths is broad and necessary.

By grasping the meaning of exponents and applying real-world examples of indices, learners can discover new problem-solving strategies and strengthen their mathematical intuition. So the next time you see a small number raised above another, remember that's the mighty index, making math both simple and powerful

Revision exercise on indices

1. Simplify

(i) $9a^2 \div 27a^{-4}$ $\left[\frac{2}{3}a^6\right]$

(ii) $(6a^{-3})\div(9a^{-4})^2$ $\left[\frac{2}{27}a^5\right]$

(iii) $\frac{2a^{-3}b^2}{7c^{-4}d^2}$ $\left[\frac{2b^2c^4}{7a^3d^2}\right]$

(iv) $(x^4yz^{-3})^2 \times \sqrt{x^{-5}y^2z} \div (xz)^{\frac{1}{2}}$ $[x^5yz^{-6}]$

(v) $\sqrt[4]{y^3}x\sqrt{y^{\frac{1}{2}}}$ $\left[y^{\frac{5}{4}}\right]$

2. Evaluate

(a) $(64)^{-\frac{3}{2}}$ [16]

(b) $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ $\left[\frac{3}{2}\right]$

(c) $\left(\frac{1}{25}\right)^{\frac{1}{2}}$ $\left[\frac{1}{5}\right]$

(d) $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ $\left[\frac{4}{9}\right]$

(e) $\left(\frac{243}{512}\right)^{-\frac{2}{3}}$ [1.6445]

3. Solve the following equations

(a) $98x^2 = 2$ [x= 0.1429]

(b) $x^{-3} = 8$ $\left[x = \frac{1}{2}\right]$

(c) $\frac{1}{32}x^3 = 8x^{-1}$[x = 4]

(d) $\frac{9}{25}x = \frac{5}{3}x^{-2}$ $\left[x = \frac{5}{3}\right]$

(e) $\frac{2}{14}x^{-2} + 14x = 0$ [x = -0.2169]

4. Solve for x

(a) $3^{2x+1} + 3 = 10(3^x)$ [x = 1 or x = -1]

(b) $2^{2x-1} + \frac{3}{2} = 2^{x+1}$[x =0, x =1.585]

(c) $7^x = 3^{1-x}$ [x=0.3608]

(d) $7x^{\frac{1}{2}}+2 = 0\left[x = \frac{4}{49}\right]$

(e) $5x^{\frac{2}{3}} = x^{-\frac{1}{3}}$ $\left[x = \frac{1}{5}\right]$

(f) $4x^{-\frac{1}{3}} = 5x^{\frac{1}{6}}$ $\left[x = \frac{16}{25}\right]$

(g) $6x^{\frac{2}{3}} - \frac{2}{3}x^{-\frac{1}{2}} = 0$ [x = 0.077]

(h) $8x^{-2} = 343x^{\frac{1}{2}}$ [x=0.003562]

5. Show that

(a) $\frac{(2^{2x}-3.2^{2x-2})(3^x-2.3^{x-2})}{3^{x-4}(4^{x+3}-2^{2x})} = \frac{1}{4}$

(b) $\frac{(1+a)^{\frac{1}{2}}-\frac{1}{3}a(1+a)^{-\frac{2}{3}}}{(1+a)^{\frac{2}{3}}} = \frac{3+2a}{3(1+a)^{\frac{4}{3}}}$

(c) $(a - a^{-1})\left(a^{\frac{4}{3}} - a^{\frac{2}{3}}\right) = \frac{a^2-a^{-2}}{a^{-\frac{1}{3}}}$

(d) $\frac{a^{\frac{1}{2}}+ab}{ab-b^2} - \frac{\sqrt{a}}{\sqrt{a}-b} = \sqrt{\frac{a}{b}}$

6. Solve

(ii) $x^{\frac{1}{3}} - 3 = 28x^{-\frac{1}{3}}$ [x =-64, x = 343]

(iii) $2x^{\frac{1}{4}} = 9 - 4x^{-\frac{1}{4}}$[x =$\frac{1}{16}$, x = 256]

(iv) $x^3 + 8 = 9x^{\frac{3}{2}}$ [x = 1, x= 4]

(v) $2x^{\frac{1}{3}} = \frac{81}{8}x^{-1}$ [x = 8.6967]

(vi) $49x^{-\frac{5}{6}} - \frac{8}{81}x^{\frac{7}{6}} = 0$ [x=22.2739]

(vii) $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 2 = 0$[x=-1]

(viii) $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$[x=-1]

(ix) $6x^{\frac{1}{3}} + 5 + x^{-\frac{1}{3}} = 0$ $\left[x = \frac{1}{2}, x = \frac{1}{3}\right]$

7. Solve for x

(a) $\sqrt{x+2} - x = 0$[x=2]

(b) $\sqrt{1+x} = 1 + \sqrt{1-x}$ $\left[x = \frac{\sqrt{3}}{2}\right]$

(c) $(3-x)^{\frac{1}{2}} = (1+x)^{\frac{1}{2}} + (2-x)^{\frac{1}{2}}$ [x = -0.92665]

(d) $\sqrt{x+6} = \sqrt{1-3x} - \sqrt{4-x}$ [-5]

8. Without using mathematical tables or calculators, find the value of

$\frac{(\sqrt{5}+2)^2-(\sqrt{5}-2)^2}{8\sqrt{5}}$ [1]

9. Find the square root of the following

(a) $6 + 2\sqrt{5}$ $[\pm(1+\sqrt{5})]$

(b) $18 - 2\sqrt{12}[\pm(\sqrt{0.695} - \sqrt{17.303})]$

10. Solve the following equations

(a) $3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$ [1, 0]

(b) $2^{2x+1} - 2^{x+1} + 1 = 2^x$ [-1, 0]

(c) $3(3^{2x}) + 2(3^x) - 1 = 0$ [-1]

(d) $3^{2x+1} - 26(3^x) = 9$ [2]

(e) $9x^{\frac{2}{3}} + 5x^{-\frac{2}{3}} = 37$ $\left[x = \frac{1}{27} or\ 8\right]$

(f) $(x^2 - 2x)^2 + 24 = 11(x^2 - 2x)$ [x= -2, -1, 3, or 4]

(g) $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

(h) $4^x - 2^{x+1} - 15 = 0$ [x=2.322]

11. Solve the simultaneous equations

$2^x + 4^y = 12$

$3(2^x) - 2(2^y)=16$ [x=3, y=1]

12. Solve the simultaneous equation

$2^x + 4^y = 12$

$3(2)^x - 2(2)^{2y} = 16$ [x= 2, y = 1]

Hence show that $(4)^x + 4(3)^{2y} = 100$

1.2 Logarithms

A logarithm is an exponent, an index or power

The logarithm of a positive quantity p to a given base q is defined as the index or power to which the bases q must be raised to make it equal to P. i.e. $\log_q p = x \text{ means that } q^x = p$ or x is the logarithm of p to base q

- x is the power (index, logarithm or exponent)
- q is the base
- p is the number (which must be positive)

Example 1

Find the values of x in the following

(a) $\log_2 8 = x$
Solution
8 = 2^3
$\therefore \log_2 8 = 3$; x = 3

(b) $\log_x 25 = 2$
Solution
25 = 5^2
⇨ $x^2 = 5^2$

$\therefore x = 5$

Example 2

Evaluate

(a) $\log_{27} 9\sqrt{3}$

Solution

Let $\log_{27} 9\sqrt{3}$ = x

$27^x = 9\sqrt{3}$

$3^{3x} = 3^2.3^{\frac{1}{2}} = 3^{\frac{5}{2}}$

Equating powers

3x = $\frac{5}{2}$

x = $\frac{5}{6}$

$\therefore \log_{27} 9\sqrt{3} = \frac{5}{6}$

(b) $\log_{\frac{1}{2}} \frac{1}{4}$

Solution

Let $\log_{\frac{1}{2}} \frac{1}{4}$ = x

$\left(\frac{1}{2}\right)^x = \frac{1}{4}$

$= \left(\frac{1}{2}\right)^2$

Equating powers x = 2

$\therefore \log_{\frac{1}{2}} \frac{1}{4} = 2$

Example 3

Given that $\log_3 x = 2\log_3 4 - \log_3 5 + \log_3 9$, find the value of x. (05marks)

$\log_3 x = \log_3 \frac{4^2 x 9}{5}$

$= \log_3 \frac{16x9}{5}$

$= \log_3 28.8$

Comparing both sides

x = 28.8

Example 4

Given that p = $\log_a (a^3 y^{-2})$ and q = $\log_a ay^2$, find the value of p + q. (05 marks)

Solution

P + Q = $\log_a (a^3 y^{-2}) + \log_a ay^2$

=$\log_a (a^3 y^{-2} x\ ay^2)$

=$\log_a (a^4)$

= $4\log_a a$

= 4 x 1 = 4

Example 5

Evaluate $\frac{\log_6 216+\log_2 64}{\log_3 243-\log_{10} 0.1}$

Solution

$\frac{\text{Log}_6 216+\log_2 64}{\log_3 243-\log_{10} 0.1} = \frac{\log_6 6^3+\log_2 2^6}{\log_3 3^5-\log_{10} 10^{-1}}$

$= \frac{3\log_6 6+6\log_2 2}{5\log_3 3- -1\log_{10} 10}$

$= \frac{3x1+6x1}{5x1+1x1}$

$= \frac{9}{6}$

$= 1.5$

Rules of logarithms

(a) (i) $\log_a a$ = 1

Proof

Let $\log_a a = x$

$a^x = a^1$

x = 1

$\therefore \log_a a = 1$

(ii) $\log_a 1$ = 0

Proof

Let $\log_a 1 = x$

$a^x = a^0$

x = 0

$\therefore \log_a 1 = 0$

(b) The power rule

$\log_a P^q = \text{q}\log_a P$

Proof

Let $\log_a P = x$

$a^x = P$

Raising each to the power q

$a^{qx} = P^q$

⇨ $\log_a P = \log_a a^{qx} = qx$

$\therefore \log_a P^q = \text{q}\log_a P$

(c) The addition/multiplication rule

$\log_a pq = \log_a p + \log_a q$

Proof

Let $\log_a p = x$ and $\log_a q = y$

$p = a^x$ and $q = a^y$

$pq = a^x . a^y = a^{x+y}$

$\log_a pq = \log_a a^{x+y} = x + y$

$\therefore \log_a pq = \log_a p + \log_a q$

(d) The subtraction/division rule

$\log_a \left(\frac{p}{q}\right) = \log_a p - \log_a q$

Proof

Let $\log_a p = x$ and $\log_a q = y$

$p = a^x$ and $q = a^y$

$\frac{p}{q} = \frac{a^x}{a^y} = a^{x-y}$

$\log_a \left(\frac{p}{q}\right) = \log_a a^{x-y} = x - y$

$\therefore \log_a \left(\frac{p}{q}\right) = \log_a p - \log_a q$

(e) Change of base

$\log_a p = \frac{\log_q p}{\log_q a}$

Let $\log_a p = x$, then $a^x = p$

⇨ $\log_q a^x = \log_q p$

$\text{x}\log_q a = \log_q p$

$x = \frac{\log_q p}{\log_q a}$

$\therefore \log_a p = \frac{\log_q p}{\log_q a}$

Example 6

Evaluate

(a) $\log_2 8\sqrt{2}$

Solution

Either: let $\log_2 8\sqrt{2}$= x

⇨ $2^x = 8\sqrt{2}$

$= 2^3 . 2^{\frac{1}{2}}$

$= 2^{\frac{7}{2}}$

$x = \frac{7}{2}$

$\therefore \log_2 8\sqrt{2} = \frac{7}{2}$

Or $\log_2 8\sqrt{2} = \log_2 \left(2^3 . 2^{\frac{1}{2}}\right)$

$= \log_2 2^{\frac{7}{2}}$

$= \frac{7}{2}\log_2 2$

$= \frac{7}{2}$

(b) $\log_a \frac{1}{a}$

Solution

Let $\log_a \frac{1}{a}$ = x
$a^x = a^{-1}$
$x = -1$
∴ $\log_a \frac{1}{a}$ = -1

Example 7

Express each of the following as a single logarithm

(a) Log 4 + log3

Solution

Log 4 + log3 = log (4 x 3)

= log 12

(b) Log 5 + log 18 – log3

Solution

Log 5 + log 18 – log3 = log$\left(\frac{5\,x 18}{3}\right)$
= log30

Example 8

Show that $\log_a p = \frac{1}{\log_p a}$. Hence solve the equation $\log_5 x + 2\log_x 5 = 3$

Solution

Let $\log_a p = x$

⇨ $a^x = p$

Introducing log to base p on both sides

$\log_p a^x = \log_p p$

$x\log_p a = 1$

$x = \frac{1}{\log_p a}$

∴ $\log_a p = \frac{1}{\log_p a}$

Then,

$\log_5 x + 2\log_x 5 = 3$

$\log_5 x + \frac{2}{\log_5 x} = 3$

Let y = $\log_5 x$

⇨ y + $\frac{2}{y}$ = 3
$y^2 - 3y + 2 = 0$
(y – 1)(y – 2) = 0

Either y = 1 or y = 2

When y = 1: $\log_5 x = 1$; x = $5^1 = 5$

When y = 2: $\log_5 x = 2$; x = $5^2 = 25$

x = 5 and x = 25

Example 9

Solve $\log_x 5 + 4\log_5 x = 4$

Expressing terms on LHS to $\log_5$.

$\frac{\log_5 5}{\log_5 x} + 4\log_5 x = 4$

$\frac{1}{\log_5 x} + 4\log_5 x = 4$

Let $\log_5 x = y$

$\frac{1}{y} + 4y = 4$

$4y^2 - 4y + 1 = 0$

(2y – 1)(2y – 1) = 0

2y = 1
y = $\frac{1}{2}$

⇨ $\log_5 x = \frac{1}{2}$

x = $5^{\frac{1}{2}} = \sqrt{5}$

Example 10

Show that 2log4 + $\frac{1}{2}\log 25 - \log 20 = 2\log 2$.

Solution

Handling the left hand side

2log4 + $\frac{1}{2}\log 25 - \log 20$

= 2log2² + $\frac{1}{2}\log 5^2 - (\log 4 + \log 5)$

= 2log2² + $\frac{1}{2}\log 5^2 - \log 4 - \log 5$

= 4log2 + $\log 5 - 2\log 2 - \log 5$

= 2log 2

Example 11

(a) (i) Find $\log_9 27\sqrt{3}$ without using tables

Solution

Let $\log_9 27\sqrt{3}$ = x

9^x =$27\sqrt{3}$

$(3^2)^x = 3^3.3^{\frac{1}{2}}$

$3^{2x} = 3^{\frac{7}{2}}$

Equating indices

2x = $\frac{7}{2}$

x =1.75

Or

Changing the base from 9 to 3

$\log_9 27\sqrt{3} = \frac{\log_3 27\sqrt{3}}{\log_3 9}$

$= \frac{\log_3 27+\log_3 \sqrt{3}}{\log_3 9}$

$= \frac{\log_3 3^3+\log_3 3^{\frac{1}{2}}}{\log_3 3^2}$

$= \frac{3+\frac{1}{2}}{2} = \frac{7}{4}$

$= 1.75$

(ii) Simplify $(\log_a b^2)(\log_b a^3)$

$$(\log_a b^2)(\log_b a^3) = (\log_a b^2)\frac{(\log_a a^3)}{\log_a b}$$

$$= (2\log_a b)\frac{(3\log_a a)}{\log_a b}$$

= 2 x 3 = 6

Or

$$(\log_a b^2)(\log_b a^3) = (2\log_a b)(3\log_b a)$$

$$= \left(\frac{2\log_{ba} b}{\log_b a}\right)(3\log_b a)$$

= 2 x 3 = 6

(b) Express $\log_{25} xy$ in terms of $\log_5 x$ *and* $\log_5 y$. Hence solve the simultaneous equations:

$\log_{25} xy = 4\frac{1}{2}$

$\frac{\log_5 x}{\log_5 y}$ = -10

Solution

(ii) By change of base rule

$$\log_{25} xy = \frac{\log_5 xy}{\log_5 25} = \frac{\log_5 x + \log_5 y}{\log_5 5^2}$$

$$= \frac{\log_5 x+\log_5 y}{2}$$

$$\therefore \log_{25} xy = \frac{\log_5 x + \log_5 y}{2}$$

Hence solving

$\log_{25} xy = 4\frac{1}{2}$

$\frac{\log_5 x+\log_5 y}{2} = \frac{9}{2}$

$\log_5 x + \log_5 y = 9$ ………………. (i)

$\frac{\log_5 x}{\log_5 y}$ = -10

$\log_5 x = -10\log_5 y$ ……………(ii)

Substituting eqn. (ii) into eqn. (i)

$-10\log_5 y + \log_5 y = 9$

$\log_5 y = -1$

y = $5^{-1} = \frac{1}{5}$

Substituting $\log_5 y$ into equation (ii)
$\log_5 x = 10$
$x = 5^{10}$
$\therefore\ x = 5^{10}$ and y = $\frac{1}{5}$

Example 12

(a) Given that $\log_b a = x$ show that b =$a^{\frac{1}{x}}$ and deduce $\log_b a = \frac{1}{\log_a b}$

Solution

$\log_b a = x$

$b^x = a$

$(b^x)^{\frac{1}{x}} = a^{\frac{1}{x}}$

$b = a^{\frac{1}{x}}$

Taking log to base a on both sides

$\log_a b = \ \log_a a^{\frac{1}{x}}$

$\log_a b = \ \frac{1}{x}\log_a a = \frac{1}{x}$

But x = $\log_b a$

$\therefore \log_a b = \frac{1}{\log_b a}$

(b) Find the value of x and y such that

(i) $\log_{10} x + \log_{10} y = 1.0$
$\log_{10} x - \log_{10} y = \log_{10} 2.5$

Solution

$\log_{10} x + \log_{10} y = 1.0$ ……………(i)

$\log_{10} x - \log_{10} y = \log_{10} 2.5$ …(ii)

Eqn. (i) + eqn. (ii)

2$\log_{10} x$ = $\log_{10} 10 + \log_{10} 2.5$

$\log_{10} x^2 = \log_{10} 25$

⇨ $x^2 = 25$
x = 5

Substituting x into eqn. (i)
$\log_{10} 5 + \log_{10} y = 1.0$

$\log_{10} y = \log_{10} 10 - \log_{10} 5$

$\log_{10} y = \log_{10} 10 \div 5 = \log_{10} 2$

y = 2

Hence x = 5 and y = 2

(ii) Simplify $2^x.3^y = 432$

Solution

$2^x.3^y = 432 = 2^4.3^3$

Comparing exponents

x = 4 and y = 3

(c) Simplify $\frac{1+\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

Solution

By rationalizing

$$\frac{1+\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(1+\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$$

$$= \frac{\sqrt{2}-\sqrt{3}+2-\sqrt{6}+\sqrt{6}-3}{2-3}$$

$$= \frac{\sqrt{2}-\sqrt{3}-1}{-1}$$

$$= 1+\sqrt{3}-\sqrt{2}$$

Example 13

Prove that $\log_6 x = \frac{\log_3 x}{1+\log_3 2}$. Given that $\log_3 2 = 0.631$, find without using tables or calculator $\log_6 4$ correct to 3significant figures

Solution

$$\log_6 x = \frac{\log_3 x}{\log_3 6} = \frac{\log_3 x}{\log_3 (2\,x3)} = \frac{\log_3 x}{\log_3 3 + \log_3 2}$$

$$= \frac{\log_3 x}{1+\log_3 2}$$

Substituting for $\log_3 2 = 0.631$

$$\log_6 x = \frac{\log_3 2^2}{1+\log_3 2} = \frac{2\log_3 2}{1+\log_3 2} = \frac{2\,x\,0.631}{1+0.631}$$

$$= 0.774$$

Application of logarithms

(i) Sound Intensity (Decibel Scale)

Use: Measures how loud a sound is.

Example: A sound 10× more intense than another is 10 dB louder.

Formula:

dB = $10.\log_{10}\left(\frac{I}{I_0}\right)$

where I is the sound intensity and I_0 is the reference intensity

(ii) Earthquake Magnitude (Richter Scale)

Use: Quantifies the energy released by earthquakes.

Example: A magnitude 6 quake is 10× stronger than a magnitude 5. Formula:

M = $\log_{10}\left(\frac{A}{A_0}\right)$

where A is the amplitude of seismic waves

(iii) Acidity (pH Scale)

Use: Measures acidity or alkalinity.

Example: pH 3 is 10× more acidic than pH 4.

Formula:

pH = $-\log_{10}(H^+)$

where [H+] is the hydrogen ion concentration.

(iv) Finance and Economics

Use: Calculates time to reach financial goals.

Example: To find how long it takes to double money at 5% interest:

t= $\frac{\log(2)}{\log(1.05)}$ =14.2 years

Revision exercise on logarithms

1. Evaluate
   (a) $\log_{\frac{1}{5}} 25\sqrt{5}$ $\left[-\frac{5}{2}\right]$
   (b) $\log_3 27$ [3]
2. Express the following as a single logarithm
   (i) Log15 - $\frac{1}{2}log9$ [log 5]
   (ii) 3log2 + 2log5 – log 20 [log 10]
3. Given that $\log_b a$ and $\log_c b = a$, show that $\log_c a = ac$
4. solve the equation
   (i) $\log_a 4 + \log_4 a^2$
   $[a = 2\ and\ a = 4]$
   (ii) $\log_{14} x = \log_7 4x$ $\left[\frac{1}{196}\right]$
   (iii) $\log_4(6 - x) = \log_2 x$ [x = 2]
5. Without using tables or calculator show that $\frac{2log9 + log8 - \log 375}{\frac{1}{3}log6 - log5^{\frac{1}{3}}}$= 9
6. If$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{16}$. Find the value of x [x = 1.6818]
7. Given$\log_a b = \log_d c$, show that $\log_c a = \log_d b$. Hence or otherwise solve the equation$\log_{9x} 64 = \log_x 4$. [x=3]
8. Solve the simultaneous equations
   $\log_{10}(y - x)$= 0
   $2\log_{10}(21 + x)$ [(x,y) =(-5, -4) or (4,5)
9. Given that $\log_2 x + 2\log_4 y = 4$. Show that xy = 16. Solve simultaneous equations
   $10\log_{10}(x + y) = 1$
   $\log_2 x + 2\log_4 y = 4$. [(x, y) =(2,8)or (8,2)
10. (a) If $\log_b a = x$, show that b = $a^{\frac{1}{x}}$ and deduce that $\frac{1}{\log_a b}$.

(b) Solve

(i) $\log_x 4 + \log_4 x^2 = 3$ $[x = 2\ or\ 4]$

(ii) $2^{2x-1} + \frac{3}{2} = 2^{x+1}$[x=0 or 1.585]

11. Prove that $\log_8 x = \frac{2}{3}\log_4 x$. Hence without using tables or calculator, evaluate $\log_8 6$ correct to three significant figure, if $\log_4 3 = 0.7925$ [0.862]
    (e) $\sqrt{x + 2} - x = 0$[x=2]
    (f) $\sqrt{1 + x} = 1 + \sqrt{1 - x}$ $\left[x = \frac{\sqrt{3}}{2}\right]$

(g) $(3-x)^{\frac{1}{2}} = (1+x)^{\frac{1}{2}} + (2-x)^{\frac{1}{2}}$ [x = -0.92665]

(h) $\sqrt{x+6}$ =$\sqrt{1-3x} - \sqrt{4-x}$ [-5]

13. Without using mathematical tables or calculators, find the value of $\frac{(\sqrt{5}+2)^2-(\sqrt{5}-2)^2}{8\sqrt{5}}$ [1]

14. Find the square root of the following

(c) 6 + 2$\sqrt{5}$ $[\pm(1+\sqrt{5})]$

(d) 18 - 2$\sqrt{12}[\pm(\sqrt{0.695} - \sqrt{17.303})]$

(e) 18 - 2$\sqrt{2}[\pm(\sqrt{0.1118} - \sqrt{17.8882})]$

15. Solve the equations

(a) $\log_x 5 + 4\log_5 x = 4$ $[\sqrt{5}]$

16. Solve the simultaneous equations

$3^x = 2^{3y+1}$; $4^{x-1} = 12^{2y+1}$

Given $\frac{log3}{log2} = \frac{8}{5}$ $\left[x = \frac{-40}{23} and\ y = \frac{-29}{23}\right]$

17. Find the value of x to 2 decimal places if $2^{3x+1} = 3^{x+2}$[1.53]

18. Show that:

2log4 + $\frac{1}{2}\log 25 - \log 20 = 2\log 2$.

19. Express $\log_{25}(xy)$in terms of $\log_5 x\ and\ \log_5 y$. Hence or otherwise solve the simultaneous equations:

$\log_{25}(xy) = 4\frac{1}{2}$

$\frac{\log_5 x}{\log_5 y} = -10$ $\left[x = 5^0 and\ y = \frac{1}{5}\right]$

20. Solve the equations

(a) $\log_x 8 - \log_x 2 = 16$[x = 2]

(b) $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{16}$

$\left[x = 8^{\frac{1}{4}} = 1.6818\right]$

21. (a) Given that $\log_2 x + 2\log_4 y = 4$, show that xy = 16. Hence solve the simultaneous equations

$\log_{10}(x+y) = 1$

$\log_2 x + 2\log_4 y = 4$

[(x, y) = (8, 2) or (2, 8)]

22. Show that $\log_a b = \frac{1}{\log_b a}$. Hence solve the simultaneous equations

$\log_a b + 2\log_b a = 3$

$\log_9 a + 2\log_9 b = 3$

$\therefore (x,y) = (27,27) or\ (9,81)$

23.

1.3 Surd

These are irrational numbers which cannot be expressed in terms of $\frac{a}{b}$ where a and b are rational. Irrational numbers may be defined as square roots of prime numbers.

Examples are $\sqrt{2}, \sqrt{3}, \sqrt{5}$ ….

Expression of root of numbers in surd form

Example 1

Write the following as the simplest surds

(i) $\sqrt{32}$ $(ii)\ \sqrt{50}$ $(iii)\sqrt{8}$ $(iv)\sqrt{27}$

Solution

(i) $\sqrt{32} = \sqrt{16\ x\ 2} = \sqrt{16}\ x\ \sqrt{2} = 4\sqrt{2}$
(ii) $\sqrt{50} = \sqrt{25\ x\ 2} = \sqrt{25}\ x\ \sqrt{2} = 5\sqrt{2}$
(iii) $\sqrt{8} = \sqrt{4\ x\ 2} = \sqrt{4}\ x\ \sqrt{2} = 2\sqrt{2}$
(iv) $\sqrt{27} = \sqrt{9\ x\ 3} = \sqrt{9}\ x\ \sqrt{3} = 3\sqrt{3}$

Addition and subtraction of surds

This is done by expressing the surds in their simplest form

Example 2

(i) $\sqrt{75} - 3\sqrt{27} + 2\sqrt{12}$

= $\sqrt{25\ x\ 3} - 3\sqrt{9\ x\ 3} + 2\sqrt{4\ x\ 3}$

= $\sqrt{25}\ x\ \sqrt{3} - 3x\sqrt{9}x\ \sqrt{3} + 2\ x\ \sqrt{4}x\sqrt{3}$

= $5\ x\ \sqrt{3} - 3\ x\ 3\ x\ \sqrt{3} + 2\ x\ 2\ x\sqrt{3}$

$(5 - 9 + 4)\sqrt{3} = 0$

(ii) $\sqrt{50} + \sqrt{2} - 3\sqrt{18} + 2\sqrt{8}$
= $\sqrt{25\ x\ 2} + \sqrt{2} - 3\sqrt{9x\ 2} + 2\sqrt{4\ x\ 2}$
=$5\sqrt{2} + \sqrt{2} - 3\ x\ 3\sqrt{2} + 2x\ 2\sqrt{2}$

$= (5 + 1 - 9 + 4)\sqrt{2} = \sqrt{2}$

Multiplication of surds

Finding the product of two surd numbers is the same as finding the root of the product of two numbers.

i.e. $\sqrt{a}\ x\ \sqrt{b} = \sqrt{ab}$

Example 3

Find the value of the following and give your answers in the simplest form

(i) $\sqrt{2}\ x\ \sqrt{2}$
(ii) $\sqrt{2}\ x\ \sqrt{20}$
(iii) $(3\sqrt{2} - 2\sqrt{3})^3$

Solution

(i) $\sqrt{2}\ x\ \sqrt{2} = \sqrt{2x2} = \sqrt{4} = 2$
(ii) $\sqrt{2}\ x\ \sqrt{30} = \sqrt{2x30} = 2\sqrt{15}$
(iii) $(3\sqrt{2} - 2\sqrt{3})^3$

Using Pascal's triangle; the coefficients of the terms in the expansion $(a + b)^3$ are 1 3 3 1

$(3\sqrt{2} - 2\sqrt{3})^3 =$

$(3\sqrt{2})^3 + 3(3\sqrt{2})^2(-2\sqrt{3}) + 3(2\sqrt{3})(-2\sqrt{3})^2 + (-2\sqrt{3})^3$

$=(54\sqrt{2}) + 3(18)(-2\sqrt{3}) + 3(12)(3\sqrt{2}) - 24\sqrt{3}$

= $162\sqrt{2} - 132\sqrt{3}$

Example 3

Express $\frac{4}{\sqrt{3}+\sqrt{2}} + \frac{4}{\sqrt{3}-\sqrt{2}}$ in form b$\sqrt{c}$ where b and c are integers. (05 marks)

$\frac{4}{\sqrt{3}+\sqrt{2}} + \frac{4}{\sqrt{3}-\sqrt{2}} = \frac{4(\sqrt{3}-\sqrt{2})+4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{8\sqrt{3}}{1}$

Hence b = 8 and c= 3

Example 4

Show that $\sqrt{\frac{25^3+5^6}{5^7-5^6}} = \frac{\sqrt{2}}{2}$

$$\sqrt{\frac{25^3+5^6}{5^7-5^6}} = \sqrt{\frac{(5^2)^3+5^6}{5^7-5^6}}$$

$$= \sqrt{\frac{5^6+5^6}{5x5^6-5^6}}$$

$$= \sqrt{\frac{2x5^6}{4\,x5^6}}$$

$$= \sqrt{\frac{2}{4}}$$

$$= \frac{\sqrt{2}}{2}$$

Example 5

Without using a calculator, evaluate $\frac{6\sqrt{10}+2\sqrt{40}}{\sqrt{2}\;x\;\sqrt{20}}$.

Solution

$$\frac{6\sqrt{10}+2\sqrt{40}}{\sqrt{2}\,x\sqrt{20}} = \frac{6\sqrt{10}+2\sqrt{4x10}}{\sqrt{2}\,x\sqrt{2x10}}$$

$$= \frac{6\sqrt{10}+2\sqrt{4}\,x\sqrt{10}}{\sqrt{2}\,x\sqrt{2}x\sqrt{10}}$$

$$= \frac{6\sqrt{10}+4\,x\sqrt{10}}{2x\sqrt{10}}$$

$$= \frac{10\sqrt{10}}{2x\sqrt{10}}$$

= 5

Division of surds

There are two types of division of surds;

- A fraction whose denominators has a single term such as $\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{3}}$, etc.
- A fraction whose denominator has double terms, such as $\frac{1}{\sqrt{2}+\sqrt{3}}, \frac{2+\sqrt{3}}{2-\sqrt{3}}, \frac{1}{1+\sqrt{5}}$, etc.
- In both cases, first eliminate the surds from the denominator. The process of eliminating surds from the denominator is called rationalization.
- In the first case rationalize the fraction by multiplying the numerator and denominator by the surd term of the denominator.

Example 5

Rationalize the following

(i) $\frac{2}{3\sqrt{5}}$

(ii) $\frac{1}{\sqrt{2}}$

(iii) $\frac{5}{\sqrt{3}}$

Solution

(i) $\frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{3\sqrt{5}\;x\;\sqrt{5}} = \frac{2\sqrt{5}}{3\;x\;5} = \frac{2\sqrt{5}}{15}$

(ii) $\frac{1}{\sqrt{2}} = \frac{1\;x\sqrt{2}}{\sqrt{2}\;x\;\sqrt{2}} = \frac{\sqrt{2}}{2}$

(iii) $\frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{3}\;x\;\sqrt{3}} = \frac{5\sqrt{3}}{3}$

In the second case rationalize by multiplying the numerator and denominator by the conjugate of the denominator.

Note

(i) The conjugate a + $\sqrt{b}$ is a - $\sqrt{b}$ and that
a - $\sqrt{b}$ is a + $\sqrt{b}$

(ii) The product of a surd function and its conjugate is equal to the difference of two squares. i.e. (a + $\sqrt{b}$)(a - $\sqrt{b}$) = $\left(a^2 - \left(\sqrt{b}\right)^2\right)$

Example 7

Rationalize the following

(i) $\frac{2}{2-\sqrt{2}}$

(ii) $\frac{1}{3-\sqrt{5}}$

(iii) $\frac{3-\sqrt{5}}{\sqrt{5}-3}$

solution

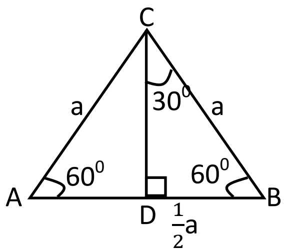
(i) $\frac{2}{2-\sqrt{2}} = \frac{2(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = \frac{2(2+\sqrt{2})}{4-2} = (2+\sqrt{2})$

(ii) $\frac{1}{3-\sqrt{5}} = \frac{1(3+\sqrt{5})}{(3-\sqrt{5})((3+\sqrt{5}))} = \frac{(3+\sqrt{5})}{9-5} = \frac{(3+\sqrt{5})}{4}$

(iv) $\frac{3-\sqrt{5}}{\sqrt{5}-3} x \frac{\sqrt{5}+3}{\sqrt{5}+3} = \frac{(3)^2-\left(\sqrt{5}\right)^2}{\left(\sqrt{5}\right)^2-(3)^2} = \frac{9-5}{5-9} = -1$

Set square angle (30^0, 45^0 and 60^0)

(a) Consider an equilateral triangle ABC of each side = a units



$ED^2 = a^2 - \left(\frac{1}{2}a\right)^2 = \frac{3a^2}{4}$

EC=$\frac{a\sqrt{3}}{2}$

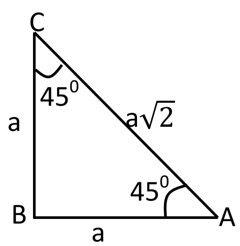
cos 60^0 = $\frac{a}{2a} = \frac{1}{2}$; sin$60^0$ =$\frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$

tan 60^0= $\frac{\sqrt{3}}{2}\ x\,\frac{2}{1} = \sqrt{3}$

cos 30^0 = $\frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$; sin$60^0$ =$\frac{a}{2a} = \frac{1}{2}$

tan 60^0= $\frac{1}{2}\ x\,\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

(b) Given a right angled isosceles triangle ABC with equal perpendicular sides each of length a units



$AC^2 = a^2 + a^2 = 2a^2$

$AC = a\sqrt{2}$

cos 45^0 = sin45^0 = $\frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$

tan 45^0 = $\frac{a}{a} = 1$

Example 8

Express without a surd in the denominator each of the following

(a) $\frac{1+\tan 30^0}{1-\tan 30^0}$

(b) $\left(\frac{1+\cos 45^0}{2-\sin 60^0}\right)^2$

Solution

(a) $\frac{1+\tan 30^0}{1-\tan 30^0} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$= \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$

(b) $\left(\frac{1+\cos 45^0}{2-\sin 60^0}\right)^2 = \left(\frac{1+\frac{\sqrt{2}}{2}}{2-\frac{\sqrt{3}}{2}}\right)^2 = \left(\frac{2+\sqrt{2}}{4-\sqrt{3}}\right)^2$

$= \frac{4+4\sqrt{2}+2}{16-8\sqrt{3}+3}$

$= \left(\frac{6+4\sqrt{2}}{19-8\sqrt{3}}\right)\left(\frac{19+8\sqrt{3}}{19+8\sqrt{3}}\right)$

$= \frac{114+48\sqrt{3}+76\sqrt{2}+32\sqrt{6}}{169}$

Application of surds

(i) Architecture and Construction

Use: Surds appear when calculating diagonal lengths or slopes.

Example: To find the diagonal of a square with side 5 meters:

Diagonal=$5^2+5^2=\sqrt{50}=5\sqrt{2}$

Why it matters: Keeps measurements exact without rounding errors.

(ii) Geometry and Trigonometry

Use: Surds are common in formulas for area, perimeter, and angles.

Example: The height of an equilateral triangle with side a is $\frac{a\sqrt{3}}{2}$

Why it matters: Helps in precise design and modeling.

(iii) Navigation and Distance Calculation

Use: Surds help calculate direct distances using Pythagoras' theorem.

Example: If a plane travels 3 km east and 4 km north, the direct distance is

$\sqrt{3^2 + 4^2}$ = $\sqrt{25}$ =5km

If the values aren't perfect squares, the result remains a surd.

(iv) Physics and Engineering

Use: Surds appear in formulas for speed, force, and energy.

Example: The speed of a wave might be expressed as $\sqrt{\frac{T}{\mu}}$.

Why it matters: Ensures accurate modeling of natural phenomena.

Revision exercise on surds

1. Simplify
   (a) $\sqrt{48}$ $[4\sqrt{3}]$
   (b) $\sqrt{162}$ $[9\sqrt{2}]$
   (c) $\sqrt{28}$ $[2\sqrt{7}]$
   (d) $\sqrt{45}$ $[3\sqrt{5}]$
   (e) $\sqrt{125}$ $[5\sqrt{5}]$
   (f) $\sqrt{147}$ $[7\sqrt{3}]$
2. Simplify the following suds
   (a) $\sqrt{8} + \sqrt{200} - 4\sqrt{18}$ $[2\sqrt{2}]$
   (b) $5\sqrt{20} + 2\sqrt{45} + 2\sqrt{5}$ $[18\sqrt{5}]$
   (c) $3\sqrt{50} + 2\sqrt{32} - 2\sqrt{75} + 2\sqrt{12} - \sqrt{27}$
   $[23\sqrt{2} - 7\sqrt{3}]$
3. Given that $4\sqrt{20} + 3\sqrt{5} - 5\sqrt{125} = x\sqrt{5}$, find the value of x [-14]
4. Find the value of the following simplifying the answer as much as possible
   (a) $(5\sqrt{2} - \sqrt{5})(3\sqrt{5} - 2\sqrt{2})[5 + 13\sqrt{10}]$
5. Express each of the following in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers
   (a) $\frac{2}{\sqrt{7}}$ $\left[\frac{2\sqrt{7}}{7}\right]$
   (b) $\frac{3}{\sqrt{2}}$ $\left[\frac{3\sqrt{2}}{2}\right]$
   (c) $\frac{14\sqrt{5}}{\sqrt{7}}$ $[2\sqrt{35}]$
6. Solve the equations
   (a) $\sqrt{(x-5)}$+ $\sqrt{x}$ = 5 [9]
   (b) $\sqrt{(1-20x)}$ - 2$\sqrt{x+1}$ = 3 $\left[x = -\frac{3}{5}\right]$
   (c) $2\sqrt{(x-1)} - \sqrt{(x+4)} = 1$
   [x = 5 or x =$\frac{13}{9}$]

Thank you Dr. Bbosa Science