S5 2024 REVISION QUESTIONS

PURE MATHEMATICS P425/1

1. Solve : $\sqrt{(4-p)} - \sqrt{(6+p)} = \sqrt{(14+2p)}$
2. Solve the simultaneous equations : x+2y=7 , $x^2 - 4x + y^2 = 1$
3. Find the number of terms in the A.P : $\frac{9}{4} + \frac{57}{20} + \frac{69}{20} + \cdots + \frac{81}{4} + \frac{417}{20}$
4. Given that $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = a + b\sqrt{6}$. Determine the values of a and b
5. The function $f(y) = Ay^2 + By + C$ *is divisible by* $y - 1$, has remainder of 2 when divided by $y + 1$ and has a remainder of 8 when divided by $y - 2$,find the values of A,B and C
6. In a GP,the third term is 32 and the sixth term is 4 ,find the first term,common ratio and the sum of the first 8 terms of the GP.
7. Solve the equation : $\log_x 5 + 4\log_5 x = \log_4 256$
8. Solve for x in the equation : $2(3^{2x}) - 5(3^x) + 2 = 0$
9. Expand $\sqrt{\frac{1+5x}{1-5x}}$ as far as the term in x^3,by taking $x = \frac{1}{9}$,evaluate $\sqrt{14}$ correct to 3sfs.
10. Given that the first two terms in the expansion $(a+bx)^4$ are 16—96x, find the values of a and b
11. Solve the equation; $x^2 + 2x - 4 = \frac{-3}{x^2+2x}$
12. Show that if x is small enough such that it's cube and higher powers can be neglected then $\sqrt{\frac{1-x}{1+x}} = 1 - x + \frac{1}{2}x^2$ and hence using $x = \frac{1}{8}$, evaluate $\sqrt{7}$ correct to 3 sfs.
13. Find the term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{2x}\right)^9$
14. The polynomial $f(x)$ leaves a remainder of 3 when divided by $x + 3$ and a remainder of 18 when divided by $x - 2$. Find the remainder when $f(x)$ is divided by $x^2 + x - 6$.
15.) The roots of the equation $25x^2 + x + 1 = 0$ are α^2 and β^2 . Find the equation with integral coefficients whose roots are $\frac{1}{\alpha}$ *and* $\frac{1}{\beta}$.
16. Find the expansion of $(4-3x^2)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^4.

17. Find the first three terms of the expansion $(2-x)^6$ and hence evaluate $(1.998)^6$ correct to 3dps

18. Solve for x and y: $\begin{matrix}\log(x+y)=1\\ 2\log y-\log(30-x)=0\end{matrix}$.

19. Without using a calculator, show that $\frac{\log\sqrt{27}+\log\sqrt{8}-\log\sqrt{125}}{\log 6-\log 5}=\frac{3}{2}$

20. Show that if n is a positive integer, then $3^{2n}+7$ is divisible by 8.

21. Prove by induction that 8^n+6 is divisible by 14

22. Prove by induction that

23. $1(2)+2(3)+3(4)+\cdots\ldots\ldots\ldots\ldots\ldots+n(n+1)\ =\frac{1}{3}n(n+1)(n+2)$

24. The polynomial $Q(x)$ is defined by $Q(x)=x^4+4x^3+ax^2+bx+5$. It known that $Q(x)$ is divisible by x^2-1, the remainder $2x+3$ is obtained. Find the values of a and b.

25. Find the value of x if $\log_a x$, $\log_a(x+3)$ and $\log_a(x+12)$ are three consecutive terms of an AP.

26. Given that the function $f(x)=ax^3-bx^2-x+6$ is divisible by x^2-1,find the values of $a\ and\ b$ hence solve $f(x)=0$

27. The ninth term of an arithmetic progression is twice the third term, and the fifteenth term is 27. Evaluate the sum of the first 25 terms of the series.

28. The roots of the equation $x^2+px+7=0$ are $\propto$ and β. Given that $\propto^2+\beta^2=22,$
find the possible values of p.

29. Prove that $\log_a x=\frac{1}{\log_x a}$. Hence solve the equation $\log_{10}x+\log_x 100=3$

30. The expression $ax^4+bx^3-x^2+2x+3$ has a remainder $3x+5$ when it is divided by x^2-x-2, find values of a and b.

31. Express $tan(45^0+x)$ in terms of $tanx$. Hence prove that; $tan75^0=2+\sqrt{3}$.

32. Prove that $\frac{Sin3\theta}{1+2cos2\theta}=Sin\theta$. Hence show that $Sin15^0=\frac{\sqrt{3}-1}{2\sqrt{2}}$

33. Prove that $\sin 4\theta=\frac{4tan\theta(1-tan^2\theta)}{(1+tan^2\theta)}$

34. Solve the equation $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{4}$

35. Solve the equation $3\cos 2\theta-7\cos\theta-2=0$for $0°\leq\theta\leq 360°$.

36. Prove that: $\frac{cos3A}{CosA}+\frac{sin3A}{sinA}=4cos2A$.

37.) Using the substitution $t = tanx$, show that $\frac{1+sin2x}{cos2x} = \tan(\frac{\pi}{4} + x)$.

38.Solve $3\,\text{Sin}\,\theta + 5\,\text{Cot}\,\theta = \text{Cosec}\,\theta$ for $0^0 \leq \theta \leq 360^0$

39.Express $55\cos\theta - 48\sin\theta$ in the form $A\cos(\theta+\beta)$ and hence find the maximum value of $\frac{1}{55\cos\theta - 48\sin\theta + 10}$.

40.Show that $\cot(A+B) = \frac{cotAcotB-1}{cotA+cotB}$

41.Solve: $\cos x - \sin x + \cos 3x - \sin 3x = 0$ for $0^o \leq x \leq \pi$.

42.Solve the equation:$\sin 2x + \sin 3x + \sin x = 0$ for $0 \leq x \leq \pi$

43.Differentiate the $y = sinx$ from first principles

44.) Line 1 has vector equation $L_1: \begin{pmatrix}5\\4\\4\end{pmatrix} + t\begin{pmatrix}-2\\-3\\4\end{pmatrix}$ and line 2 $L_2: \begin{pmatrix}1\\-4\\14\end{pmatrix} + \mu\begin{pmatrix}0\\-2\\2\end{pmatrix}$. Show that these lines intersect and give the coordinates of the point of intersection.

45.Find t such that $a = \begin{pmatrix}-1\\5\end{pmatrix}$ and $b = \begin{pmatrix}2\\t\end{pmatrix}$ are perpendicular

find the equation of the plane through A(-1,2,0) B(3,1,1) and C(1,0,3) in Cartesian equation.

46. $A(-3,\ 0)$ and $B(3,\ 0)$ are fixed points. Show that the locus of a point $P(x,\ y)$ which moves such that $PB = 2PA$ is a circle

47.The two lines $bx + ay = 32$ and $2x - 3y = 4$ are perpendicular and intersect at $(8,4)$.Find the values of a and b

48.Show that the curve whose parametric equations are $x = 5 + \sqrt{3}\cos\theta$ and $y = -3 + \sqrt{3}\sin\theta$ is a circle

49.The circumference of a circle is increasing at a rate of $\frac{3}{2}cms^{-1}$.Find the rate at which the area is increasing when it's radius is $2cm$.

50.Find the gradient of the curve $x^2 - 2xy + 3x + 2y^2 = 2$ at the point $(1,2)$ and hence show that the equation of the normal to the curve is $6x - y - 4 = 0$

51.Use the concept of small changes to show that $\frac{1}{\sqrt{0.97}} \approx \frac{1015}{1000}$

52.The point $(2,1)$ lies on the curve $Ax^2 + By^2 = 11$ where A and B are constants. If the gradient of the curve at that point is 6. Find the values of A and B

53. The line $y = -4x + 12$ cuts the $x - axis\ and\ y - axis$ at $A\ and\ B$ respectively. Find the locus of a point $P(x, y)$ such that it's distance from point A is one third it's distance from point B.

54. The lines $\boldsymbol{r_1}\ and\ \boldsymbol{r_2}\ have\ equations\ \ \boldsymbol{r_1} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and

$\boldsymbol{r_2} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ respectively. Show that the two lines intersect and hence find the coordinates of the point of intersection

55. Using the concept of small increments, show that $tan61.5 \approx \frac{1}{30}(30\sqrt{3} + \pi)$

56. An inverted cone with semi vertical angle 60° is receiving drips of water at a rate of $9cm^3min^{-1}$.Find the rate at which the water level is increasing when the water level is 6cm

57. If $y = \sqrt{x}$,show from first principles that $\frac{\Delta y}{\Delta x} = \frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}$. Hence find $\frac{dy}{dx}$.

58. If $y = \frac{x}{3x+2}$,show that $\frac{dy}{dx} = \frac{1}{8}$ at $x = \frac{2}{3}$

59. Show that the locus of a point which moves so that the sum of squares of its distances from $(-2, 0)\ and\ (2, 0)\ is\ 26$ is a circle

60. Given that $x = \frac{t}{\sqrt{t^2+1}}$ and $y = \frac{1}{\sqrt{t^2+1}}$ are parametric equations of a curve,find the cartesian equation of the curve.

GOOD LUCK

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