# TURNING EFFECT OF FORCE (MOMENTS)

Moment of a force is also called the measure of the turning effect of a force.

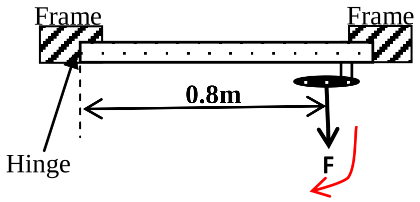
**Moment of a force** is a product of a force and the perpendicular distance of the line of action of the force from the fulcrum (pivot).

$$\begin{pmatrix}\textbf{Moment of}\\ \textbf{a force}\end{pmatrix} = \textbf{Force} \times \begin{pmatrix}\textbf{Perpendicular distance of}\\ \textbf{the line of action of the}\\ \textbf{force from the pivot}\end{pmatrix}$$

SI unit of moment is a newton meter (Nm). Moment of a force is a vector quantity.
Examples of the turning effects of a force;

- Opening or closing a door
- Children playing on a see-saw
- Bending of the fore arm of a hand

**Example;1**
A force of 12N is applied to open a door handle, which is 0.8m from the hinges of the door. Calculate the moment of the force produced.

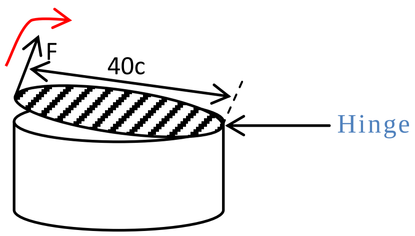



**Solution**
Taking moments about the Hinge;

$$Moment\ of\ Force\ = Force, F \times Perp.distance$$
$$Clockwise\ moment = 12 \times 0.8$$
$$\underline{Clockwise\ moment = 9.6Nm}$$

**Example;2**
The moment of a force is 4Nm in the clockwise direction when the lid of a tin is opened. Calculate the vertical force applied, if the perpendicular distance from the hinges is 40cm.

**Solution**




Taking moments about the Hinge;

$$\text{Moment of Force} = \text{Force, F} \times \text{Perp. distance}$$
$$4 = \text{F} \times 0.4$$
$$\text{F} = \frac{4}{0.4}\text{N}$$
$$\text{F} = 10\text{N}$$

Thus the vertical force applied is 10N

From above, it can be noted that:

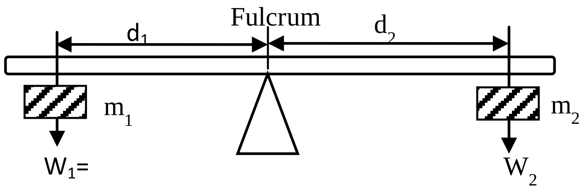
- The greater turning effect of a force occurs when the force acts on an object at a right angle.
- It is easier to close the door by pushing it at a point as far away from the hinges as possible. Because the force applied can easily balance with the reaction at the hinges.

**Factors affecting moments**
The moment of the force depends on the:-
i) magnitude of a force
ii) Perpendicular distance from the turning point (fulcrum).

**Law or principle of moments**
This states that when body is in a state of equilibrium the sum of clockwise moments about any point is equal to the sum anticlockwise moments about the same point.

$$\begin{pmatrix}\textbf{Sum of clockwise}\\ \textbf{moments}\end{pmatrix} = \begin{pmatrix}\textbf{Sum of anticlockwise}\\ \textbf{moments}\end{pmatrix}$$

**Experiment to verify the principle of moments.**




The metre rule is balanced horizontally on a knife-edge and its centre of gravity, G noted.

Un equal masses $m_1$ and $m_2$ are hung from cotton loops on either sides of the rule.

The distances of the masses are then adjusted until the rule balances horizontally once again. The distances $d_1$ and $d_2$ are measured and recorded.

The experiment is repeated several times and the results tabulated including values of $w_1d_1$ and $w_2d_2$.

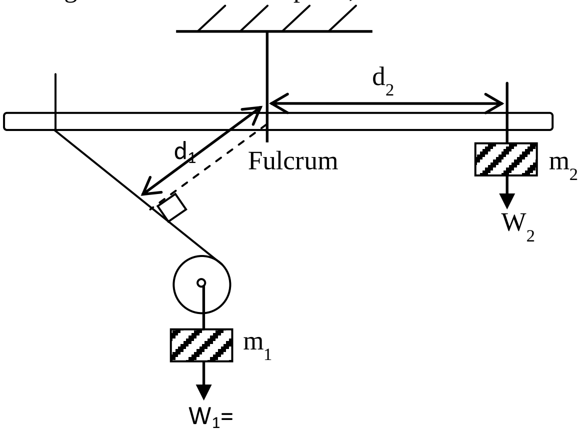
It is found that; $\mathbf{w_2d_2 = w_1d_1}$. Where $w_2d_2$ is the clockwise moment and $w_1d_1$ is the anticlockwise moment. This verifies the principle of moments.

The points from which moments are being taken acts as the pivot and the moments of force at that point is zero (0).

$$\begin{pmatrix}\text{Clockwise}\\ \text{moments}\end{pmatrix} = \begin{pmatrix}\text{clockwise}\\ \text{force}\end{pmatrix} \times \begin{pmatrix}\text{Perpendicular}\\ \text{distance from pivot}\end{pmatrix}$$

$$\begin{pmatrix}\text{Anti}-\\ \text{clockwise}\\ \text{moments}\end{pmatrix} = \begin{pmatrix}\text{Anti}-\\ \text{clockwise}\\ \text{force}\end{pmatrix} \times \begin{pmatrix}\text{Perpendicular}\\ \text{distance from pivot}\end{pmatrix}$$

Taking moments about the pivot;




Clockwise moments $=W_1 \times d_1 = W_1d_1$
Anticlockwise moments $=W_2 \times d_2 = W_2d_2$
And by the law of moment;
Sum of clockwise moments = sum of anti-clock moment about any point. Thus, $W_1d_1 = W_2d_2$

***Note:** When calculating moments about a point (pivot) all distances should be measured from that point.*

**Finding the mass and weight of uniform body**

- ✓ When body is uniform, the mass or weight must act at the centre.
- ✓ A metre rule is marked from 0-100cm mark. If it is uniform, then its mass/weight must act in the middle, which is 50cm mark.
- ✓ The mass or weight is calculated by applying the principle of moment.

**Example2**:
A metre-rule suspended from the centre of gravity is in equilibrium, i.e. balanced at G, when forces of $W_1$, $W_2$ and $W_3$, act at distances of a, b and c respectively from the pivot.

(i) Draw a labeled diagram to show all the forces acting on the metre-rule.

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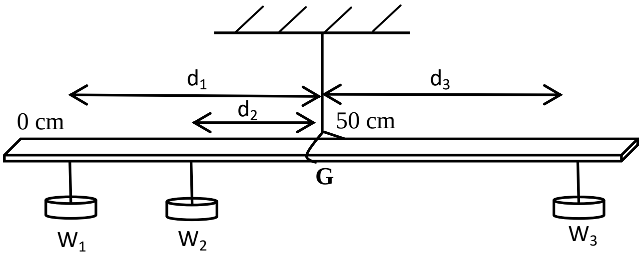
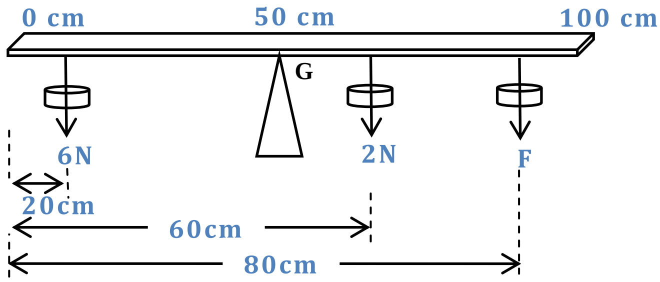



(ii) Write an expression for the sum of the moments.
Taking moments about the pivot;
Sum of Anticlockwise moments$=W_1 \times d_1 + W_2 \times d_2$
Sum of Clockwise moments$=W_3 \times c$

Applying the principle of moments;
Sum of clockwise moments = Sum of anticlockwise moments
$W_3 \times c = W_1 \times a + W_2 \times b$

**Example3**:
A uniform metre rule is pivoted at its centre and three forces of 6N, 2N and F act at distances of 20cm, 60cm, and 80cm respectively from the zero mark. If the metre rule balances horizontally, find the value of F.

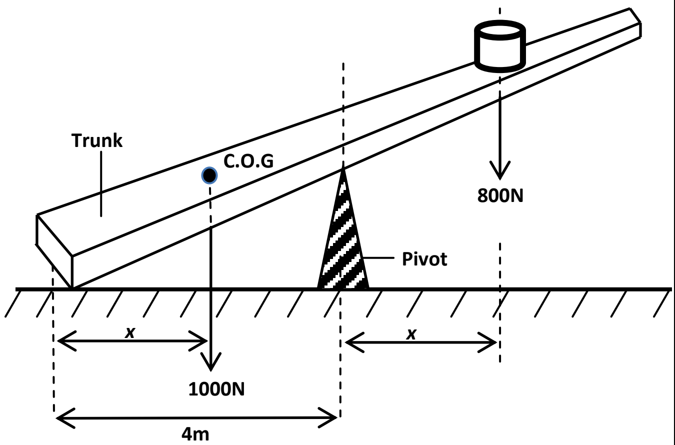
**Solution**




Taking moments about the pivot;
Sum of Anticlockwise moments$=6 \times 30 = 180$Ncm
Sum of Clockwise moments$=2 \times 10 + F \times 30$
$=(20+30F)$ Ncm
Applying the principle of moments;
Sum of clockwise moments = Sum of anticlockwise moments
$(20+30F)$ Ncm $=180$Ncm
$30F = 160$
$F = 5.3$ N

**Example 4:**
A non-uniform tree trunk of weight 1000N is placed on a pivot, 4m from the thick end. A weight of 800N is placed on the other side of the pivot, at a distance equal to that from the thick end to the centre of gravity, just tips off the tree trunk. How far is the weight from the thick end?

**Solution:**



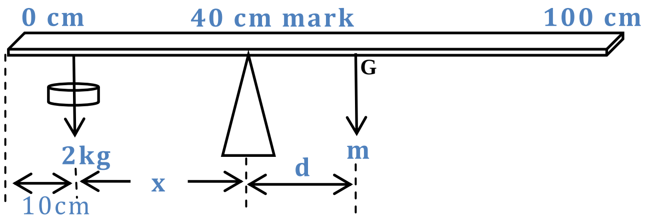

Let the distance from the thick end to the Centre of gravity (C.O.G) be ***x***.

Taking moments about the pivot;

Applying the principle of moments;
Sum of clockwise moments = Sum of anticlockwise moments
$(800 \times x)$ Nm $=1000 \times (4 - x)$ Nm
$800x = 4000 - 1000x$
$1800x = 4000$
$\frac{1800x}{1800} = \frac{4000}{1800}$
$x = 2.2$m

Thus the heavy weight, is (4+2.2)m = 6.2m from the thick end.

**Example: 1**
A uniform metre rule is suspended from 40cm marking as shown in the diagram below. Find the mass of the metre rule if it's in equilibrium.




Taking moments about the pivot;
$\boldsymbol{x} = (40 - 10) = 30cm$
$\boldsymbol{d} = (50 - 40) = 10cm$

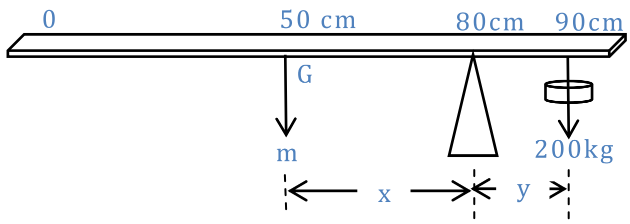
Applying the principle of moments;
Sum of clockwise moments = Sum of anticlockwise moments

$$(2 \times x) = m \times d$$
$$2(30) = 10m$$
$$60 = 10m$$
$$\frac{\not{10}m}{\not{10}} = \frac{60}{10}$$

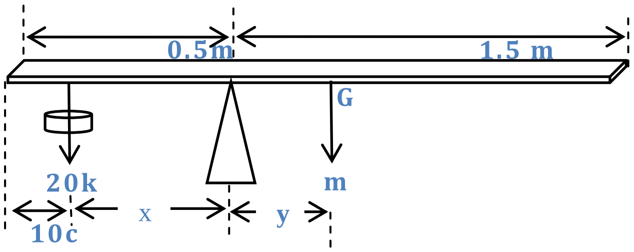
m = 6kg
Thus, the mass of the metre rule is 60kg

**Example 2:** A uniform metre rule pivoted at 10cm mark balance when a mass of 400g is suspended at 0cm mark. Calculate the mass of the metre rule. **(Ans: m=100g)**

**Example 3:** The diagram below is a metre rule pivoted at 80cm mark. Calculate the mass of the metre. **(Ans: m=67g)**




**Example 4:** A uniform beam 2m long is suspended as shown below. Calculate the mass of the metre. **(Ans: m=16kg)**




**Interpreting the question in diagram form.**
✓ the diagram for anybody should be drawn in the form.
✓ if the body is uniform, its mass or weight will act from the centre of gravity which is obtained by,

$$\text{C.o.g} = \frac{L}{2}$$

i.e. For a uniform metre rule, which is marked from 0-100cm, the centre of gravity from which the mass or weight acts is,

$$\text{C.o.g} = \frac{L}{2} = \frac{100cm}{2} = 50.0cm$$

✓ Then the required value is calculated from the principle of moment.

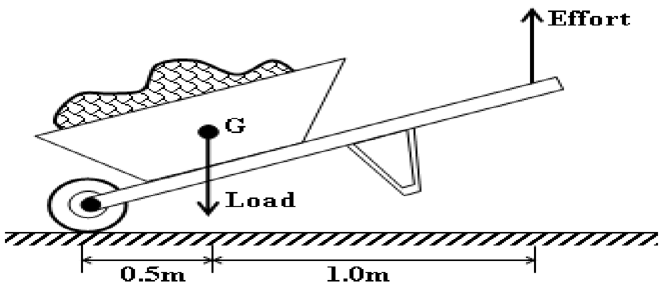
**Example:**5 A uniform metre rule is pivoted at 30cm mark. It balances horizontally when a body of mass 20g is suspended at 25cm mark.

a) Draw a force diagram for the arrangement.
b) Calculate the mass of the metre rule
**(Ans: m=5g)**

**Example: 6** A uniform half-metre rule is pivoted at 15cm mark and balances horizontally when a body of 40g is hanging from 2cm mark.
i) Draw a diagram of the arrangement.
ii) Calculate the mass of the metre rule.
**(Ans: m=52g)**

**Example: 7** A uniform rod AB of length 5cm is suspended at 2m from end A. if the mass of the rod is 10kg. Calculate the mass of the body, which must be suspended at 1m from end A so as for the rod to balance horizontally.
**(Ans: m=5kg)**

**Example: 8**A hand cart of length 1.5 m, has the centre of gravity at length 0.5 m from the wheel when loaded with 50kg as shown below.




If the mass of the hand cart is 10 kg, find the effort needed to lift the hand cart.

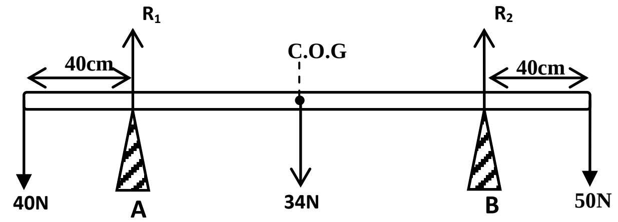
**Condition for Body to be in Equilibrium Under action of parallel forces.**

When a number of parallel forces act on a body such that the body attains equilibrium, then the following conditions must be met or fulfilled:
(i) The sum of the forces is one direction is equal to the sum of forces in the opposite direction.
(ii) The sum of the clockwise moment about any point is equal to sum of the anti-clockwise moments about the same point.

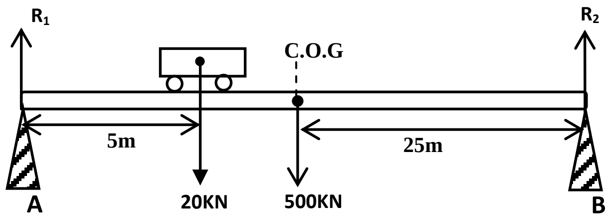
The above conditions are useful in calculations involving two unknown forces. The following steps should be taken.
(i) An equation for sum of force in one direction equaling to sum of forces in the opposite direction is written.
(ii) Moments should be taken about one of the unknown force. Where by the sum of anticlockwise moment is equal to sum of the clockwise moments.

**Example:**
A uniform wooden beam of length 2m and weight 34N rests on two supports A and B placed at 40cm from either end of the beam. Two weights of 40N and 50N are suspended at the end of the beam.
(i) Draw a diagram to show the forces acting on the beam.
(ii) Calculate the reactions at the supports.

**Solution:**
**(a)**




**Example 1:** A truck of weight 20KN is driving across a uniform 50m long bridge of weight 500KN as shown below.




Calculate the reactions at "A" and "B" if the bridge is in equilibrium sum of forces in one direction = sum of forces in opposite direction.

**Solution**
Sum of upward forces=sum of downward forces

$$R_1 + R_2 = 20KN + 500KN$$

$R_1 + R_2 = 520KN$……………………….(i)

Since $R_1$ and $R_2$ are, unknown forces so moments can be taken about either $R_1$ or $R_2$ .

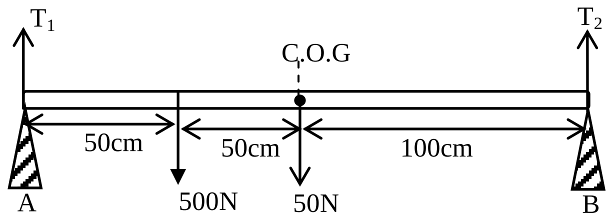
Taking moments about $R_1$:
Sum of clockwise moments = Sum of anticlockwise moments

$$(20 \times 5)+(500\times25) = R_2\times 50$$
$$100+12500= 50R_2$$
$$12600= 50R_2$$
$$\frac{\cancel{50}R_2}{\cancel{50}} = \frac{12600}{50}$$

$\underline{R_2 = 252KN}$
Substituting for $R_2 = 252KN$ into equation (i), gives;
$R_1 + 252 = 520$
$R_1 = 268KN$
Thus, the Reactions $R_1$ and $R_2$ respectively are 268KN and 252KN.

**Example 2:** Two laborers "A" and "B" carry between them a load of 500N on a uniform pole of weight 50N. if the pole is 2m long, and the load is 50cm from A towards B.

(a) Draw a diagram to show the force acting on the poles.




(b) Find the tension on A and B

**Solution**
Sum of upward forces=sum of downward forces

$$T_1 + T_2 = 500N + 50N$$

$T_1 + R_2 = 550N$……………………….(i)

Since $T_1$ and $T_2$ are, unknown forces so moments can be taken about either $T_1$ or $T_2$ .

Taking moments about $R_1$:
Sum of clockwise moments = Sum of anticlockwise moments

$$(500 \times 50)+(50\times100) = T_2\times 200$$
$$25000+5000= 200T_2$$
$$30000= 200T_2$$
$$\frac{\cancel{200}T_2}{\cancel{200}} = \frac{30000}{200}$$

$\underline{T_2 = 150N}$
Substituting for $T_2 = 150N$ into equation (i), gives;
$T_1 + 150 = 550$
$T_1 = 400N$
Thus, the Tensions $T_1$ and $T_2$ respectively are 400N and 150N.

(c) Find the fraction of the total weight that is supported by B.

$$\text{Fraction}=\frac{\text{Weight supported by B}}{\text{Total weight}}$$

$$\text{Fraction}=\frac{150}{550}$$

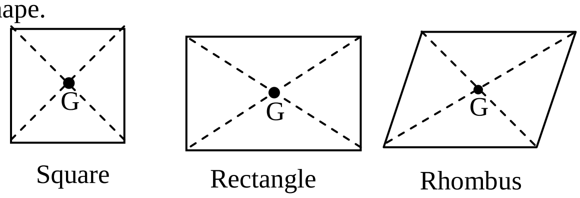
$$\text{Fraction}=\frac{3}{11}$$

**Centre of gravity**
Centre of gravity is the point of application of the resultant force due to the earth's attraction on it. All bodies are made of a large number of tiny equal particles each of mass "m" and pulled towards the earth with a force "**mg**". If the total mass of the entire particle in a body is "m" then the resultant force of gravity on the body is mg and it acts vertically downwards at the point G.

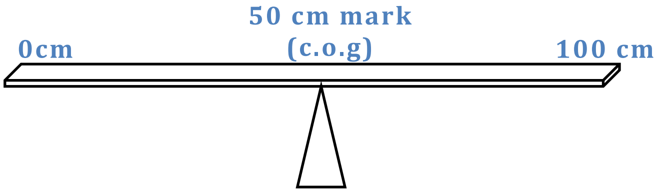
The centre of gravity or centre of the mass is a fixed point in the object where the resultant weight, (force of gravity) seems to act. If the centre of gravity is taken to, any other point of support is not zero.

Centre of gravity or regularly shaped object.

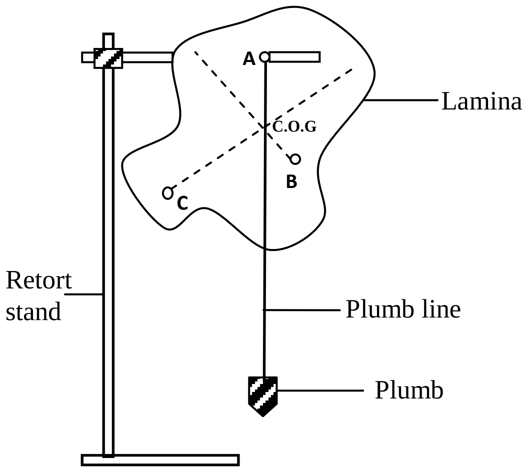
The mass or weight is evenly distributed and its centre or gravity is in the middle, which is at the geometric centre of the shape.




e.g. uniform metre rule.




**Finding the centre of gravity of an irregularly shaped object (lamina).**




Marking holes: Three holes "A", "B" and "C" are made in the object at the edges far away from each other.

Marking the cross lines: The object is suspended on a retort stand from each of the holes and plumb (or pendulum bob) is used to trace the centre of gravity by marking a line on the object tracing the plumb line thread when swinging stops.

Repeating: The experiment is repeated with the object hung at B and C and cross lines marked. The point C.O.G at which all the lines cross is the centre of gravity of the body.

**1:5:2 STABILITY:**

Stability is the difficulty with which a body topples.

When a body is at rest, it is said to be in a state of equilibrium or stability.

**Types or states of stability or Equilibrium**

Some bodies are in a more stable state than others.

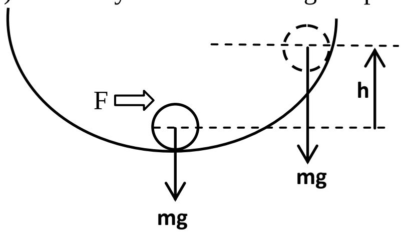
There are three types or states of equilibrium or stability:

a) stable equilibrium
b) unstable equilibrium
c) neutral equilibrium

**(a) Stable equilibrium:**

This is the type of equilibrium where, if the body is slightly tilted and then released:

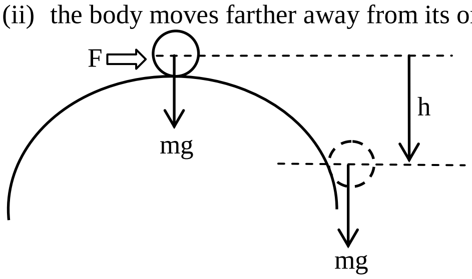
(i) the centre of gravity of the body is raised.
(ii) the body returns to its original position.




**(b) Unstable equilibrium:**

This is the type of equilibrium where, if the body is slightly tilted and then released:

(i) the centre of gravity of the body is lowered.
(ii) the body moves farther away from its original position.



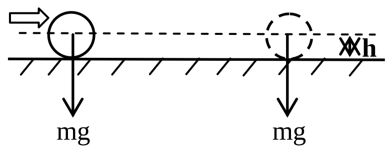

**(c) Neutral equilibrium:**

This is the type of equilibrium where, if the body is slightly tilted and then released:

(i) the centre of gravity of the body is neither raised nor lowered.
(ii) the body stays in its new position orbit just rolls on before stopping.

Example:

A ball on a flat surface.




**How to increase the stability of a body.**

The stability of a body can be increases by:

(i) Lowering the centre of gravity by putting more weight at the base.
(ii) Increasing the area of the base.
(iii) If the body is slightly displaced and then released.

**Exercise: See UNEB**

| | |
|---|---|
| 2003 Qn. 5 | 1993 Qn.14 |
| 1987 Qn.10 | 2000 Qn.11 and Qn.2 |
| 1988 Qn.2 and Qn.7 | 2002 Qn.11 |
| 1989 Qn.15 and Qn.38 | 2003 Qn.5 |
| 1991 Qn.30 | 2007 Qn.17 and Qn.5 |

## 1: 6. MACHINES

A machine is a device on which a force applied at one point, is used to overcome a force at another point.
A machine is a device, which simplifies works by magnifying the effort.
.
**Principle of machines:**
It states that a small force (effort) moves over a large distance to produce a bigger force that moves the load over a small distance.
Effort: Is the force applied at one point of a machine to overcome the load.
Load: Is the force, which is overcome by the machine using the effort.

**Mechanical Advantage (M.A):**
This is the ratio of load to effort.
i.e.; $\text{M.A} = \frac{\text{Load}}{\text{Effort}}$

**Note:-**Mechanical advantage has no units.
-M.A is the number of times the load is greater than the effort. Alternatively, it gives the number of times the machine magnifies the effort.

**Velocity ratio (V.R):**
This is the ratio of the distance moved by the effort to the distance moved by the load.
i.e.; $\text{V.R} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$
**Note:** It is the ratio of the velocity of the effort to the velocity of the load in the same time.
It is independent of friction.

**Efficiency ($\eta$):**
This is the ratio of work output to the work input expressed as a percentage.
i.e.; $\text{Efficiency } (\eta) = \frac{\text{Work output}}{\text{Work input}} \times 100\%$
$\text{Work out put} = \text{Load} \times \text{load distance}$
$\text{Work input} = \text{Effort} \times \text{Effort distance}$

$$\text{Efficiency} = \frac{\text{Load} \times \text{load distance}}{\text{Effort} \times \text{Effort distance}} \times 100\%$$
$$= \frac{\text{Load}}{\text{Effort}} \times \frac{\text{Load distance}}{\text{Effort distance}} \times 100\%$$
$$= \text{M.A} \times \frac{1}{\text{V.R}} \times 100\%$$

**Efficiency** $(\eta) = \frac{\mathbf{M.A}}{\mathbf{V.R}} \times \mathbf{100\%}$

**NOTE:**
**The efficiency of a machine system is always less than 100% because of;**
- ❖ Friction in the moving parts of the machine.
- ❖ Work wasted in lifting useless weights like movable parts of the machine.

**The efficiency can be improved by;**
- ❖ Oiling or greasing the movable parts.
- ❖ Using lightweight materials for movable parts.

**SIMPLE MACHINES:**
A simple machine is a device that work with one movement and change the size and direction of force.

Examples of simple machines:

| | |
|---|---|
| 1.Lever system<br>2.Wheel and Axle machine<br>3. Gear system<br>4. Pulley systems | 5.Screws<br>6.Inclined Planes<br>7. Wedges |

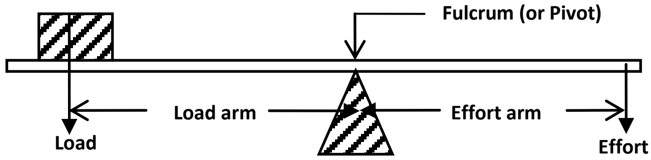
### (a) LEVER SYSTEM:
A lever is a rigid bar, which is free to move about a fixed point called fulcrum or pivot.
It works on the principle of moments.

**Classes of levers:**

| Class of lever | Position of F,E,L | Examples |
|---|---|---|
| $1^{st}$ | **F** is between E and L | Pair of scissors |
| $2^{nd}$ | **L** is between E and F | Wheel barrow |
| $3^{rd}$ | **E** is between F and L | Human arm |

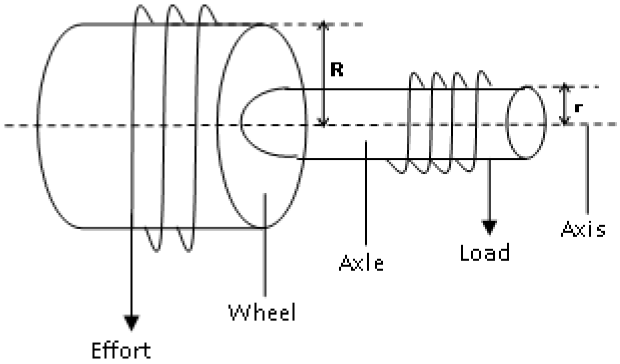
| Class of lever | Examples |
|---|---|
| (i) First class lever:<br>Is a lever system where the fulcrum (or pivot) is between the load and the effort. | • See-saw<br>• Pair of scissors<br>• Pair of pliers<br>• Weighing scale<br>• Claw Hummer |
| (ii) Second class lever:<br>Is a lever system where the load is between the fulcrum (or pivot) and the effort. | • Wheel barrow<br>• Nutcracker<br>• Bottle opener |
| (iii) Third class lever:<br>Is a lever system where the Effort is between the fulcrum (or pivot) and the load. | • Fishing rod<br>• Pair of tongs<br>• Human arm<br>• Spade<br>• Forceps |

**NOTE:** -Load arm is the distance of the load from pivot.
-Effort arm is the distance of effort from pivot.
-$\mathbf{M.A \approx V.R \Rightarrow \frac{Load}{Effort} \approx \frac{Effort\ arm}{Load\ arm}}$
Hence, a lever system is more efficient compared to other machines.




### (b) WHEEL AND AXLE MACHINE :

This consists of two wheels of different radii on the same axis. The axle has the same attachment on the wheel.
The effort is applied to the wheel and a string attached to the axle raises the load.



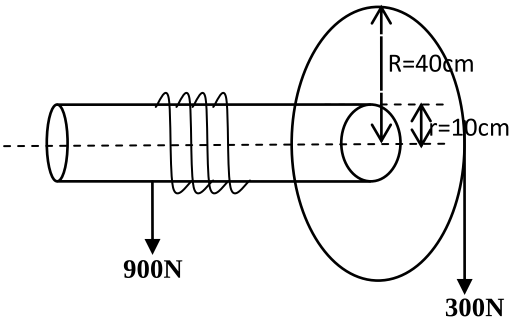

For a complete turn or rotation;

- The effort moves through a distance equal to the circumference of the wheel. $\mathbf{C = 2\pi R}$, R= radius of wheel.
- The load moves through a distance equal to the circumference of the axle. $\mathbf{C = 2\pi r}$, r= radius of axle.
- Thus, from; V.R = $\frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$

$$= \frac{2\pi R}{2\pi r}$$

$$\mathbf{V.R} = \frac{\mathbf{R}}{\mathbf{r}}$$

**Example1:**
The figure below shows a wheel land axle system, which uses an effort of 300N to raise a load of 900N using an axle of radius 10cm.




Calculate the; (i) velocity ratio
(ii) Efficiency of the system

**Solution:**

R=40cm, r=10cm; L=900N, E=300N;

Thus, from; V.R = $\frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{2\pi R}{2\pi r}$

(i)
$$V.R = \frac{R}{r} = \frac{40}{10} = 4$$
Thus, the velocity ratio is 4.

(ii).
$$M.A = \frac{\text{Load}}{\text{Effort}} = \frac{900}{300} = 3$$
**M.A = 3**

$$\text{Efficiency } (\eta) = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{3}{4} \times 100\%$$
η = 75% .
Thus, efficiency is 75%.

**Example2:**
A wheel and axle machine is constructed from a wheel of diameter 20cm and mounted on an axle of diameter 4cm.
(a) Calculate the;
(i) Velocity ratio of the machine
(ii) Greatest possible value of mechanical advantage.
(b) Explain why the mechanical advantage is likely to be less than this value.

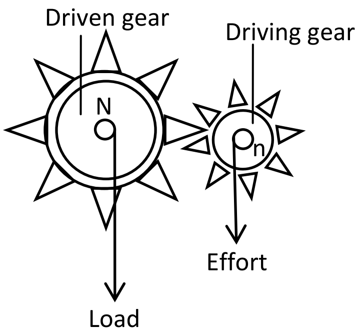
**Solution:**
D =20cm$\Rightarrow R = 10$cm, d = 4cm$\Rightarrow r = 2$cm
(a)(i)
$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{2\pi R}{2\pi r}$$
$$= \frac{R}{r}$$
$$= \frac{10}{2}$$
V.R = 5.
Thus, the velocity ratio is 5.

(a)(ii)
For the greatest (or maximum) mechanical advantage, the system is 100% efficient.
Hence M.A=V.R= 5

(b) The M.A is likely to be less than 5 because work needs to be done against friction

**Example3:**
A common windlass is used to raise a load of 480N by application of an effort 200N at right angles to the handle. If the crank is33cm from the axis and the radius of the axle is 11cm, calculate the;
(i) Velocity ratio. (Ans: V.R=3)
(ii) Efficiency of the windlass. (Ans: η =80%)

## (c) GEAR SYSTEM:
A gear is device consisting of toothed wheels.
These are rigidly fixed to the axis and turn with their axis.




They change direction and speed of rotation when the effort applied is not changed.
The direction of the driven gear is opposite to that of the driving gear.
The number of rotations of the gear wheels depends on the ratio of number of teeth and the radii of the wheels.

The effort and the load are applied on the shafts connected to the gear wheels. A large V.R is obtained only when the effort is applied on a small gear so that it drives the large gear.

$$\text{V.R} = \frac{\text{Number of teeth on driven gear}}{\text{Number of teeth on driving gear}} = \frac{\mathbf{N}}{\mathbf{n}}$$

**Example 1:**
Two gearwheels A and B with 20 and 40 teeth respectively lock into each other. They are fastened on axles of equal diameters such that a weight of 400N attached to a string wound around one-axle, raises a load of 600N attached to a string wound around the other axle. Calculate the:
(a) Velocity ratio of the system when; (i) A drives B
(ii) B drives A
(b) Efficiency when; (i) A drives B
(ii) B drives A

**Solution:**
(a) N=40cm, n=20cm
L=600N, E=400N
(i)Thus, from; $\text{V.R} = \frac{\text{Number of teeth on driven gear}}{\text{Number of teeth on driving gear}} = \frac{\text{N}}{\text{n}}$

$\text{V.R} = \frac{N_B}{n_A}$

$= \frac{40}{20}$

<u>V.R = 2.</u>
<u>The velocity ratio is 2.</u>

(ii)
$\text{V.R} = \frac{N_A}{n_B}$

$= \frac{20}{40}$

<u>V.R = 0.5.</u>
<u>The velocity ratio is 0.5.</u>

(iii) M.A = $\frac{\text{Load}}{\text{Effort}} = \frac{600}{400} = 1.5$
V.R = 2.

Efficiency $(\eta) = \frac{\text{M.A}}{\text{V.R}} \times 100\%$

$\eta = \frac{1.5}{2} \times 100\%$

<u>η = 75%</u>

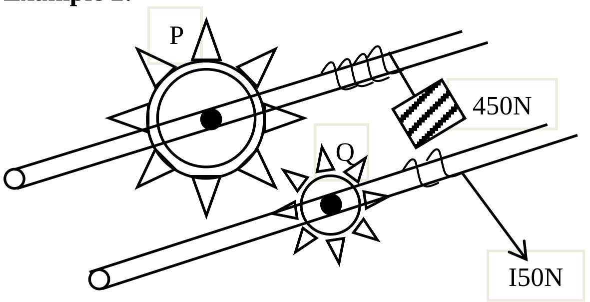
(ii) M.A=1.5
V.R =2

Efficiency $(\eta) = \frac{\text{M.A}}{\text{V.R}} \times 100\%$

$\eta = \frac{1.5}{0.5} \times 100\%$

<u>η = 300%</u>

**Example 2:**



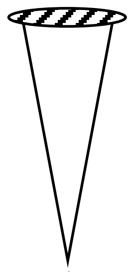

Two gear wheels P and Q with 80 and 20 teeth respectively, lock each other. They are fastened on axles of equal diameters such that a weight of 150 N attached to a string wound around one-axle raises a load of 450N attached to a string wound around the other axle. Calculate the;
(i) Velocity ratio of the gear system. (Ans: V.R=4)
(ii) Efficiency of the system. (Ans: η =75%)

**Example: 3**
Two gear wheels P and Q with 25 and 50teeth respectively lock into each other. They are fastened on axles of equal diameters such that a weight of 400N attached to the string wound around one axle raises a load of 600N attached to a string wound around the other axle. Calculate the:
(i) Velocity ratio and efficiency when Q drives P.
[Ans: V.R = 0.5, Efficiency = 300%]

(ii) Velocity ratio and efficiency when P drives Q.
[Ans: V.R = 2, Efficiency = 75%]

## (d) SCREW MACHINE :

A screw is a nail or bolt with threadlike windings.
It is like a spiral stair case.
It is an essential feature of machines like the vice and the screw jack.



- The distance between any two successive threads of a screw is called a **Pitch.**
- An effort is applied on a handle like in a vice or in a car jack.
- For a complete turn (or rotation) of the effort, the load moves through a distance equal to 1pitch while the effort moves a distance equal to the circumference of the handle

.

$$\text{V.R} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{\text{circumference of handle}}{\text{1Pitch}}$$

$$\mathbf{V.R} = \frac{\mathbf{2\pi} \boldsymbol{l}}{\mathbf{1Pitch}}$$

**Example 1:**
In a screw jack, the length of the handle is 56cm and a pitch of 2.5mm. It is used to raise a load of 2000N. Calculate the;
(i) Effort required to raise the load. (Ans: E = 1.42N).
(ii) V.R (Ans: V.R = 1408).
(iii) Efficiency of the screw, hence explain the significance of your value of efficiency. (Ans: η =100%)

**Example 2:**
A load of 800N is raised using a screw jack whose lever arm is 49cm has a pitch of 2.5cm.If it is 40% efficient, Find the
(i) V.R
(ii) M.A

**Example 3:**
A certain screw machine has a pitch of 3.5mm. The effort is applied using a handle, which is 44cm long. Calculate its velocity ratio. (Ans: V.R = 3.95)

**Example 4:**
A screw jack with a lever arm of 56 cm, has threads which are 2.5mm apart is used to raise a load of 800N. If its 25% efficient, find the;

(i) Velocity ratio (Ans: V.R = 1408)
(ii) Mechanical advantage (Ans: M.A = 352)

**Solution:**

(a) Radius (lever arm), $l$ =56cm, Pitch of a screw $= \frac{2.5}{10} =$ 0.25cm

L= 800N.

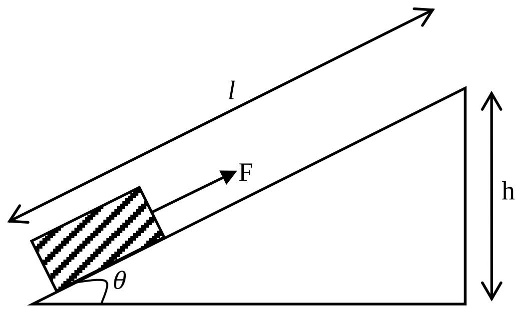
$$\text{V.R of a screw} = \frac{2\pi l}{1\text{Pitch}}$$

$$\text{V.R of a screw} = \frac{2 \times 3.14 \times 56}{0.25}$$

= 1406.72

### (e) INCLINED PLANE

An inclined plane is a slope, which allows a load to be raised more gradually by using a smaller effort than when lifting vertically upwards.

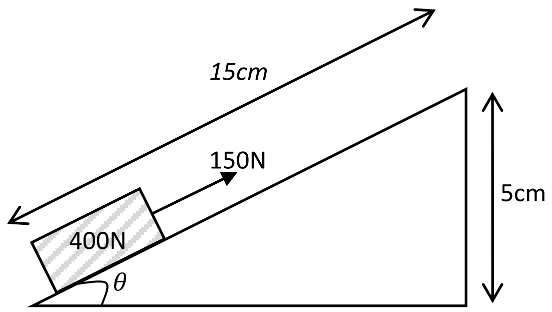



$$\text{V.R} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{\text{length of the plane}}{\text{height of the plane}} = \frac{\mathbf{l}}{\mathbf{h}}$$

$$\text{OR: V.R} = \frac{\mathbf{1}}{\mathbf{\sin\theta}}$$

**Example**:

A load of 400N is pulled along an inclined plane as shown below.




Calculate the;

**Solution:**

(i) V.R (=3)

l=15cm, h=5cm

(i)Thus, from;

$$\text{V.R} = \frac{\text{length of the plane}}{\text{height of the plane}} = \frac{\boldsymbol{l}}{\boldsymbol{h}}$$

$$\text{V.R} = \frac{\boldsymbol{l}}{\boldsymbol{h}}$$

$$\text{V.R} = \frac{\mathbf{15}}{\mathbf{5}}$$

V.R = 3.

Thus, the velocity ratio is 3.

(iii) M.A(=5N)

L=400N, E=150N

$$\text{M.A} = \frac{\text{Load}}{\text{Effort}} = \frac{400}{150}$$

$= 2.67$

(iv) Efficiency

$$(\eta) = \frac{\text{M.A}}{\text{V.R}} \times 100\%$$

$$\eta = \frac{2.67}{3} \times 100\%$$

η = 88.9%

(v) Work input

Work input $=$ Effort $\times$ Effort distance

Work input $= 150 \times 15$

$= 2250$J

(vi) Work out put

Work out put $=$ Load $\times$ load distance

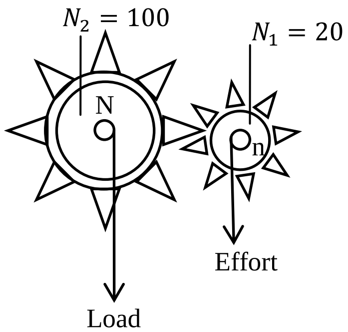
Work out put $= 400 \times 5$

$= 2000$J

**Practice Question:**

**1.** A wooden plank, 3m long is used to raise a load of 1200N through a vertical height of 60cm. If the friction between the load and the plane is40N, calculate the:
   (i) effort required [Ans: E = 280N]
   (ii) Mechanical advantage [Ans: M.A = 4.29]

**2.** In the gear system in figure 3 below $N_1$ and $N_2$ are the number of teeth on the wheels. The efficiency of the gear system is 60%.




Find the;
   (i) Velocity Ratio.
   (ii) Load that can be raised by an effort of 200N.
   (iii) Explain why its preferred to use a longer ladder to a shorter ladder when climbing a tree.

## (f) PULLEY SYSTEM:

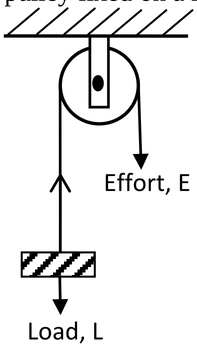
A pulley is a wheel with a grooved rim over which a string passes.

**<u>Types of pulleys</u>.**

(i) Single fixed pulley
(ii) Single movable pulley
(iii) Block and tackle pulley system

(i) **Single fixed pulley**

This is the type of pulley fixed on a rigid support.




It is applied in:

- ❖ Raising a flag
- ❖ Lifting building materials during construction

Here, -load distance = effort distance

-tension is the same throughout the string.

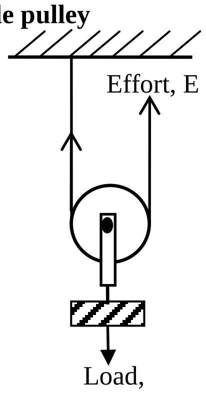
-If no friction is considered, Load = Effort. Hence M.A = $\frac{\text{Load}}{\text{Effort}} = \frac{E}{E} = 1.$ (since L = E)

However, in practice the mechanical advantage and V.R of a single fixed pulley is less than **one**. Because of the following;

(i) Some energy is wasted in overcoming friction.

(ii) Some energy is wasted in lifting useless loads like threads.

**(ii) Single movable pulley**




Here, the effort distance is twice the load distance.

Here, -load distance = 2 x effort distance

-tension is the same throughout the string.

-If no friction is considered, Load = Effort. Hence M.A = $\frac{\text{Load}}{\text{Effort}} = \frac{2E}{E} = 2.$ (since L = 2E)

***At balancing;***

***Sum of upward force = sum of downward forces***

$L = E + E$

<u>**L = 2E**</u>

M.A = $\frac{\text{Load}}{\text{Effort}} = \frac{2E}{E} = 2.$ (since L = 2E)

M.A and V.R of a single movable pulley is two

However, in practice, the M.A. of a single movable pulley is less than **two**. Because of the following reasons;

(i) Some energy is wasted in overcoming friction.

(ii) Some energy is wasted in lifting useless loads like threads.

A single movable pulley is more advantageous than a single fixed pulley. In that, for a single movable pulley the effort required to raise a load is less that the load.

**(ii) Block and tackle pulley system**

This is consists of two blocks each having one or more pulleys, combined together to form a machine. This is done in order to have high velocity ratio and a higher mechanical advantage.

It is applied in:

- ❖ Cranes
- ❖ Brake downs
- ❖ lifts

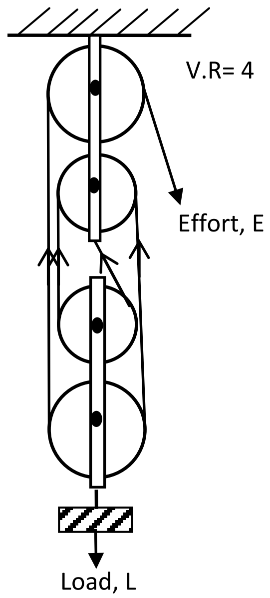
} For raising heavy loads

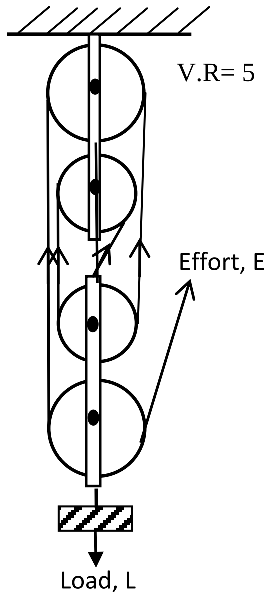
**Note:** (i) The number of portions of the string supporting the lower block is equal to the velocity ratio of the system.

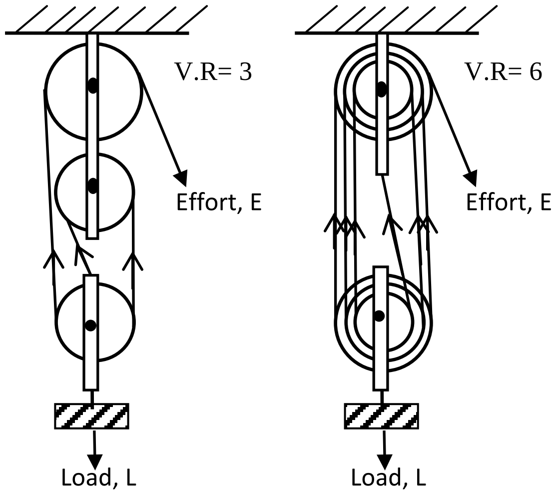
(ii) The effort applied is equal to the tension in each string supporting the movable block.

E.g. If the effort is 6N, the tension in each string is also 6N.

(iii) For an odd number of pulleys in a system, the upper block contains one more pulley than the lower block. In addition, the string starts from the lower block.



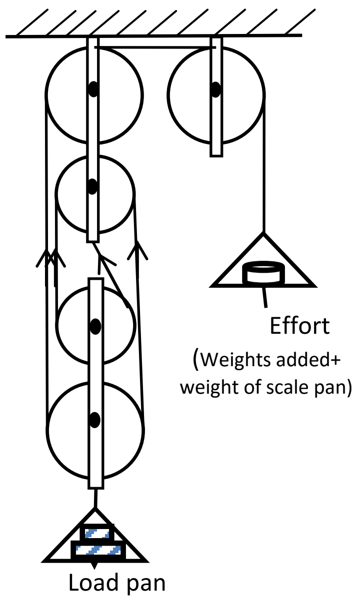






**Passing the string**

- ❖ If the number of pulley wheels is odd, then the string should be tied down to the movable block.
- ❖ For even number of pulley wheels, the string should be tied up to the fixed block.

**Experiment to measure mechanical advantage and efficiency of pulley system.**




Determining effort: A known load is place on the load pan and knows weights are added to effort pan until the load just rises steadily when given a gentle push.

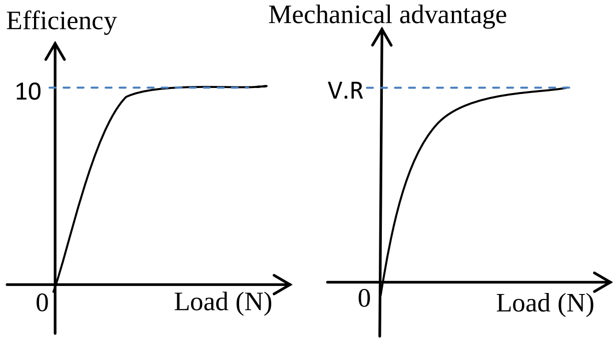
Repeating: The experiment is repeated with different loads and the results are recorded in table shown below:

V.R=……….

| Load (N) | Effort (N) | $M.A=\frac{Load}{Effort}$ | $Efficiency=\frac{M.A}{V.R} \times 100$ |
|---|---|---|---|
| ………. | …… | ………… | …………….. |

Drawing the graph:

From the table a graph of efficiency or mechanical advantage against the load is plotted.




**Explanation of the shape of the graphs:**

- ❖ As the load increases, the efficiency also increases
- ❖ This is because the weight of the movable pulley block and friction become very small compared to the load.

**Note:**

In practice, the movable block has some weight (w) and there is friction (F). These two together with the load (L) act downwards and they become part of the total downward forces.

Thus, the efficiency does not increase beyond 100% because;

i) some energy is wasted on overcoming friction

ii) Some energy is wasted on lifting useless loads like movable pulley blocks.

Therefore at Equilibrium;

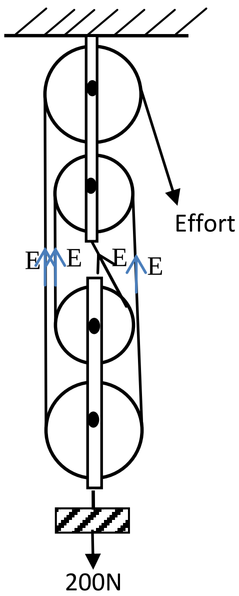
**Sum of upward forces = sum of downward force**

$$\begin{bmatrix}\textbf{Sum of tensions} \\ \textbf{suporting lower} \\ \textbf{block, (V.R) E}\end{bmatrix} = \textbf{Load(L)+Weight(W)+Friction(F)}$$

$$\textbf{V.R=L+W+F}$$

**Example 1:**

Below is a pulley system of mass 0.4kg, and there is friction of 5N




(a) Calculate the;

(i) Velocity ratio of the system

V.R= $\left(\begin{array}{c}\text{Number of portions of the string}\\ \text{suporting the movable block}\end{array}\right)$

$V.R = 4$

(ii) Effort required to raise the load.

**Solution**

Data

L=200N, m =0.4Kg, F=5N, E=?,

W=mg = 0.4 x 10

W=4N

Sum of upward forces = sum of downward force

$E + E + E + E = L + W + F$

$4E = L + W + F$

$4E = 200 + 4 + 5$

$4E = 209$

$\frac{4E}{4} = \frac{209}{4}$

$E = 52.25N$

(iii) Mechanical advantage of the system

$M.A = \frac{\text{Load}}{\text{Effort}}$

$= \frac{200}{52.25}$

$M.A = 3.83$

(b) If the load is raised through 6m, calculate the distance the effort moves at the same time.

**Example 2:**

Data

L.D =6m, E.D =?

$V.R = \frac{\text{Effort distance}}{\text{Load distance}}$

$4 = \frac{E.D}{6}$

$E.D = 4 \times 6$

$E.D = 24m$

**Example 2:**

A pulley system has two pulleys on the bottom block. A load of 1000N is hung from the bottom block, it is found that an effort of 300N to raise the load.

(i) How much energy is supplied, if the effort moves through 5m?

**Solution**

Data

L=1000N, E=300N, E.D =5m

Work in put $= \text{Effort} \times \text{Effort distance}$

$= 300 \times 5$

$= 1500N$

(ii) If the effort moves through 5m, find how far the load rises.

**Solution**

Data

E.D =5m, V.R=4, L.D=?

$V.R = \frac{\text{Effort distance}}{\text{Load distance}}$

$4 = \frac{5}{L.D} \Longrightarrow 4L.D = 5 \Longrightarrow L.D = \frac{5}{4} \Rightarrow \mathbf{L.D = 1.25m}$

(iii)Find how much energy is gained by the load if the effort moves through 5m.

Work out put $= \text{Load} \times \text{distance}$

$= 300 \times 5$

$= 1500N$

**Example 2:**

A pulley system of velocity ratio 3 is used to lift a load of 100N. The effort needed is found to be 60N.

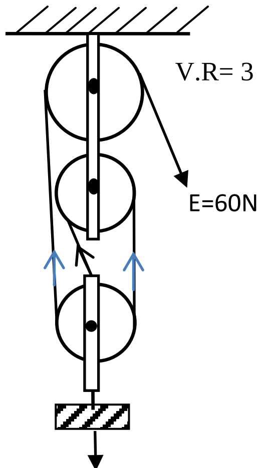
(a) Draw the arrangement of the pulley system.

**Solution**

Velocity ratio is odd. then;

Number of pulley wheels on each block $= \frac{\text{Velocity ratio}}{2} = \frac{3}{2} =$ 1 remainder 1.

The remainder wheel is added to fixed block.




(b) Calculate the efficiency of the system.

**Solution**

| | |
|---|---|
| V.R =3<br>$M.A = \frac{\text{Load}}{\text{Effort}}$<br>$= \frac{100}{60}$<br>M.A= 1.67 | Efficiency= $\frac{M.A}{V.R} \times 100\%$<br>Efficiency= $\frac{1.67}{3} \times 100\%$<br>Efficiency = 55.56% |

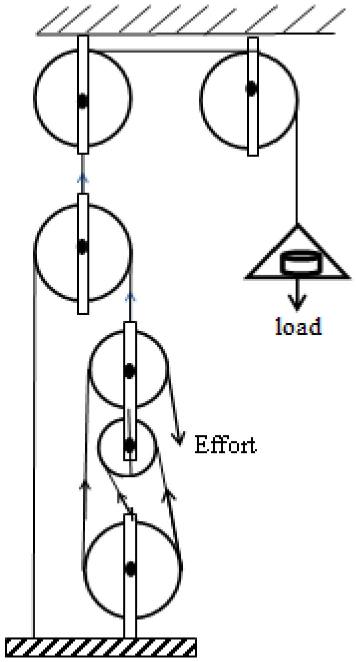
**Coupled machines**

If two or more machines are, coupled machines such that the output of one is connected to the input of the other, the overall performance is summed up by:

Overall -$V.R = V.R_1 + V.R_2$

-$M.A. = M.A_1 + M.A_2$

-$Eff = Eff_1 + Eff_2$




The diagram above shows a pulley system used by a sailor for hoisting. Calculate the:

(a) Velocity ratio of the system

**<u>Solution</u>**

Velocity ratio of lower block = 4
Velocity ratio of middle = 2
Velocity ratio of upper block = 1
<u>Overall V.R = 4 + 2+ 1 = 7</u>

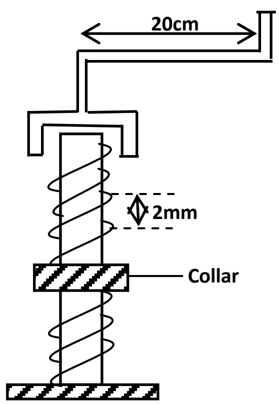
(b) The effort required to lift the load if the efficiency of the system is 75%.

**<u>Solution</u>**

| | |
|---|---|
| Efficiency= $\frac{\text{M.A}}{\text{V.R}} \times 100\%$ <br> 75%= $\frac{\text{M.A}}{7} \times 100\%$ <br> 0.75= $\frac{\text{M.A}}{7}$ <br> M.A =0.75×7 <br> <u>M. A = 5.25</u> | Then from; <br> M. A= $\frac{\text{Load}}{\text{Effort}}$ <br> 5.25= $\frac{1500}{\text{E}}$ <br> 5.25E= 1500 <br> <u>E = 285.7N</u> |

**Example:**

The diagram below shows a screw jack being used to lift a car in order that a wheel may be charged.




If the car bears down on the car with a force of 5000N and that efficiency of a screw jack is 15%. Calculate the;

a) V.R.

Given; Radius, $r = 2\text{cm} = \frac{2}{100} = 0.02\text{m}$

Pitch, $P = 2\text{mm} = \frac{2}{1000} = 0.002\text{m}$

| | |
|---|---|
| Then; <br> V.R= $\frac{\text{Effort Distance}}{\text{Load Distance}}$ <br> V.R= $\frac{2\pi r}{Pitch}$ <br> V.R= $\frac{2(3.14)(0.02)}{0.002}$ <br> <u>V.R=62.8</u> <br><br> Mechanical Advantage <br><br> Given; <br> Efficiency=15%, <br> V.R= 62.8 <br> Then; <br> Effi = $\frac{\text{M.A}}{\text{V.R}} \times 100\%$ <br> 15%= $\frac{\text{M.A}}{62.8} \times 100\%$ <br> $0.15 = \frac{\text{M.A}}{62.8}$ <br> M. A = 0.15(62.8) <br> <u>M. A = 9.42</u> | (b)The effort required to turn the handle <br><br> M.A= $\frac{\text{Load}}{\text{Effort}}$ <br> 9.42= $\frac{5000}{\text{E}}$ <br> 9.42E=5000 <br> $\frac{\cancel{9.42}\text{E}}{\cancel{9.42}} = \frac{5000}{9.42}$ <br> <u>E=530.79N</u> <br><br> (c)Work done by the operator in order to raise the side or the car by 25cm. <br><br> Eff = $\frac{\text{Work output}}{\text{Work input}} \times 100\%$ <br> Work output=Load × Load distance <br> Work output=5000× $\left(\frac{25}{100}\right)$ <br> Work output=1250J |

NB: Work input is the work done by the effort. Sometimes it is considered as the work done by operator.

$$\text{Efficiency} = \frac{\text{Work output}}{\text{Work input}} \times 100\%$$

$$15\% = \frac{1250}{\text{W}_{\text{in}}} \times 100\%$$

$$0.15 = \frac{1250}{\text{W}_{\text{in}}}$$

$$0.15\text{W}_{\text{in}} = 1250$$

<u>$W_{in} = 8333.33\text{J}$</u>

In general;
Work wasted = work input - work output
= 8333.33 – 1250
=<u>7083.33J</u>

**From above, it is noted that work input is greater than workout put due to;**

i) some work wasted in lifting useless loads,
ii) Some work wasted in reducing friction.

**Note:** For the screw the velocity ratio is very high because the length of the handle is very big compared to the pitch of the screw.
However the efficiency is very low. Usually lower than 50%. This is because friction is very high so the screw cannot run back if left.

**Exercise : See UNEB**

| | |
|---|---|
| **1999 Qn.2** | **1998 Qn.6** |
| **1994 Qn.8** | **2006 Qn.4** |
| **1987 Qn.36** | **1992 Qn.6** |
| **1988 Qn.34** | **2001 Qn.42** |
| **1991 Qn.26** | **2007 Qn.1** |

# WORK, ENERGY AND POWER

**WORK**

**Work** is the product of the force applied and the distance moved by the point of application of the force in the direction of the force.

Note that the distance moved has to be in the direction of the applied force. It is common that a force may be applied to move an object to the right, but instead the object moves to the left.

The force in this case has not done any work.

**Work done = Force, F × Displacements**

**W= FS**

The S.I unit of work done is a **joule** (J)

Definition:

A **joule** is the work done when the point of application of a force of 1N, moves through a distance of 1m in the direction of the force.

**Example:1**

1. Calculate the work done when a force of 9000N acts on a body and makes it move through a distance of 6m.

**Solution**

Force, F= 9000N

Distance,s = 6m

Work done = Force,F × Displacement,S

**W= F×S**

W= 9000×6

W= 54000J

**Note:**

If an object is raised vertically or falling freely, then the force causing work to be done is weight.

Force = Weight = mass, m × acceleration due to gravity,g

Force = Weight= mg

Thus, the work done against gravity is given by;

**Work done = Weight × height**

**Work done = mgh**

Where m is mass in kg, h is distance in metres and sometimes, it is height.

**Example:2**

A block of mass 3kg held at a height of 5m above the ground is allowed to fall freely to the ground. Calculate the work done.

**Solution**

Given, mass, m = 3Kg, Distance, s = 5m

Force F= Weight,W =mass × g

F= mg

= 3×10

F = 30N

Work done = Force,F × Displacement,S

**W= F×S**

W= 30×5

W= 150J

**Example: 3**

A man of mass 80kg runs up a staircase of 10stairs, each of vertical height 25cm. Find the work done against gravity.

**Solution:**

Given, mass m = 80Kg,

Distance, $h = 25\text{cm} = \frac{25}{100} = 0.25\text{m}$

Total Distance, $h_T = 0.25\text{m}\times 10\text{stairs}$

Total Distance, $h_T = 2.5\text{m}$

Then;

Work done = Weight × height

Work done = mgh

= 80×10×2.5

Work done = 2000J

**Example: 4**

A crane is used to to raise 20 tons of concrete to the top floor of a building 30m high. Calculate the total work done by the crane.

**Solution:**

Given, mass m = 20tonnes=20×1000=20,000Kg,

Distance, h = 30m

Then;

Work done = Weight × height

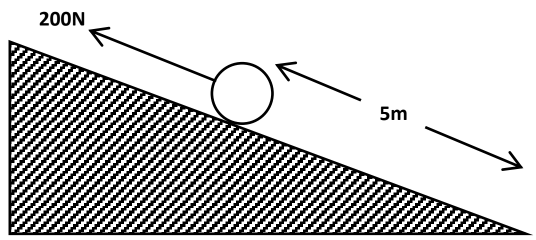
Work done = mgh

= 20,000×10×30

Work done = 6,000,000J

**Example: 5**

The figure bellow shows a bale of hay being pulled up an inclined plane with a force of 200N. The bale moves down the incline to a distance of 5m.




(i) Calculate the work done by the force.

**Solution:**

Work done = Force,F × Displacement,S

= 200N×(-5m)

Work done = -1000J

(ii) Explain your answer.

The distance moved by the bale, was in a direction opposite to that of the force applied hence a negative displacement.

The negative in the answer therefore means that the bale did the work instead of the force applied.

**1:7:2.ENERGY**

Energy is the ability or capacity to do work.

The S.I unit of work done and energy is a joule (J).

**Sources of energy:**
The raw material for the production of energy is called the energy source.
There are two types of energy sources.

(a) Non-renewable sources of energy
These are energy sources, which cannot be replaced when they get used up.
Examples of non- renewable sources of energy
(i) Fossil fuels; these are formed from plant remains that died million years ago. They include; coal, petroleum oil, natural gas, e.t.c.
(ii) Nuclear fuels; these are fuels found in radioactive elements which may be occurring naturally such as Uranium.
These fuels can be used in nuclear reactions to produce electricity.

Advantages of non-renewable source of energy.
- They have high energy density. I.e. a lot of energy can be produced from a small quantity.
- They are readily available as demand increases.

Disadvantages of non-renewable source of energy.
- They are highly polluting.

(b) Renewable sources of energy
These are energy sources which can be replaced when they get used up. They can never get exhausted.
Advantage:
They are non-polluting.

Examples of renewable sources of energy.
(i) Solar energy:This is the form of energy which reaches the earth in form of heat and light.
It can be harvested using solar panels and transformed into electrical energy, which is used for many purposes.
It is also used in direct low temperature heating.
(ii) Wind: Wind can be harvested using giant windmills, which can turn electrical generators to produce electrical energy, which is a more useful form.
(iii) Running water: Running water is used in hydro-electricity plants to turn giant turbines, which produce electrical energy.
The water will always flow hence a renewable source. Tides can also be used to generate electricity in this way.
(iv) Geothermal energy: Water is pumped to hot underground rocks where it's heated and then forced out through another shaft where it can turbines.

**Forms of energy**
Energy can exist in the following forms;
a) Chemical energy:
Chemical energy is the form of energy a body has due to the nature of its atoms and molecules and the way they are arranged.
In the combination of atoms to form compounds, there is gain or loss of energy. This energy is stored in the compound as chemical energy.
If the atoms in such compounds are rearranged to form a new compound, this energy is released. E.g. If sugars in the human body are burnt, a lot of chemical energy is released.

b) Nuclear energy:
This is the energy released when atomic nuclei disintegrate during nuclear reactions.
In nuclear reactions, the energy, which holds the nuclear particles together (Binding energy), is released.
There are two types of nuclear reactions i.e. fission (Where large nuclei break to form smaller ones) and fusion (Where smaller nuclei combine to form larger ones). In both cases, large amounts of energy are released.

c) Electrical energy (Electricity):
This is the form of energy which is due to electric charges moving from one point of a conductor to another.
This form of energy is most easily converted to other forms, making it the most useful form.

d) Light energy:
This is the form of energy which enables us to see. Light is part of a wider spectrum of energy called the electromagnetic spectrum. Light consists of seven visible colors, of red, orange, yellow, green, blue, indigo and violet. We are able to see because the eye is sensitive to the colors.

e) Heat energy:
Heat is a form of energy, which results from random movement of the molecules in the body.
It is responsible for changes in temperature.
When a body is heated or when heat energy of the body increases;
(i) The internal kinetic energy of the molecules increases leading to a rise in temperature.
(ii) The internal potential energy of molecules increases leading to expansion and change of state of the body.

f) Sound energy:
This is the energy which enables us to hear.
Like light, sound is also a form of wave motion, which makes particles to vibrate. Our ears are able to detect sound because it produces vibrations in the ear.
g) Mechanical energy:
This is the energy of motion.
**Mechanical energy=kinetic energy+Potential energy**

There are two forms of mechanical energy.
(i) Kinetic energy:- This is the energy possessed by a body due to its velocity or motion.

$$\text{Kinetic energy}=\frac{1}{2}(\text{mass})\times(\text{velocity})^2$$

$$\mathbf{K.E} = \frac{1}{2}mv^2$$

(ii) Potential energy:- This is the energy possessed by a body due to its position or condition.
It is equal to the work done in putting the body in that position or condition.
A body above the earth's surface has an amount of gravitational potential energy equal to the work done against gravity.
Weight is the force of gravity acting on a body.
Weight = mg.

$$\begin{pmatrix}\textbf{Gravitational}\\\textbf{Potential energy}\end{pmatrix} = (\textbf{mass}) \times \begin{pmatrix}\textbf{acceleration}\\\textbf{due to gravity}\end{pmatrix} \times \begin{pmatrix}\textbf{Height above}\\\textbf{the ground}\end{pmatrix}$$

$$\mathbf{P.E = mgh}$$

**Conservation of Energy.**

The principle of conservation of energy:

It states that 'energy is neither created nor destroyed' but can be changed from one form to another.

In any system, the total original energy is equal to the total final energy. For example, electrical energy is changed to light energy in the bulb. However, the bulb also feels hot because some of the energy is changed to heat.

Therefore, light energy plus the heat energy is equal to the electrical energy supplied.

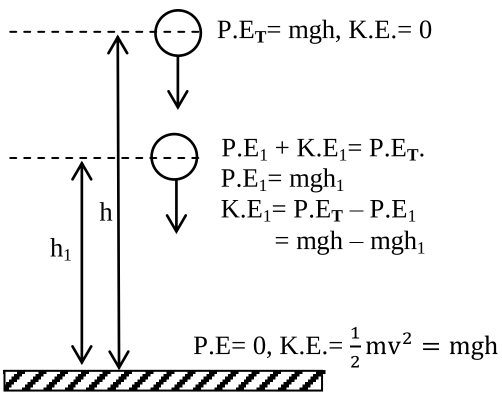
Thus from this principle, we conclude that;

- No new energy is created
- Total existing energy is not destroyed
- Energy is only changed from one form to another.

As energy is changed from one form or state to another, an energy converter (Device) is required to ease the conversion. Examples of such devices are shown in the table below.

| **Energy Change** | **Converter** |
|---|---|
| Chemical to electrical | Cells or Batteries |
| Light to Electrical | Solar panels |
| Electrical to light | Electric lamps e.g. bulbs |
| Electrical to heat | Cooker or flat iron, etc. |
| Heat to Electrical | Thermocouple |
| Electrical to sound | Loud speakers |
| Sound to Electrical | Microphones |
| Electrical to Kinetic | Electric motors |
| Kinetic to Electrical | Electric generators |

Conservation of mechanical energy:




A body of mass m at a height h above the ground, has a potential energy, P.E = mgh. At this point, the velocity of the body is $0ms^{-1}$ hence it has no kinetic energy. (K.E. = 0J).

When the body is released, it begins to fall with an acceleration g. The velocity of the body thus increases as the height, h decreases. The body therefore gains kinetic energy at the expense of potential energy.

When the body is just reaching the ground, the height, h is zero (h = 0m) while its velocity is given by;

$v^2 = u^2 + 2as; \qquad where\ s = h, a = g\ and\ u = 0$

$v^2 = 0^2 + 2gh$

$v^2 = 2gh$

$\boldsymbol{v = \sqrt{2gh}}$

Thus, its kinetic energy as it reaches the ground is given by; $K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{2gh}\right)^2 = mgh$

Hence, $\qquad \mathbf{K.E = \frac{1}{2}mv^2 = mgh}$

Therefore; **Gain in K.E = Loss in P.E.**

The above illustration shows that energy is conserved. Mechanical energy is continually transformed between kinetic and potential energy.

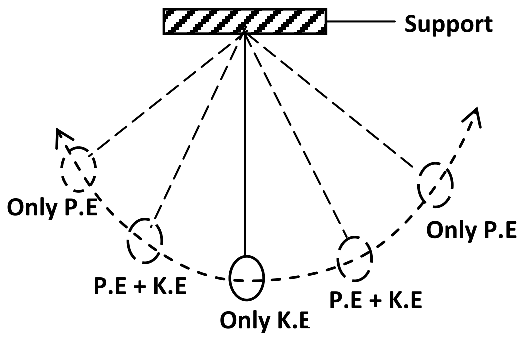
**A swinging pendulum.**

The transformation of energy between kinetic and potential energy can also be seen in a swinging pendulum.

At the end (extremes) of the swing, the energy of the pendulum bob is only potential.

As it passes the central position, it has only kinetic energy.

In other positions between the extreme ends and the central position, it has both potential and kinetic energies.




Mechanical energy=P.E + K.E

As the bob falls from the left towards the central position, it gains K.E at the expense of P.E.

As it rises from the central position towards the left end, it gains P.E at the expense of K.E.

**Example**:

A ball of mass 200g falls freely from a height of 20m above the ground and hits a concrete floor and rebounds to a height of 5m. Given that g = $10ms^{-1}$, find the;

i) P.E of the ball before it fell.
ii) Its K.E. as it hits the concrete.
iii) Velocity with which it hits the concrete.
iv) K.E as it rebounds.
v) Velocity with which it rebounds.
vi) Velocity when it has fallen through a height of 15m.

**Solution:**

**(i)**
P.E=mgh (h=height from which the ball is dropped)
P.E=0.2×10×20
P.E=40J

**(ii)**
As it hits the concrete, Total P.E is converted to K.E
K.E=$\frac{1}{2}mv^2$= mgh
(h=height from which the ball is dropped)
K.E=0.2×10×20
K.E=40J

**(iii)**
As it hits the concrete, Total P.E is converted to K.E
K.E=$\frac{1}{2}mv^2$= mgh
(h=height from which the ball is dropped)
$\frac{1}{2}mv^2$= mgh
$v^2$= 2gh.
$v = \sqrt{2gh}$
$v = \sqrt{2(10)(20)}$
$v = \sqrt{400}$
$v = 20ms^{-1}$

**(iv)**
As the bounces from the concrete, the K.E used to move the it from the bottom to the height $h_1$ is converted to P.E at $h_1$and it is momentarily at rest.
K.E$_1$=$\frac{1}{2}mv_1^2$= mgh$_1$
($h_1$=height to which the ball bounces).
K.E$_1$=0.2(10)(5)
K.E$_1$=10J

**(v)**
As the bounces from the concrete, the K.E used to move it from the bottom to the height $h_1$ is converted to P.E at $h_1$and it is momentarily at rest.
K.E$_1$=$\frac{1}{2}mv_1^2$= mgh$_1$
($h_1$=height to which the ball bounces).
$\frac{1}{2}(0.2)v_1^2$=0.2(10)(5)
$0.1v_1^2$= 10
$v_1^2 = 100$
$v_1 = \sqrt{100}$
$v_1 = 10ms^{-1}$

(vi) As it falls from the top, Total P.E at the top is converted to some K.E and some P.E in
Falling to the height $h_1$.
K.E$_T$=$\frac{1}{2}mv_1^2$+ mgh$_1$
($h_1$=height of the ball from ground).
40=$\frac{1}{2}(0.2)v_1^2$+ 0.2(10)(5)
40= $0.1v_1^2$+ 10
30= $0.1v_1^2$
$v_1^2 = 300$
$v_1 = \sqrt{300}$
$v_1 = 17.32ms^{-1}$

**Example 1**:
Calculate the kinetic energy of a 2Kg mass trolley traveling at 400m per second.
$Given; m = 2kg, v = 400ms^{-1}$
K.E=$\frac{1}{2}mv^2$
K.E=$\frac{1}{2}(2)(400)^2$
K.E=160,000J

**Example 2:**
A 5Kg mass falls from a height of 20m. calculate the potential energy lost.
$Given; m = 5kg, h = 20m$
P.E=$mgh$
P.E=5(10)(20)
P.E=1000J

**Example 3:**
A 200g ball falls from a height of 0.5m. Calculate its kinetic energy just before hitting the ground.
$Given; m = 200g = \frac{200}{1000} = 0.2kg, h = 0.2m$
K.E gained = P.E lost
K.E=mgh
K.E=0.2(10)(0.5)
K.E=1J

**Exercise:**
1. A block of mass 2 kg falls freely from rest through a distance of 3m.
i) Find the K.E of the block. (Ans: =60J)
K.E gained = P.E lost
ii) Potential energy (Ans: =60J)
iii) The velocity with which the body hits the ground. (K.E gained = P.E lost).

2. A body falls freely through 3m. Calculate the velocity with which it hits the ground.(Ans: = 7.75ms$^{-1}$)

3. 100g steel ball falls from a height of 1.9m on a plate and rebounds to a height of 1.25m. Find the;

(i) P.E of the ball before the fall. (Ans: =1.8J)
(ii) Its K.E. as it hits the plate. (Ans: =1.8J)
(iii) Its velocity on the plate. (Ans: =6ms$^{-1}$)
(iv) Its K.E as it leaves the plate on rebound. (Ans: =1.25J)
(v) Its velocity of rebound. (Ans: =5ms$^{-1}$)

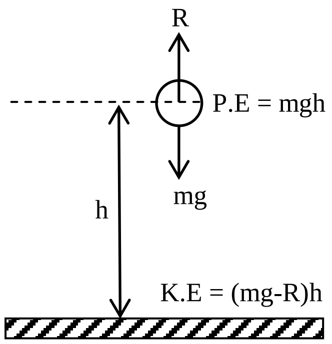
For a body not falling freely but as it falls it experiences air resistance then the kinetic energy gained by the body just before it hits the ground is calculated from:
$$\mathbf{K.E\ gained = (mg - R)h}$$

Where mg is the weight of the body, R is the air resistance and h is the height above the ground.

**Example 4:**
A 20kg body falls from 1.8m above the ground. If the air resistance is 0.9N.
(i) Calculate the kinetic energy just before hitting the ground.

**Solution**

$Given; m = 20kg, R = 0.9N, h = 1.8m, K.E. = ?$




K.E gained=(mg-R)h

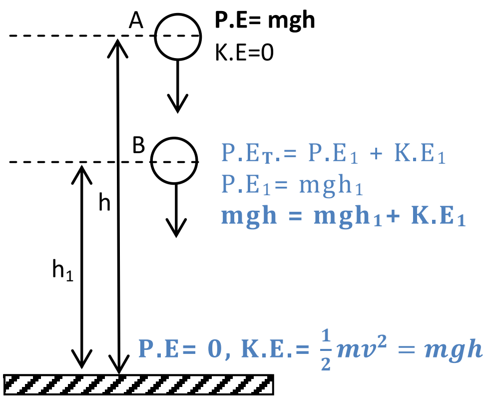
K.E gained=(20×10 - 0.9)×1.8
=(200 - 0.9)×1.8
=(199.1)×1.8
K.E gained=358.38J

(ii) Calculate energy lost due to air resistance
Total energy at h,=mgh
=20×10×1.8
Total energy at h,=360J

Energy lost due to air resistance=360J-358.38J
Energy lost due to air resistance=1.62J

**Note:**
Energy lost due to air resistance can also be calculated from;
Energy lost due to air resistance=Work done against R
Energy lost due to air resistance=Force×Height
Energy lost due to air R=Air resistance×Height
=0.9×1.8
Energy lost due to air R=1.62J

**Calculating the kinetic energy at any point for a body falling freely.**

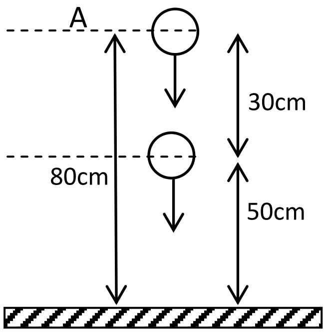



At A the body has all potential energy. So the energy at A is mgh = Total energy.

At B the body has a mixture of kinetic energy and potential energy.
$P.E_T = K.E_1 + P.E_1$
$mgh = K.E_1 + mgh_1$
$\mathbf{mgh = \frac{1}{2}mv_1^2 + mgh_1}$
Where "$\mathbf{h_1}$" is the height above the ground.

**Example 5:**
A stone of 150g is dropped from a height of 80m. Calculate the;
(i) Kinetic energy when it is 50m above the ground.

**Solution:**
$m = 150g = \frac{150}{1000} = 0.15kg$, $h_1 = 50m$, $h = 80m$.




(i)
$mgh = K.E_1 + mgh_1$
$0.15(10)(80) = K.E_1 + 0.15(10)(50)$
$120 = K.E_1 + 75$
$K.E_1 = 120 - 75$
$\underline{K.E_1 = 45J}$

(ii) Its velocity when its 50m above the ground.

Method 1 :
$K.E_1 = \frac{1}{2}mv_1^2$
$45 = \frac{1}{2} \times 0.15 \times v_1^2$
$90 = 0.15v_1^2$
$600 = v_1^2$
$\sqrt{600} = v_1$
$\underline{v_1 = 24.49ms^{-1}}$

Method 2 :
Given; $a = 10ms^{-2}$, $u = 0ms^{-1}$
Where h is the height fallen through.

Then using the third equation of motion, we have;
$v^2 = u^2 + 2ah$
$v^2 = 0^2 + 2(10)(30)$
$v^2 = 600$
$v = \sqrt{600}\ ms^{-1}$
$\underline{v = 24.49\ ms^{-1}}$

(iii) Its kinetic energy when it has fallen through 50m.
Given; $g = 10ms^{-2}$, $h = 80$, $h_1 = (80-50) = 30m$, K.E=?
Where h is the height above the ground.
Then from;
$mgh = K.E_1 + mgh_1$
$0.15(10)(80) = K.E_1 + 0.15(10)(30)$
$120 = K.E_1 + 45$
$K.E_1 = 120 - 745$
$\underline{K.E_1 = 75J}$

### 1:7:3. POWER

Power is the rate of doing work. Or
Power is the rate of transfer of energy.
**Note:** Work done is the same as energy transferred.

$$\textbf{Power} = \frac{\textbf{Work done}}{\textbf{Time taken}} = \frac{\text{Energy transfered}}{\text{Time taken}}$$

Where work done=Force×Distance

$$\text{Power} = \frac{F \times d}{t} = F \times \frac{d}{t} = FV$$

$$\textbf{Power} = \textbf{F.V}$$

Where V is the velocity of the body.

$$\textbf{Power} = \frac{\textbf{mgh}}{\textbf{t}}$$

Where mg is the weight of the body and h the height.

The S.I unit of power is **watt** (W).
**1watt=1Js$^{-1}$**

<u>Definition:</u>
**A watt** is the rate of transfer of energy of one joule per second.
Or It is the rate of doing work of 1joule in one second.

**Example 1:**
An engine raises 20kg of water through a height of 50m in 10 seconds. Calculate the power of the engine.
**Solution:**

$$\text{Power} = \frac{mgh}{t}$$

$$\text{Power} = \frac{20(10)(50)}{10}$$

$$\underline{\text{Power} = 1000W}$$

**Example 2:**
An electric bulb is rated 100W. How much electrical energy does the bulb consume in 2hours.
**Solution:**

$$\text{Power} = \frac{\text{Energy used}}{\text{time taken}}$$

$$100 = \frac{\text{Energy used}}{2 \times 60 \times 60}$$

$$\text{Enery used} = 100(2 \times 60 \times 60)$$

$$\underline{\text{Enery used} = 720{,}000J}$$

**Example 3:**
A man uses an electric motor whose power output is 3000W for 1hour. If the motor consumes $1.44\times10^7$J of electricity in that time, find the efficiency of the motor.
**Solution:**
Given;$P_{out}$=3000W, t=1hr=1×60×60=3600s.
$\text{Energy}_{input}=1.44\times10^7$J

$$\text{power input} = \frac{\text{Energy input}}{\text{time taken}}$$

$$P_{in} = \frac{E_{in}}{t}$$

$$P_{in} = \frac{1.44\times10^7}{36000}$$

$$P_{in} = 4000W$$

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} \times 100\%$$

$$\text{Efficiency} = \frac{3000}{4000} \times 100\%$$

$$\underline{\text{Efficiency} = 75\%}$$

**For machines**

Power input is the power created by effort.

$$\text{Power input} = \frac{\text{Work input}}{\text{Time taken}} = \frac{\text{Effort}\times\text{Effort Distance}}{\text{Time taken}}$$

Power output is the power created by the load.

$$\text{Power output} = \frac{\text{Work output}}{\text{Time taken}} = \frac{\text{Load}\times\text{Load Distance}}{\text{Time taken}}$$

**Example 4:**
An effort of 250N raises a load of 1000N through 5m in 10 seconds. If the velocity ratio is five, Calculate the:
i) Power input
ii) Efficiency
**Solution:**

| | |
|---|---|
| **(i)**Given;<br>Effort=250N, Load=1000N, V.R=5, L.D=5m, t=10s<br>$V.R = \frac{E.D}{L.D} \Leftrightarrow 5 = \frac{E.D}{5}$<br><u>E.D=25m</u><br>$\left(\begin{matrix}\text{Power}\\ \text{input}\end{matrix}\right) = \frac{\text{Work input}}{\text{Time taken}}$<br>$= \frac{\text{Effort}\times\text{Effort Distance}}{\text{Time taken}}$<br>$= \frac{250\times25}{10}$<br><u>Power input=625 W</u> | $\left(\begin{matrix}\text{Power}\\ \text{input}\end{matrix}\right) = \frac{\text{Work output}}{\text{Time taken}}$<br>$= \frac{\text{Load}\times\text{Load Distance}}{\text{Time taken}}$<br>$= \frac{1000\times5}{10}$<br><u>Power output=500 W</u><br>**(ii)**<br>$\text{Eff} = \frac{\left(\begin{matrix}\text{Power}\\ \text{output}\end{matrix}\right)}{\left(\begin{matrix}\text{Power}\\ \text{input}\end{matrix}\right)} \times 100\%$<br>$\text{Efficiency} = \frac{500}{625} \times 100\%$<br><u>Efficiency = 80%</u> |

### INTERNAL COMBUSTION ENGINE

A heat engine is a machine, which changes heat energy obtained by burning fuel to kinetic energy (mechanical energy)

Engines are always less than 100% efficient because:-
h) Some of the energy is lost in overcoming friction between walls of the cylinder and pistons.

ii) Some heat energy is lost to surrounding due to conduction.
iii) Some of the energy is also wasted in lifting useless loads like pistons.

**Qn1:** A pulley system of V.R six is used to lift a load of 250N through a distance of 3m. If the effort applied is 50N, calculate how much energy is wasted.

**Qn2**: A girl of mass 40kg runs up a stair case in 16 seconds. If each stair case is 20 cm high and she uses $100\ Js^{-1}$ . Find the number of stairs. [Ans: 20]

**See UNEB**

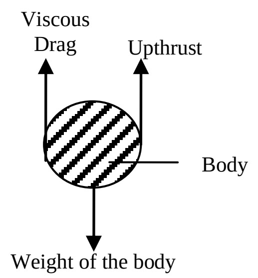
| | |
|---|---|
| 1994 Qn. 17 | 2006 Qn.7 |
| 1989 Qn. 29 | 1997 Qn.5 |
| 2007 Qn.33 | 1995 Qn.9 |
| 1987 Qn.3 and Qn.24 | 1991 Qn.11 |
| 1993 Qn.4 and 18 | 1992 Qn.11 |
| 1997 Qn.10 | 2003 Qn.15 |
| 1999 Qn.2 and Qn.8 | 2007 Qn.6 |
| 2000 Qn.23 | 1993 Qn.4 |
| 2001 Qn.26 | 2005 Qn.45 |

## MOTION IN FLUIDS

When a body falls through a fluid it will be acted on by three forces namely:

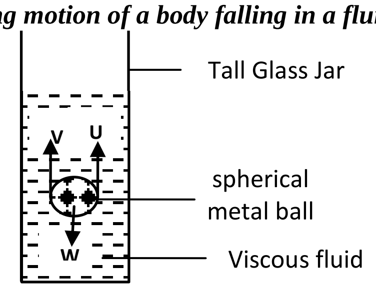
i) weight of the body
ii) viscous force (Viscous drag)
iii) up thrust

**Directions of the above forces.**
i) Weight of body: down ward direction towards earth.
ii) Up thrust: upward direction
iii) Viscous force; direction opposite to that of motion




**Direction of motion**
The direction of motion is determined by direction of the viscous force, which is a force that opposes motion like in the above body the direction of motion is down ward because the viscous force is acting in upward direction.

***Describing motion of a body falling in a fluid***



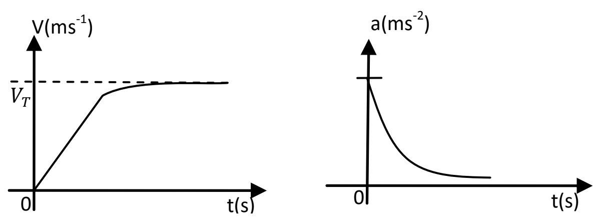

-As the body falls, it accelerates first with a net force (resultant force) given by the equation.

**F=W – (v+u) Or F=W – v – u**

-As the body continues to fall, it attains a maximum uniform velocity called ***terminal velocity*** when weight of body (W) = Viscous force (v) + up thrust (u)

**W = v + u Or W – v= u**

**Terminal velocity** is the uniform velocity attained by a body falling through a fluid when the net force on the body is zero such that: **Weight = Viscous force + up thrust**




**Velocity and acceleration graphs for a body falling in a fluid.**

**BERNOULLI'S PRINCIPLE**

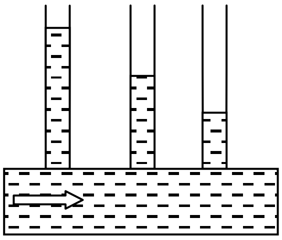
*It states that when the speed of the fluid increases, the pressure in the fluid decreases and vice versa.*

Liquids flowing in a pipe have three kinds of energies, namely;

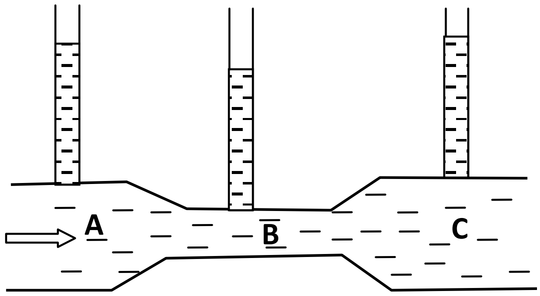
- ✓ kinetic energy
- ✓ potential energy
- ✓ pressure energy

the sum of these three energies is a constant.

a) Liquid



When the liquid flows through the uniform tube, the level goes on decreasing as shown in the diagram, the faster the liquid, the lower the pressure.



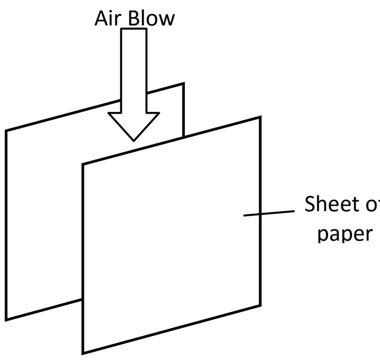

The pressure falls in the in the narrow part B but rises again in the wider part C. This is because, since B is narrow, the speed at which the liquid moves through it is higher, hence the fall in pressure.

Note:

- ✓ Fluid pressure changes with the rate of flow in the pipe
- ✓ Speed of water is greater at the constriction
- ✓ The order of pressure in the tubes decreases in the order A, C and B.

b) Gases

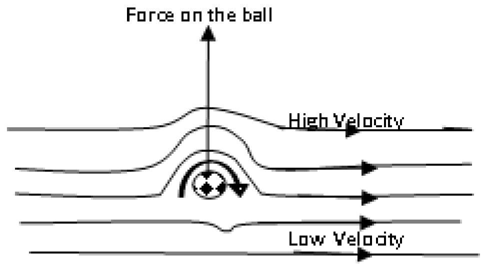
Bernoulli Effect in an air stream can be shown by blowing air between two sheets.



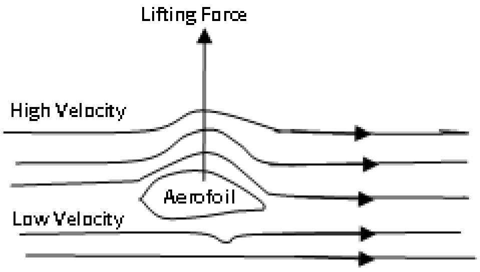

When air is blown the two sheets come together because the air between them moves faster resulting in decrease of pressure between them.

***Application of Bernoulli's principle***

i) When the fluid comes out of a jet, the speed increases as the pressure decreases.
ii) At the jet the gas comes out at high speed so the pressure is low at the jet. This results in air to be drawn in.
iii) A spinning ball takes a curved path because the ball drag air around causing air to pass more rapidly over one side than the other. This results in pressure difference that causes a resultant force on the ball.




iv) An aero plane wing called aero foil is shaped so that air has to travel farther and so faster on the top than underneath. This results in a pressure difference that causes a resultant up ward force on the wing, thus it lifts.




v) When two large vehicles pass each other, a force of attraction is experienced. This is because:
The speeding vehicles drag layers of air along with them. As these layers of air pass each other at high speed, they cause a pressure decrease.
This results in the vehicles being pushed towards each other.

**Fluid flow**

A fluid is a liquid or gaseous substance.

There are two types of fluid flow namely:

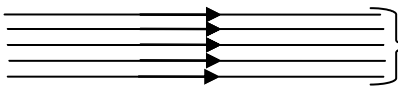
i) Stream line flow
ii) Turbulent flow

***Stream line flow or Steady flow or Laminar flow***

Is the type of fluid flow where all the fluid particles that pass a given point follow the same path at the same speed.

Stream line flow occurs where the slope falls gently so that the fluid flows slowly and uniformly.
It is obtained by making the;

- ✓ Diameter of the pipe wide
- ✓ Fluid flow slowly and uniformly.



***Turbulent flow***

Is type of fluid flow in which the speed and direction of the fluid particles passing any given point vary with time.

Turbulent flow occurs where the slope is so steep, such as at a water fall and when there is a constriction.
Due to constriction or steep slope, water tends to flow very fast and so disorderly.
It is obtained by making the;

- ✓ Diameter of the pipe narrow
- ✓ Fluid flow very fast and disorderly, by lying the pipe steeply.

**Differences between streamline and turbulent flow.**

| *Turbulent* | *Streamline* |
|---|---|
| Is due to steep slope or constriction so that water flows very fast and disorderly throughout. | Is due to slope falling gently so that the water flows slowly and uniformly throughout. |

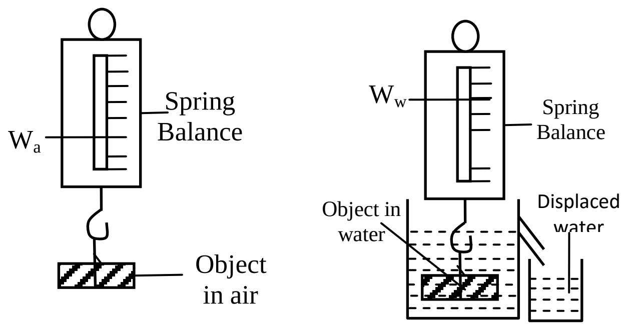
## 1: 9. ARCHIMEDES' PRINCIPLE

***Up thrust*** is an upward force due to the fluid resisting being compressed. When any object is immersed or submerged into fluid its weight appears to have been reduced because it experiences an up thrust from the fluid.

**Archimedes principle** states that when a body is either wholly or partly submerged in a fluid the up thrust is equal to the weight of fluid displaced.i.e

$$\boldsymbol{Upthrust} = \begin{pmatrix} \boldsymbol{Weight\ of} \\ \boldsymbol{displaced\ fluid} \end{pmatrix} = (\boldsymbol{V_f \rho_f})\boldsymbol{g}.$$

**Experiment: To verify Archimedes principle**




- ❖ Weight $W_a$ of object in air

An object is weighed in air using a spring balance to obtain Wa.

- ❖ Weight $W_w$ of object in water

The object is weighed when completely immersed in water using a spring balance to obtain $W_w$ and the displacement water collected in beaker as shown below.

- ❖ Weight of displaced water

By using a spring balance the beaker is weighed with the displaced water when it's empty

- ❖ Up thrust "U" = $W_a$-$W_w$

It is found that weight of displaced water is equal to up thrust. Thus Archimedes' principle.

**Calculations involving Archimedes principle**

For any calculation involving Archimedes' principle the following should be noted:
i) The body should be completely immersed or submerged.
ii) The weight of the body when completely immersed or submerged is called its apparent weight.
The apparent weight is less than the weight of the body because when the body is immersed it experiences an up thrust.

$$\begin{bmatrix} Apparent\ loss \\ in\ weight \end{bmatrix} = \begin{bmatrix} Weight\ of \\ body\ in\ air \end{bmatrix} - \begin{bmatrix} Weight\ of \\ body\ in\ fluid \\ or\ \begin{pmatrix} Apparent \\ weight \end{pmatrix} \end{bmatrix}$$

$$\boldsymbol{Upthrust, U = W_a - W_f}$$

iii) According to Archimedes Principle;

$$[Upthrust] = \begin{bmatrix} Weight\ of \\ displaced\ fluid \end{bmatrix} = \begin{bmatrix} Apparent\ loss \\ in\ weight \end{bmatrix}$$
$$= \boldsymbol{W_a - W_f}$$
$$= V_f \rho_f g$$

Where $V_f$ *and* $\rho_f$ are the volume and density of the displaced fluid.
For a body completely immersed or submerged fully, according to the displacement method,

$$\begin{bmatrix} Volume\ of \\ immersed\ boddy \end{bmatrix}; V_b = \begin{bmatrix} Volume\ of \\ displaced\ fluid \end{bmatrix}; V_f$$

Weight = mg and m =$V\rho$ where "m" is mass in kg, V is volume, and $\rho$ is density.
But up thrust = weight of displaced fluid.
= $m_f$ g (where $m_f$ is mass of displaced fluid)

***Up thrust = ($V_f \rho_f$) g***

***Up thrust = $V_f V_f g$***

Where "V" is volume of displaced fluid "$\rho$" is density of fluid.

**Example: 1**

A glass blocks weight 25N. When wholly immersed in water, the block appears to weigh 15N. Calculate the Up thrust.

**Solution**

$W_a = 25N$; $W_f = 15N$;
Upthrust $= W_a - W_f$
$= 25 - 15$
Upthrust = 10N

**Example 2:**

A metal weighs 20 N in air and 15N when fully immersed in water Calculate the:

i) Up thrust;
ii) Weight of displaced water
iii) Volume of Displaced Water

**Solution**

| $W_a = 20N$; $W_f = 15N$; | |
|---|---|
| i) Up thrust; | ii) Weight of displaced water |

| | |
|---|---|
| $\text{Upthrust} = W_a - W_f$<br>$= 20 - 15$<br>$\underline{\text{Upthrust} = 5N}$ | Weight of displaced fluid<br>= up thrust<br>= 5N |
| iii) Volume of Displaced Water<br>Weight of displaced fluid = up thrust<br>Weight of displaced fluid = up thrust = $\boldsymbol{V\_\rho_f g}$ .<br>$Upthrust = V_f \rho_f g$ .<br>$5 = V_f \times 1000 \times 10$.<br>$5 = 10000V_f$.<br>$V_f = \frac{5}{10000}$<br>$V_f = 5 \times 10^{-4} m^3$ | |
| iv) Volume of the metal<br>Volume of the metal = Volume of displaced fluid<br>$= 5 \times 10^{-4} m^3$ | |
| v) Density of the metal<br>$W_a = 20N$<br>$W_a = V_b \rho_b g$<br>$20 = (5 \times 10^{-4}) \times \rho_b \times 10$<br>$20 = 0.005\rho_b$<br>$\rho_b = \frac{20}{0.005}$<br>$\rho_b = 4000 \text{ kgm}^{-3}$ | |

**Example 3:**
An iron cube of volume 800cm$^3$ is totally immersed in (a)Water (b) oil of density 0.8gcm$^{-3}$.Calculate the up thrust in each case. Density of water =1000 kgm$^{-3}$
$V_b = 800cm^{-3}$ = 800/(100x100x100) kgm$^{-3}$;

**Solution**
(a) Up thrust in water
$\rho_f$ =1gcm$^{-3}$=1000 kgm$^{-3}$
$V_f=V_b= 800cm^{-3}$ = 800/(100x100x100) kgm$^{-3}$;
Up thrust = weight of displaced water
$Upthrust = V_f \rho_f g$

$$\text{Upthrust} = \left(\frac{800}{100 \times 100 \times 100}\right) \times 1000 \times 10$$

$\underline{\text{Upthrust} = 8\ N}$

(b) Upthrust in the oil
$\rho_f$ = 0.8 gcm$^{-3}$=0.8x1000kgm$^{-3}$=800 kgm$^{-3}$

Up thrust = weight of displaced water
$\text{Upthrust} = V_f \rho_f g$

$$\text{Upthrust} = \left(\frac{800}{100 \times 100 \times 100}\right) \times 800 \times 10$$

$\underline{\text{Upthrust} = 6.4\ N}$

**Note**: the greater the density, the greater the up thrust. The apparent weight of a body is less in fluids of greater density.

**Example 3:**
An iron cube, mass 480g and density 8g/cm$^3$ is suspended by a string so that it is half immersed in oil of density 0.9g/cm$^3$. Find the tension in string.

**Solution**
m = 480g, $\rho_b$= 8 gcm$^{-3}$
$W_a$= mg = $(\frac{480}{1000}) \times 10$ = 4.8N

$\rho_f$ =0.9gcm$^{-3}$= 0.9 x 1000 = 900 kgm$^{-3}$

$$V_b = \frac{m_b}{\rho_b} = \frac{480}{8} = 60\ m^3$$

Since its half –immersed then: $V_f$ of oil = $\frac{1}{2} \times 60$ = 30cm$^3$
Up thrust = weight of displaced fluid
$\text{Upthrust} = V_f \rho_f g$

$$\text{Upthrust} = \left(\frac{30}{100 \times 100 \times 100}\right) \times 900 \times 10$$

$\underline{\text{Upthrust} = 0.27\ N}$

Tension in string = Apparent weight ($W_f$)

$$Upupthrust = W_a - W_f$$
$$0.27 = 4.8 - W_f$$
$$W_f = 4.8 - 0.27$$
$$W_f = 4.53\ N$$

Thus Tension in string = 4.53 N

**Application of Archimedes principle**

**(a) Relative density of a solid**

By Archimedes principle, the apparent weight is equal to the weight of water displaced by the solid. The volume of this water displaced is the same as the volume of the solid.
But *apparent loss in weight of solid in water = Wa – Ww*

$$\text{RelativeDensity(R. D)}$$
$$\mathbf{R.D} = \frac{\text{Weight of solid in air}}{\text{Apparent loss in weight of solid in water}}$$
$$= \frac{\text{Weight of solid in air}}{Upthrust\ in\ water} = \frac{\mathbf{w_a}}{\mathbf{W_{a-w_w}}}$$

$W_a$- $W_w$ = Up thrust: Where; $W_a$ is weight of solid in air. $W_w$ is weight of solid in water

**Example**
A glass block weighs 25N. When wholly immersed in water the block appears to weigh 15N. Calculate the relative density.

**Solution**

| | |
|---|---|
| $W_a = 25N$; $W_f = 15N$;<br>$\text{Upthrust} = W_a - W_f$<br>$= 25 - 15$<br>$\underline{\text{Upthrust} = 10N}$ | $R.D = \frac{w_a}{W_{a-w_w}}$<br>$R.D = \frac{25}{10}$<br>$\underline{R.D = 2.5}$ |

**(b) Relative density of liquid**

This is determined by using a solid. This solid sinks in water and in the liquid for which the relative density is to be determined.
A solid of weight $W_a$ is weighed when completely immersed in the liquid to obtain $W_l$. The solid is then weighed when completely immersed in water to obtain Ww.
So $helative\ Density\ of\ the\ liquid\ (R.D)$ is given by;

$$R.D = \frac{\text{Apparent loss in weight of solid in liquid}}{\text{Apparent loss in weight of solid in water}}$$

$$= \frac{\text{Upthrust in liquid}}{\text{Upthrust in water}} = \frac{\mathbf{w_a - w_l}}{\mathbf{w_a - \boldsymbol{w_w}}}$$

**Example**

A metal weighs 25N in air. When completely immersed in liquid it weighs 15N and it weighs 20N when completely immersed in water. Calculate the relative density of the liquid.

**Solution**

$W_a = 25N; W_l = 15N; W_w = 20N$:

$$\text{Relative density of liquid} = \frac{W_a - W_l}{W_{a-}W_w} = \left(\frac{25-15}{25-20}\right) = \frac{10}{5}$$

$$= 2.0$$

## FLOATATION

When a stone is placed on water, it sinks because its weight is greater than the up thrust. When a cork is held below the surface of water, it rises on release. This is because the up thrust on the cork is greater than its weight.

A piece of wood neither rises nor sinks but floats because the up thrust on the piece of wood and its weigh just balance so it experiences no net force.

In general a body floats because up thrust is equal to weight of the body. A body will sink because up thrust on it is less than the weight of the body.

**The principle of flotation states** *that: A floating body displaces its own weight of fluid i.e. for a floating body; weight of body = weight of displaced fluid*

$\boldsymbol{W_b = W_f}$

Where $W_a$ is weight of body floating, $W_f$ is weight of displaced fluid.

$\boldsymbol{W_f = V_f \rho_f g}$

Where $V_f$ is volume of displaced fluid

$$\boldsymbol{W_b = V_b \rho_b g}$$

Where $V_b$ is volume of floating body, $\boldsymbol{\rho_b}$ is density of floating body "g" is acceleration due to gravity

In general: $\boldsymbol{V_b \rho_b g = V_f \rho_f g}$

Thus: $\boxed{\boldsymbol{m_b = m_f}}$

**Example** 1:

A piece of cork of volume $100cm^3$ is floating on water.

If the density of the cork is $0.25gcm^{-3}$. Calculate the volume of cork immersed in water.

**Solution**

(a) Upthrust in water

$\rho_f = 1gcm^{-3} = 1000\ kgm^{-3}$ : $\rho_b = 0.25gcm^{-3} = 0.25 \times 1000kgm^{-3}$

$V_b = 100cm^{-3} = 100/(100\times100\times100)\ kgm^{-3}$;

Up thrust = weight of displaced water

Upthrust $= V_f \rho_f g$

Upthrust $= \left(\frac{800}{100 \times 100 \times 100}\right) \times 1000 \times 10$

Upthrust = 8 N

Upthrust $= V_f \rho_f g$

$m_b = V_b \rho_b$

$$V_f \times 1000 = \left(\frac{100}{100 \times 100 \times 100}\right) \times (0.25 \times 1000) \times 10$$

$1000V_f = 2.5$

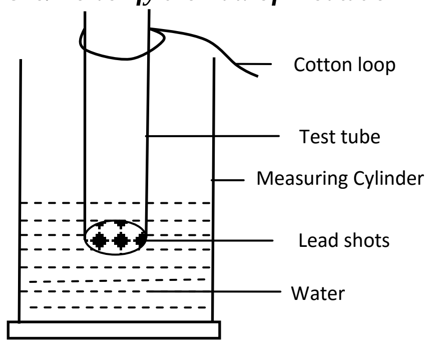
$V_f = 0.000025\ m^3 \text{ or } 2.5 \times 10^{-5}\ m^3$

**Exercise:**

1. A glass block weighs 25N in air. When wholly immersed in water, the block weighs 15N. Calculate the;
   (i) up thrust on the block [10N]
   (j) density of the glasskgm$^{-3}$ [2500kgm$^{-3}$]
2. A piece of iron weighs 555N in air. When completely immersed in water, it weighs 530N and weighs 535N when completely immersed in alcohol. Calculate the relative density of alcohol. [R.D=0.8]

3. UNEB 1991. Qn. 7
4. UNEB 1990. $P_2$ Qn. 5

**Experiment:** ***To verify the Law of Floatation***




**Procedure**

- A test tube is placed in a measuring cylinder containing water and the original reading of the water level ($V_1$) is noted.
- Lead shots are added to the test tube until it floats up right and the new water level ($V_2$) is noted.

  Volume of displaced water $= (V_2 - V_1)cm^3$

  Weight of displaced water $= \rho_w(V_2 - V_1)g$

- The test tube together with the shots is removed from the cylinder and weighed using a spring balance. (The cotton loop helps to attach it to the balance hook). Their weight is recorded, **W**.

  $(\text{Weight of lead shots + testtube}) = W_a$

**Observation**

- The weight of lead shots and test tube is equal to the weight of displaced water.

$$\left(\begin{matrix} Weight\ of\ lead \\ shots + testtube \end{matrix}\right) = \left(\begin{matrix} \text{Weight of} \\ \text{displaced water} \end{matrix}\right)$$

**Conclusion**

- From the above observation, it is noticed that the law of floatation is verified.

**Application of the law of floatation**

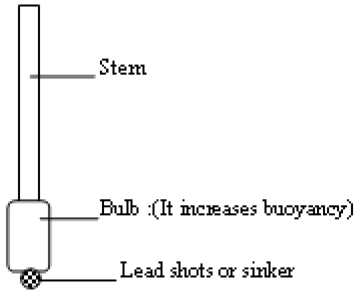
| | |
|---|---|
| (i) Ahydrometer<br>(ii) Submarines | (iii) Ships<br>(iv) Balloons |

***(i) A hydrometer***

The relative density of any liquid may be found using *a **hydrometer.***

-It is used to test the purity of milk.

-It is used to test R.D of a car battery acid.
This consists of a float with along stem. A heavy weight is placed beneath the float to keep the hygrometer up right. The higher the hydrometer float the higher the relative density of the liquid.



(Enable the hydrometer)

*(ii)* ***Submarines***
The average density of submarines is varied by means of ballast tanks. For the submarines to float, the ballast tanks are filled with air. To sink the submarines, the tanks are filled with water causing average density to rise higher than that of water.

***(iii) Ships***
***Why ships float.***
Ships float on water, although they are made from iron and steel which are denser than water. This is because a steel or iron ship is made hollow and contains air. So the average density of the ship is less than that of water.

The loading lines called plimsoul marks on the sides show the level to which it can be safely loaded under different conditions.
Weight of displaced water ($W_w$) = weight of the ship ($W_s$) + weight of the cargo ($W_c$).

$$\mathbf{W_w = W_s + W_c}$$

***(iv) Balloons***
These are airships used in meteorological measurements.
A balloon filled with hydrogen weighs less than the weight of air it displaces.
The up thrust being greater than its weight, a resultant up ward force on the balloon causes it to rise.
The balloon continues to rise up until the upthrust acting on it is equal to the weight of the balloon plus its content and then it floats.
The lifting power of the balloon is calculated from the formula:
$U = W_{balloon} + W_{hydrogen} + W_{load}$
$U = m_b g + V_h \rho_h g + m_l g$

$\text{Upthrust in air} = \text{Weight of displaced air}$
$\text{Upthrust in air} = \rho_a V_a g$

**Example: 1**
A balloon has a capacity $10m^3$ and is filled with hydrogen. The balloon's fabric and the container have a mass of 1.25kg. Calculate the maximum mass the balloon can lift.
{Density of hydrogen = 0.089kgm$^{-3}$: density of air=1.29kgm$^{-3}$}
**Solution:**

Volume of balloon, $V_b = 10m^3$
Density of hydrogen, $\rho_h = 0.089kgm^{-3}$
Density of air, $\rho_a = 1.29kgm^{-3}$
Volume of air displaced, $V_a$ = Volume of balloon, $V_b = 10m^3$
Volume of hydrogen, $V_h$ = Volume of balloon, $V_b = 10m^3$
Mass of balloon and container, = 1.25kg
Let the mass of the load = x

$\text{Upthrust} = \text{Weight of balloon} + \text{weight of } H_2 + \text{load}$
$U = m_b g + V_h \rho_h g + m_l g$
$V_a \rho_a g = m_b g + V_h \rho_h g + m_l g$
$10 \times 1.29 \times g = 1.25 \times g + 0.089 \times 10 \times g + x \times g$
$x = 10.76kg$

**Relationship between density of a floating body, density of a liquid and fraction submerged**

$$\begin{pmatrix}\text{Density of} \\ \text{floating object}\end{pmatrix} = \begin{pmatrix}\text{Fraction} \\ \text{submerged}\end{pmatrix} \times \begin{pmatrix}\text{Density of} \\ \text{liquid}\end{pmatrix}$$

**Exercise:**
1. A rubber balloon of mass 5g is inflated with hydrogen and held stationary by means of a string. If the volume of the inflated balloon is 0.005m$^3$, find the tension in the string.

(Assume hydrogen is a light gas, and density of air = 1.25kg$^{-3}$): [Ans: 1.25 X 10$^{-2}$N]

| | |
|---|---|
| 2. UNEB: 1995. Qn. 7 | 5. UNEB: 1989. Qn. 4 |
| 3. UNEB: 1988. Qn. 11 | 6. UNEB: 2001. Qn.2 |
| 4. UNEB: 2000. Qn. 40 | |

# LINEAR MOTION

**Distance**: Is the space between two points.
**Displacement:** Is the distance moved in a specified direction.
The S.I unit of distance and displacement is **metre** or **m**
Distance is a scalar quantity while displacement is a vector quantity.

**Speed:** Is the rate of change of distance. Or It is distance moved in a unit time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time taken}}$$

**Velocity**; is the rate of change of displacement. Or It is speed in a specified direction.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

The S.I unit of speed and velocity is **metre per second. (m/s)** or **($ms^{-1}$).**
Speed is a scalar quantity while Velocity is a vector quantity.

**Differences between velocity and speed**

| Velocity | speed |
|---|---|
| -Vector quantity | -scalar quantity |
| - displacement/time taken | -distance/time taken |

**Types of velocities**

- **initial velocity u'**

Is the velocity with which a body starts motion in a given time interval.

**Note;**
**1**. For a body starting from rest the initial velocity "u' must be zero i.e. u = 0 $ms^{-1}$
**2**. For a stationary body starting motion means that the body is starting from rest u = 0 $ms^{-1}$
**3.** For a body traveling with a certain velocity, *x*, the initial velocity for such a body will be x so, u = *x* $ms^{-1}$ e.g. a car traveling at 20 $ms^{-1}$, has u =20 $ms^{-1}$

- **Final velocity v'**

The velocity with which a body ends motion for a given time.
**Note**: if a body is brought to rest, then the final velocity is zero i.e., v = 0m/s.eg; A body traveling at 20m/s is uniformly brought to rest in 2s. Then; v = 0m/s.
The units of velocity must include m/s or km/hr or cm/s.

**Average velocity**:

$$\text{Average Velocity} = \frac{\text{Final velocity} + \text{Initial velocity}}{2}$$

$$\text{Average Velocity} = \frac{\text{V} + \text{u}}{2}$$

**Uniform velocity**
Is the constant rate of change of displacement.
OR
Uniform velocity is when a body makes equal displacements in equal time intervals.

When a body moves with uniform velocity, initial velocity (u) must be equal to final velocity, v. i.e. V = u.
**E.g.** A car traveling with uniform velocity of 20m/s has u=20m/s. V=20m/s.

When a body moves with uniform velocity, its acceleration is zero. (i.e. a = 0).

**Acceleration (a)**
Is the rate of change in velocity with time.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$\textbf{Acceleration}, \quad \mathbf{a} = \frac{\mathbf{V} - \mathbf{u}}{\mathbf{t}}$$

Change in velocity=final velocity (V)-initial-velocity (U)
The S.I unit for change in velocity is **$m/s^2$** or **$ms^{-2}$**.

**Uniform acceleration**
Uniform acceleration is the constant rate of change in velocity with time.
**OR:**
Uniform acceleration is when a body moves with equal change in velocity in equal time intervals.

When a body moves with uniform acceleration, the final velocity is not equal to initial velocity.

**Example.**
A car starts from rest and it accelerates to 10m/s. Calculate the change in velocity.
U=0m/s
V=20m/s
$\text{Change in velocity} = \text{v} - \text{u}$
$\text{Change in velocity} = 20 - 0$
$= 20\text{ms}^{-1}$

Note: the velocity to which a body is accelerating becomes the final velocity for that given time interval.

**Differences between velocity and acceleration**

| Velocity | Acceleration |
|---|---|
| i) S.I unit is $ms^{-1}$<br>ii) Is the rate of change of displacement | i) S.I unit is $ms^{-2}$<br>ii) Is the rate of change of velocity with time? |

**Equations Of Motion**

The units of acceleration must always be $m/s^2$ and units' m/s or km/hr are for velocity.

**1st Equation of motion**

From the definition of acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

, $$a = \frac{V - u}{t}$$

$at = v - u$

$\mathbf{v = u + at}$

This is called the first equation of motion.

**Example1**

A car started from rest it accelerates uniformly for 5s at a rate of $4m/s^2$. Calculate the final velocity.

Solution

| | |
|---|---|
| Given<br>u=0 $m/s$<br>v=0 $m/s$<br>a=$4ms^{-2}$<br>t=5s<br>v=? | From;<br>$v = u + a\sim$<br>$v = 0 + (4)(5)$<br>$v = 20$<br>$\underline{v = 20ms^{-1}}$ |

**Example 2**.

A body starting from rest is accelerated to 30m/s in two seconds. Calculate the acceleration of the body.

Solution

| | |
|---|---|
| Given<br>u=0m/s<br>v=30m/s<br>t=2s | From;<br>$v = u + at$<br>$30 = 0 + a(2)$<br>$30 = 2a$<br>$15 = a$<br>$\underline{a = 15ms^{-2}}$ |

**Example 3.**

A body starts from rest and accelerated uniformly at $2m/s^2$ for 3s. Calculate the final velocity.

Solution

| | |
|---|---|
| Given<br>u=0m/s<br>a=$2m/s^2$<br>t=3s | From;<br>$v = u + at$<br>$v = 0 + (2)(3)$<br>$v = 6$<br>$\underline{v = 6ms^{-1}}$ |

**Example 4.**

A body traveling at 10m/s is accelerated uniformly for 3 seconds at $5m/s^2$. Calculate the velocity at the end of the third second.

Solution

| | |
|---|---|
| Given<br>u=10m/s<br>a=$5m/s^2$<br>t=3s | From;<br>$v = u + at$<br>$v = 10 + (5)(3)$<br>$v = 10 + 15$<br>$\underline{v = 25ms^{-1}}$ |

**Example: 5.**

A body traveling at 20m/s is accelerated for 4s at $5m/s^2$. Calculate the average velocity.

Solution

| | | |
|---|---|---|
| Given<br>u=20m/s<br>a=$5m/s^2$<br>t=4s | From;<br>$v = u + at$<br>$v$<br>$= 20$<br>$+ (5)(4)$<br>$v = 20 + 20$<br>$\underline{v = 40ms^{-1}}$ | Then from ;<br>$\text{Average Velocity} = \frac{V + u}{2}$<br>$\text{Average Velocity} = \frac{40 + 20}{2}$<br>$\text{Average Velocity} = \frac{60}{2}$<br>$\underline{\text{Average Velocity} = 30ms^{-1}}$ |

**Example: 5.**

A car travels with a uniform velocity of 20m/s for 6s. Calculate its acceleration.

Solution

| | |
|---|---|
| Given<br>u=20 $m/s$<br>v=20 $m/s$<br>a=?<br>t=6s | From;<br>$v = u + at$<br>$20 = 20 + a(6)$<br>$20 = 20 + 6a$<br>$6a = 0$<br>$\underline{a = 0ms^{-2}}$ |
| *From the above example, it can be noted that for a body moving with uniform velocity, its acceleration is zero because the change in velocity becomes zero as initial velocity is equal to final velocity*. | |

**Example: 6.**

A car traveling at 90km/hr is uniformly brought to rest in 40 seconds. Calculate the acceleration.

Solution

| | |
|---|---|
| Given<br>u=90km/hr =$\frac{90\times1000}{1\times60\times60} = 25m/s$<br>a=?<br>t=40s<br>v=0 m/s | From;<br>$v = u + at$<br>$0 = 25 + (a)(40)$<br>$0 = 25 + 40a$<br>$-40a = 25$<br>$\underline{a = -0.625ms^{-2}}$ |
| ***Note****: If the value obtained for acceleration is negative, it implies that the body is decelerating or retarding. This occurs when there's a decrease in velocity.* | |

**2nd Equation of motion**

Displacement: "***S*** or ***x***" is length moved in specified direction

From the definition of Displacement.

$Displacement = (Average\ velocity) \times (Time)$

$s = \left(\frac{v+u}{2}\right) \times t \quad Where$: $v=u+at$

$$s = \left(\frac{u + at + u}{2}\right) \times t$$

$$s = \left(\frac{2u + at}{2}\right) \times t$$

$$s = \left(\frac{2ut + at^2}{2}\right)$$

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

This is called the second equation of motion. This equation is mainly used when the question involves distance and time.

**Example: 1**
A body starts from rest and accelerates uniformly at $2ms^{-2}$ for 3s. Calculate the total distance travelled.

Solution

| Given | From; |
|---|---|
| u=0 $m/s$<br>a=$2ms^{-2}$<br>t=3s<br>s=? | $s = ut + \frac{1}{2}at^2$<br>$s = (0)(3) + \frac{1}{2}(2)(3^2)$<br>$\underline{s = 9\ m}$ |

**Calculations involving deceleration or Retardation**
*When calculating a problem involving deceleration; it should be remembered that the value of "a" should be negative.*

**Example 2**:
A body moving at 40m/s decelerates uniformly for 20s at $3m/s^2$. Calculate distance covered.

Solution

| Given | From; |
|---|---|
| u=40 $m/s$<br>a=$-3ms^{-2}$<br>t=20s<br>s=? | $s = ut + \frac{1}{2}at^2$<br>$s = (40)(20) + \frac{1}{2}(-3)(20^2)$<br>$s = 800 - 600$<br>$\underline{s = 200\ m}$ |

**Example 3**:
A car traveling at 40m/s is uniformly decelerated to 25m/s for 5s. Calculate the total distance covered.

Solution

| Given | From; | Then, from: |
|---|---|---|
| u=40 m/s<br>v=25 m/s<br>a=?<br>t=5s<br>s=? | $v = u + at$<br>$25 = 40 + 5a$<br>$5a = 25 - 40$<br>$5a = -15$<br>$\underline{\underline{a = -3ms^{-2}}}$ | $s = ut + \frac{1}{2}at^2$<br>$s = (40)(5) + \frac{1}{2}(-3)(5^2)$<br>$s = 200 - 37.5$<br>$\underline{s = 162.5\ m}$ |

**Third Equation of motion**
From:
$Displacement = (Average\ velocity) \times (Time)$.
Making ***'t'*** the subject of the formula in the first equation of motion and substituting it in here, we get;

$$Displacement, s = \left(\frac{v+u}{2}\right) \times \left(\frac{v-u}{a}\right)$$

$$s = \frac{(v+u)(v-u)}{2a}$$

$$2as = v^2 - u^2$$

$\boldsymbol{v^2 = u^2 + 2as}$
This is called the third equation of motion
This equation is applied when time is not given and not required.

**Example 1:**
Calculate the final (maximum) velocity of a body traveling at 4m/s. When it accelerates at $2m/s^2$ and covers a distance of 5m.

Solution

| Given | From; |
|---|---|
| u=4 $m/s$<br>v=?<br>a=$2ms^{-2}$<br>s=5m | $v^2 = u^2 + 2as$<br>$v^2 = 4^2 + 2(2)(5)$<br>$v^2 = 16 + 20$<br>$v^2 = 36$<br>$\boldsymbol{v = 6ms^{-1}}$ |

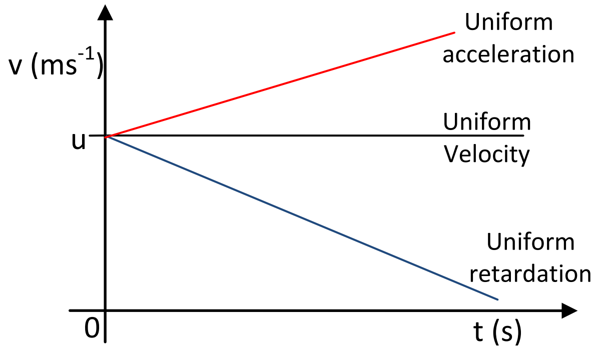
**Example 2:**
A body traveling at 90km/hr is retarded to rest at $20m/s^2$. Calculate the distance covered.

Solution

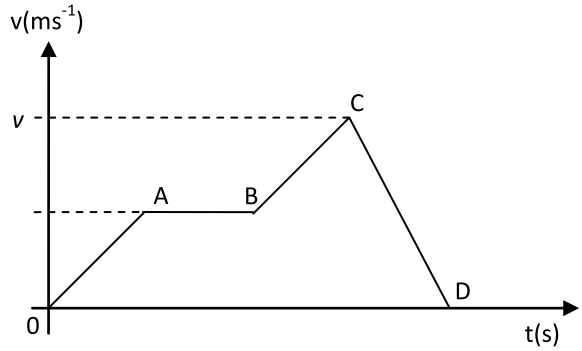
| Given | From; |
|---|---|
| u=90km/hr=$\frac{90\times1000}{1\times60\times60} = 25m/s$<br>a=$-20m/s^2$<br>v=0 m/s<br>s=? | $v^2 = u^2 + 2as$<br>$0^2 = (25)^2 + 2(-20)s$<br>$0^2 = 625 - 40s$<br>$40s = 625$<br>$\frac{40s}{40} = \frac{625}{40}$<br>$\underline{s = 15.625m}$ |

**Graphical presentation of uniform velocity and uniform acceleration.**
Uniform velocity can be represented on a 2 type of graphs.
i) Velocity against time graph
ii) Distance against time.

**i) velocity against time graph**




**Note**:
When a body maintains the same speed, it implies that it moves with uniform velocity.




OA- uniform acceleration
AB- uniform velocity
BC- uniform acceleration
CD- uniform deceleration or uniform retardation

❖ The slope of a velocity time graph gives the acceleration of the body. i.e.**:**

$$\mathbf{slope, s = \frac{Change\ in\ v}{Change\ in\ t} = acceleration}$$

❖ The Area under any stage or section of velocity time graph gives the distance covered during that time.

**Drawing a velocity against time graph**

This involves the following steps:

- ✓ Divide the motion into stages basing on the timing.
- ✓ Obtain the initial velocity (u) and final velocity (v) for each stage.
- ✓ The final velocity for one stage becomes the initial for the next stage.

**Example 1:**

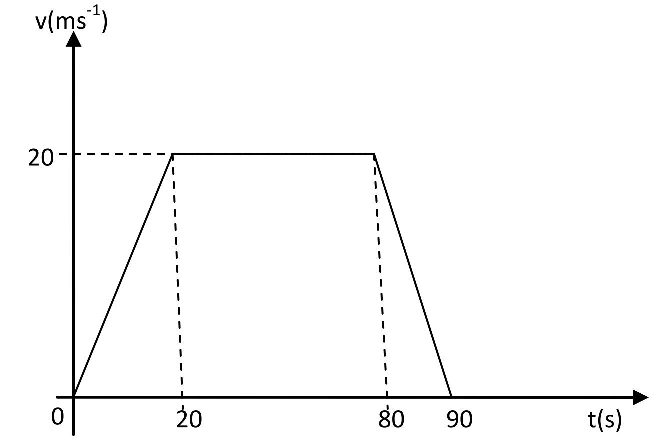
A cyclist starts from rest and accelerate uniformly at $1m/s^2$ for 20s. Then he maintains the maximum speed so reached for 1 minute and finally decelerates to rest uniformly for 10s.

**i)** Draw a velocity against time graph for the body.
**ii)** Calculate the total distance travelled.

Solution

| **Stage1** | **Stage 2** | **Stage 3** | Then from; |
|---|---|---|---|
| $u=0\ ms^{-1}$<br>$a=1ms^{-2}$<br>t=20s<br>v=? | u<br>$= 20ms^{-1}$<br>$a= 0\ ms^{-2}$<br>t=<br>1min=60s | $u = 20ms^{-1}$<br>$v= 0\ ms^{-1}$<br>t = 10s | $v = u + at$<br>$v = 0 + (1)(20)$<br>$v = 20ms^{-1}$ |

**(i) A velocity against time graph for the motion.**




**ii)** Total distance travelled

**Stage 1:**

$$\text{Distance} = \frac{1}{2}bh$$
$$\text{Distance} = \frac{1}{2} \times 20 \times 20$$
$$\text{Distance} = 200m$$

**Stage 2:**

$$\text{Distance} = lw$$
$$\text{Distance} = 60 \times 20$$
$$\text{Distance} = 1200m$$

**Stage 3:**

$\text{Distance} = \frac{1}{2}bh$

$\text{Distance} = \frac{1}{2} \times 10 \times 20$

$\text{Distance} = 100m$

**Then Total Distance is;**

=Stage1 + Stage2 + Stage3

=200m + 1200m + 100m

**= 1500m**

**Example 2:**

A car traveling at 10m/s is uniformly accelerated for 4s at $2m/s^2$. It then moves with a constant speed for 5s after which it is uniformly brought to rest in another 3s.

**i)** Draw a velocity against time graph.
**ii)** Calculate the total distance travelled.

Solution

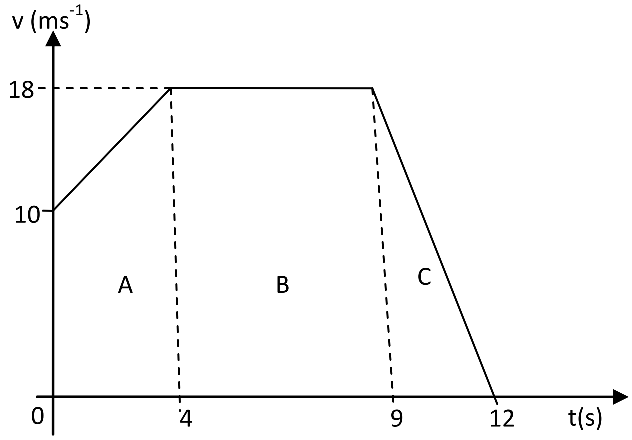
| **i) Stage A** | **Stage B** | **Stage C** |
|---|---|---|
| $u=10\ ms^{-1}$<br>$a=2\ ms^{-2}$<br>t=4s | $u = 18ms^{-1}$<br>$a = 0\ ms^{-2}$<br>t =5s<br>v=? | $u = 18ms^{-1}$<br>$v = 0\ ms^{-1}$<br>t = 3s |

Then from;

$v = u + at$

$v = 10 + (2)(4)$

$v = 18ms^{-1}$




**i)** Total distance travelled

| | |
|---|---|
| **Stage A:**<br>$\text{Distance} = \frac{1}{2}h(a + b)$<br>$\text{Distance} = \frac{1}{2} \times 4 \times (10 + 18)$<br>$\text{Distance} = 56\ m$ | **Stage B:**<br>$\text{Distance} = lw$<br>$\text{Distance} = 5 \times 18$<br>$\text{Distance} = 90\ m$ |
| **Stage C:**<br>$\text{Distance} = \frac{1}{2}bh$<br>$\text{Distance} = \frac{1}{2} \times 3 \times 18$<br>$\text{Distance} = 27\ m$ | **Then Total Distance is;**<br>=Area (A+B+C)<br>=56 m + 90 m + 27 m<br>**= 173 m** |

**Note:** Distance covered during stage A can also be obtained by dividing the area A into a triangle and a rectangle and then finding the sum of the two areas.

ie;$A_1 = \frac{1}{2}bh = \frac{1}{2} \times 4 \times 8 = 16m$

$A_2 = lw = 4 \times 10 = 40m$

Thus: Area, A = Area $A_1$+ Area $A_2$

= 16 m + 40 m

**= 56m**

**Example 3:**

A body moving with uniform velocity:

A car travels at a velocity of 20m/s for 6s. It is then uniformly brought to rest in 4s.

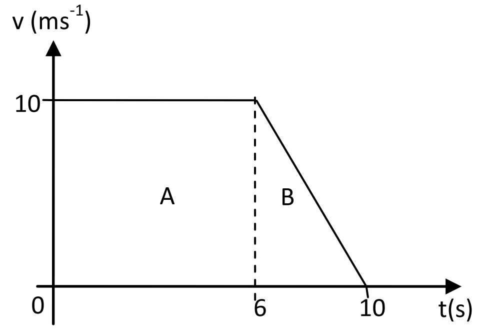
**i)** Draw a velocity against time graph.
**ii)** Calculate the retardation
**iii)** Find the total distance traveled
**iv)** Calculate the average speed of the body

Solution

| **i) Stage A** | **Stage B** | Then from; |
|---|---|---|
| $u=20\ ms^{-1}$<br>$a=0\ ms^{-2}$ | $u = 20ms^{-1}$<br>a =? | $v = u + at$<br>v =20+ (a) (4) |

t=6s t = 4s -4a=20

v=20 $ms^{-1}$ $\underline{a=-5ms^{-2}}$




**ii)** The retardation

Retardation or deceleration occurs in region B.

| | |
|---|---|
| u=20 $ms^{-1}$<br>v=0 $ms^{-1}$<br>t = (10 - 6)<br>t=4s | Then from;<br>$v = u + a$<br>$0 = 20 + (a)(4)$<br>$-4a = 20$<br>$a = -5ms^{-2}$<br>Thus the retardation is $\mathbf{5ms^{-2}}$ |

**iii)** Total distance travelled

| | |
|---|---|
| **Stage A:**<br>$Distance = l \times w$<br>$Distance = 6 \times 10$<br>$Distance = 60\ m$ | **Stage B:**<br>$Distance = \frac{1}{2}bhDistance$<br>$= \frac{1}{2} \times 4 \times 10Distance$<br>$= 20\ m$ |
| **Then Total Distance is;**<br>=Area (A+B)<br>=60 m + 20 m<br>**= 80 m** | |

**iv)**

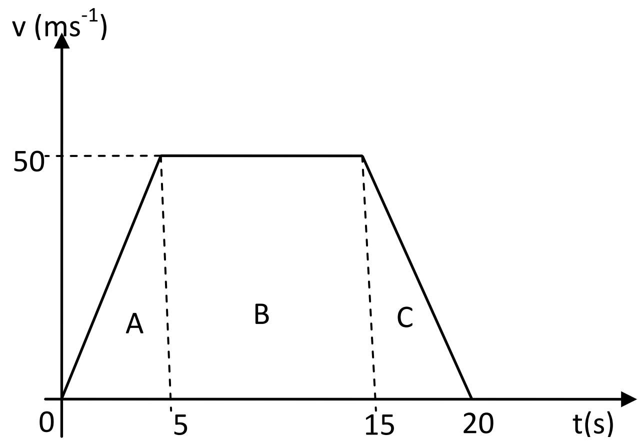
$$\text{Average speed} = \frac{\text{Total Distance travelled}}{\text{Total Time Taken}}$$

$$\text{Average speed} = \frac{80\text{ m}}{10\text{ s}}$$

$$\text{Average speed} = \mathbf{8ms^{-1}}$$

**EXERCISE**

**Qn1.** The graph below shows motion of abode of mass 2kg accelerating from rest.




(a) Describe the motion of the body

(b) Use the graph to calculate the:

(i) total distance covered

(ii) Distance covered when moving with uniform velocity.

(iii) acceleration

(iv) retardation

(v) average retarding force

Solution

| **For A** | **For B** | **For C** |
|---|---|---|
| u=0$ms^{-1}$;v=50$ms^{-1}$<br><br>Then from;<br>$v = u + at$<br>50<br>$= 0 + (a)(5)5a$<br>$= 50a = 10\ ms^{-2}$ | u=50$ms^{-1}$;<br>v=50$ms^{-1}$<br>Then from;<br>$v = u + at$<br>50<br>$= 50 + (a)(5)5a$<br>$= 50\ \ a$<br>$= 00\ ms^{-2}$ | u=50$ms^{1}$;v=0$ms^{-1}$<br><br>Then from;<br>$v = u + at$<br>0<br>$= 50 + (a)(5)$<br>$-5a = 50\ \ a$<br>$= -10\ ms^{-2}$ |

**Description of the motion**

- The body *accelerates uniformly at* 10 $ms^{-2}$ from *rest* to 50 $ms^{-1}$ for the *first* 5s.
- It then moves with a *uniform velocity* of 50 $ms^{-1}$ for the *next* 10s. (Or it maintains it for next 10s).
- It finally *decelerates or retards uniformly* at 10 $ms^{-2}$ from 50 $ms^{-1}$ to *rest* in the last 5s.

**a)** i) Total distance= Area A +Area B + Area C

Area A = $\frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 50$

= 125 m

Area B = *lw*

=10 × 50

= 500 m

Area A = $\frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 50$

= 125 m

Total distance= Area A +Area B + Area C

= 125m + 500m +125m

= 750 m

ii) Distance covered when moving with uniform

Area B = *lw*

=10 × 50

= 500 m

iii) Acceleration

**For A**

u= 0 $ms^{-1}$

v=50 $ms^{-1}$

t = 5s

$$v = u + at$$
$$50 = 0 + (a)(5)$$
$$5a = 50$$
$$a = 10\text{ ms}^{-2}$$

iv) Retardation or Deceleration

**For C**

u= 50 $ms^{-1}$

v=0 $ms^{-1}$

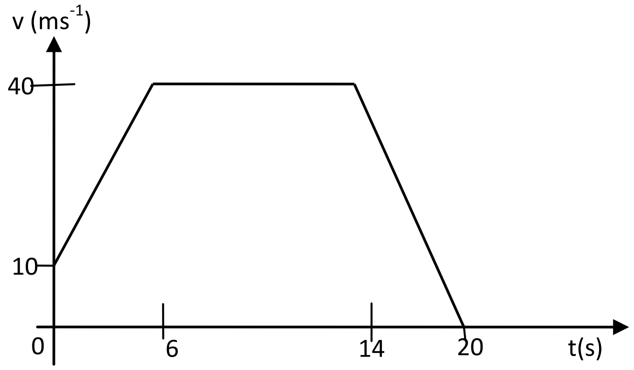
t = 5s

$$v = u + at$$
$$0 = 50 + (a)(5)$$
$$-5a = 50$$
$$a = -10\ ms^{-2}$$

Thus the retardation is $\mathbf{10\ ms^{-2}}$

i) Average retarding force
Mass, m = 2kg
Retardation = 10 $ms^{-2}$
Then from;
Force = mass × acceleration
Average retarding force = mass × retardation
= 2 × 10
= 20 N

**Qn2.** The graph below shows motion of a body of mass 3kg.Use it to answer the questions that follow.

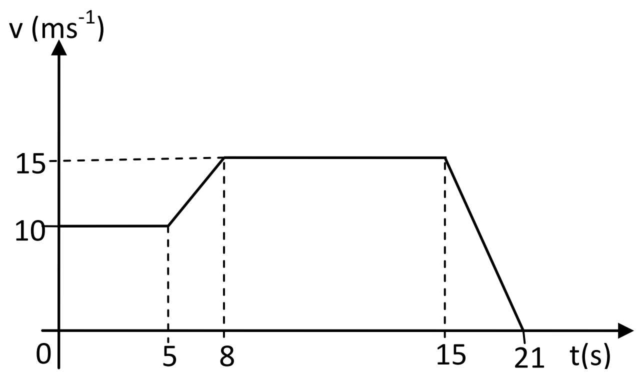



a) Describe the motion of the body
b) Use the graph to calculate the:
(i) Distance covered during acceleration. (150 m)
(ii) Distance covered when moving at constant velocity. (320 m)
(iii) total distance covered.(590 m)
(iv) acceleration. ($a = 5\ ms^{-2}$)
(v) retardation. ($a = -6.67\ ms^{-2}$)
(vi) average accelerating force. ($F = 15\ N$)

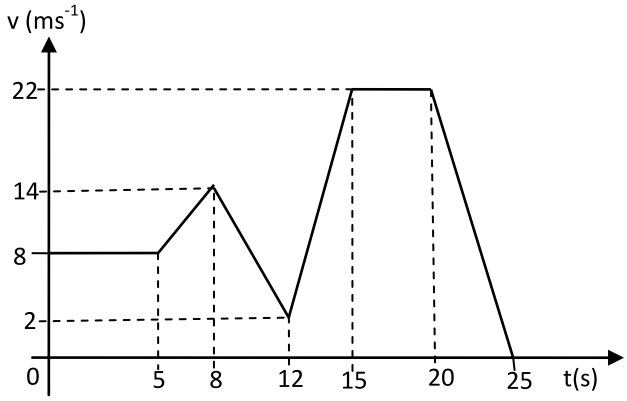
**Qn3**. The graphs below show motion of bodies. Use them to answer the following questions.

a) Describe the motion of the body
b) Use the graph to calculate the:
i) Total distance covered.
ii) Average velocity
iii) Acceleration.
iv) Retardation.

**(A)**




**(B)**

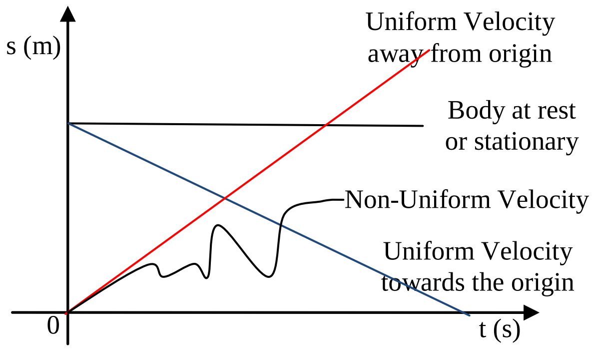


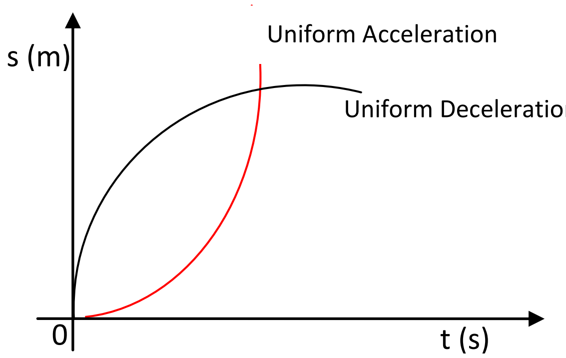

**Qn4**. A body accelerates uniformly from rest at 3ms$^{-2}$ for 4 seconds. Its velocity then remains constant at the maximum value reached for 7 seconds before retarding uniformly to rest in the last 5 seconds. Calculate the:
i) uniform velocity ($v = 12\ ms^{-1}$)
ii) total distance travelled ($= 138\ m$)
iii) retardation ($a = -2.4\ ms^{-2}$)
iv) average velocity for the journey.($v = 8.63\ ms^{-1}$)

**Qn5.** A body moves from rest at a uniform acceleration of 2ms$^{-2}$.
a) Sketch a velocity time graph for the motion of the body.
b) Find:
i) its velocity after 5 seconds. ($v = 10ms^{-1}$)
ii) how far it has gone in this time. ($s = 25\ m$)
iii) how long it will take the body to be 100 m from the starting point. ($t = 10\ s$)

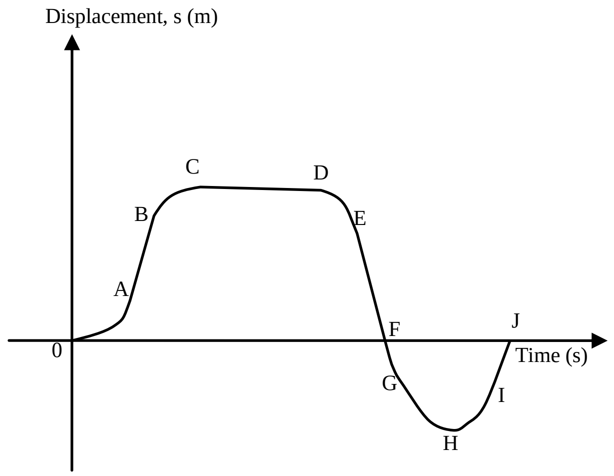
*Non-uniform acceleration is when the rate of change of velocity with time is not constant.*

**ii) Displacement against time graph**



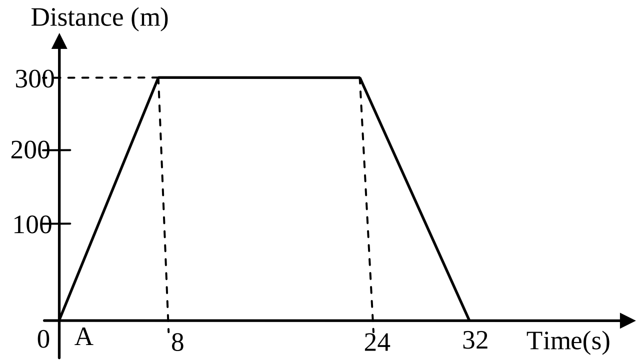



**Describing the motion on a displacement time graph**




**Along:**
OA- accelerating
AB- moving with uniform velocity away from the origin
BC- Decelerating
CD- Stationary
DE- Accelerating and moving toward the origin
EF- Moving with uniform velocity
FG- Moving with uniform velocity in opposite direction to the original direction.
GH- Decelerating
At "H"- Momentarily stationary
HI- Accelerating and moving back towards the origin.

**Example 1:**
The graph below shows the variation of distance with time for a body.




a) Describe the motion of the body
b) Calculate the:
   i) acceleration of the body
   ii) maximum velocity attained by the body

Solution

**For A**

s = 300 m32$a$ = 0
t = 8s
Speed, u= $\frac{distance}{time}$
u= $\frac{300}{8}$
u =37.5ms$^{-1}$

$300 = 37.5(8) + \frac{1}{2}(a)(8^2)$

$a = 0\ ms^{-2}$

Then from;

$$s = ut + \frac{1}{2}at^2$$

**Description of the motion**

- The body starts from A and moves 300m with a uniform velocity of 37.5 ms$^{-1}$ for the first 8seconds.
- It then a rests for the next 16 seconds.
- It finally returns to A with the uniform velocity of 37.5 in the last 8 seconds.

**Exercise:**
**Qn: 1; [UNEB 1997 Paper II Qn.2]**
Two vehicles A and B accelerate uniformly from rest. Vehicle A attains a maximum velocity of $30ms^{-1}$ in 10s while B attains a maximum velocity of $40ms^{-1}$ in the same time. Both vehicles maintain these velocities for 6s before they are decelerated to rest in 6s and 4s respectively.

(i) Sketch on the same axes, velocity time graphs for the motion of the vehicles.
(ii) Calculate the velocity of each vehicle 18s after the start. ($v_A = 20ms^{-1}$ and $v_B = 20ms^{-1}$)
(iii) How far will the two vehicles be from one another during the moment in (ii) above? ($S_A = 380m$ and $S_B = 500m$: $S_{AB} = 120m$)

**Qn. 2 See : UNEB:**

| | |
|---|---|
| 1993.Qn.25 and Qn.5 PII<br>1996.Qn.1 Paper II<br>2000.Qn.1 Paper I | 1994. Qn.10 and Qn.26<br>1987. Qn.25 |

**MOTION UNDER GRAVITY (FALLING BODIES)**
In a vacuum, all bodies fall at the same rate. However, in atmosphere different bodies fall at different rate because the air resistance is greater to light objects.

**Acceleration due to gravity, g.**
*Acceleration due to gravity is the change in velocity with time for body falling freely under the force of gravity.*
**Note:** Acceleration due to gravity varies from place to place because:

- ❖ The earth is not a perfect sphere
- ❖ The earth is always rotating

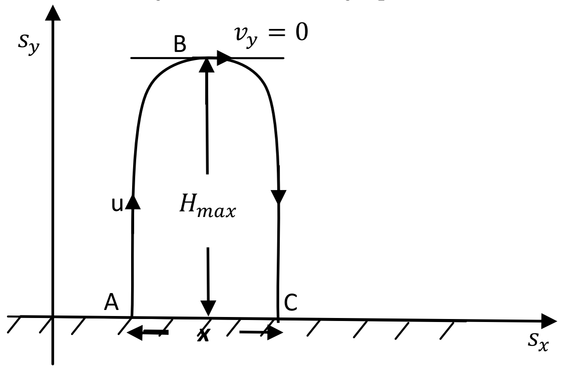
***All bodies thrown upwards or falling freely in the earth's surface, have a constant acceleration called Acceleration due to gravity***, i.e: $\boldsymbol{a = g = 10\ ms^{-2}}$.

***Since the gravitational force acts vertically down wards, i.e. accelerates all objects down wards towards the earth's surface. Thus for downward motion (falling objects),*** $\boldsymbol{a = +g = +10\ ms^{-2}}$. ***And for upward motion (objects thrown upwards),*** $\boldsymbol{a = -g = -10\ ms^{-2}}$.

**Projectile motion**

**A projectile** is a particle which has both vertical and horizontal motions when thrown in air.

Consider a body thrown vertically upwards from A.




In projectiles, the horizontal and vertical motions are handled separately but simultaneously. The horizontal velocity of the body in motion remains the same throughout since there is no acceleration due to gravity in the horizontal.

First equation of motion

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} u \\ u \end{pmatrix} + \begin{pmatrix} 0 \\ g \end{pmatrix} t \qquad \Rightarrow \begin{cases} v_x = u \\ v_y = u + gt \end{cases}$$

Second equation of motion

$$\begin{pmatrix} s_x \\ s_y \end{pmatrix} = \begin{pmatrix} u \\ u \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ g \end{pmatrix} t^2 \qquad \Rightarrow \begin{cases} s_x = ut \\ s_y = ut + \frac{1}{2}gt^2 \end{cases}$$

Third equation of motion

$$\begin{pmatrix} v_x^2 \\ v_y^2 \end{pmatrix} = \begin{pmatrix} u^2 \\ u^2 \end{pmatrix} + 2\begin{pmatrix} 0 \\ g \end{pmatrix}\begin{pmatrix} s_x \\ s_y \end{pmatrix} \quad \Rightarrow \begin{cases} v_x^2 = u^2 \\ v_y^2 = u^2 + 2gs_y \end{cases}$$

*Where* $\boldsymbol{g = +10\ ms^{-2}}$*.for downward motion (freely falling objects or objects dropped from a height), If a body is dropped from a height, then* $\boldsymbol{u = 0\ ms^{-1}}$ *hence.*

$$\boldsymbol{S = \frac{1}{2}gt^2 \ \ and \ \ v = gt \ \ or \ \ v = \sqrt{2gs}}\ .$$

.

*Alternatively the principle of conservation of energy may be used for a freely falling body.*

*i.e.:* $\frac{1}{2}cv^2 = mgh \Rightarrow \boldsymbol{v = \sqrt{(2gh)}}$**.**

$\boldsymbol{g = -10\ ms^{-2}}$*.for upward motion (objects thrown upwards),*

**Maximum Height,** $\boldsymbol{s_y = \mathrm{H_{max}}}$**:** Is the highest vertical distance attained by a projectile.

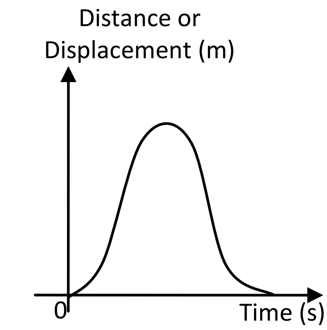
At $\mathrm{H_{max}}$ ,$v_y = 0$:

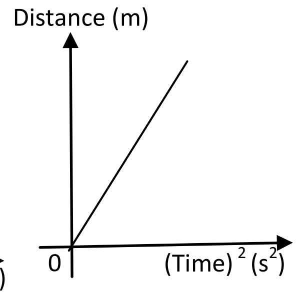
**Time of Flight, T**: Is the total time taken for a projectile to move from origin until it lands. This time is twice the time taken to reach the maximum height.

If **t** is the time taken to reach the maximum height, then

**T=2t**

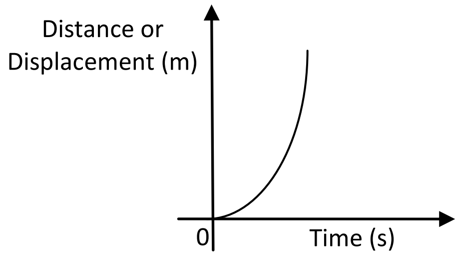
**Distance - Time or Displacement –Time Graphs for a body thrown vertically upward.**



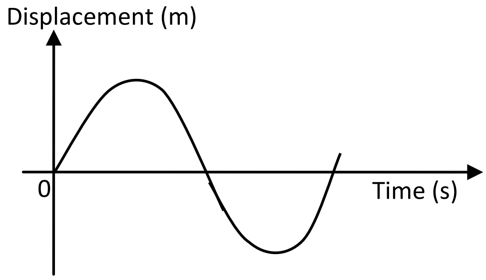

$$\mathbf{S = ut - \frac{1}{2}gt^2}$$

Distance- Time or Displacement –Time Graph for a body falling freely from rest.

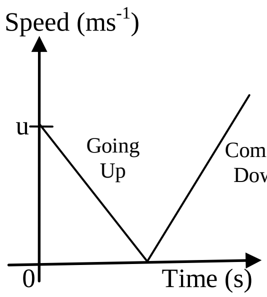


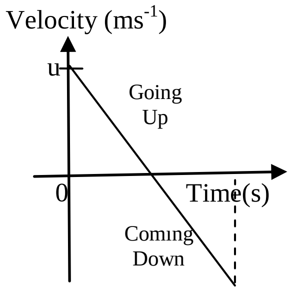

$$\mathbf{S = ut + \frac{1}{2}gt^2}$$

**Displacement –Time Graph for a body thrown vertically upwards from appoint above the ground.**




**Speed- Time and Velocity –Time Graphs For a body thrown vertically upwards.**

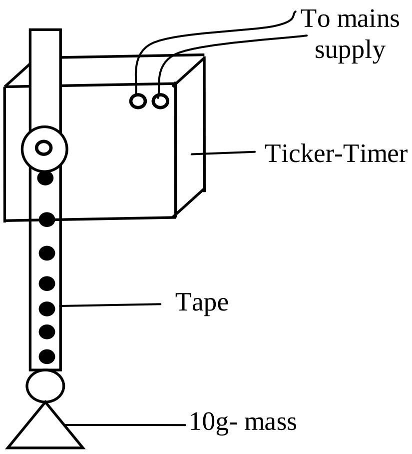



The speed of the object decreases as it goes higher. At maximum height reached the speed is zero because the object is momentarily at rest and when the object starts to fall the speed increases.

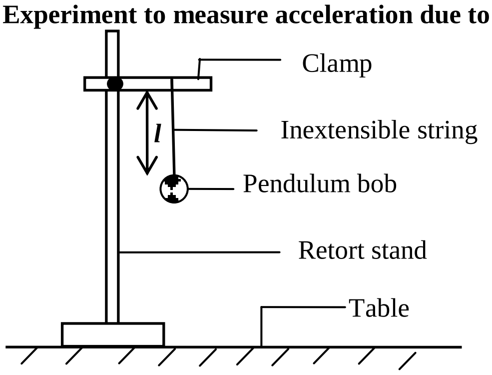
The velocity decreases upwards and it is zero at the maximum height. However the velocity increases downwards and negative because of the change in direction.

**Experiment to measure acceleration due to gravity**.




- A tape is passed through a ticker-timer and attached to a 10g- mass.
- The ticker-timer makes dots on the tape at an interval determined by the frequency of the mains supply. i.e. $T = \frac{1}{f}$. This is the time taken to make **one space** (2 dots).
- The distance **S** between the first dot to the last dot made just before the mass hits the ground is measured using a metre-rule.
- The time, **t** taken to make n- spaces in distance S is calculated from: $t = nT$.
- The acceleration due to gravity, g is then calculated from $\mathbf{S = \frac{1}{2}gt^2}$

**Experiment to measure acceleration due to gravity**

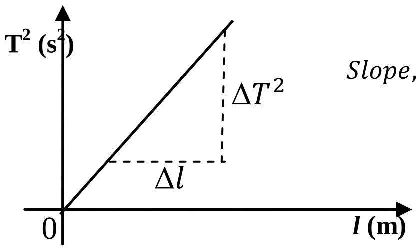



- A pendulum bob is suspended from a clamp using an inextensible string as shown in the diagram above.
- The length of the string of the pendulum bob '$l$' is adjusted such that *l=0.3m*.
- The bob is the slightly displaced through a small angle and released.
- The stop clock is started and the time taken to make 20 oscillations (20T) is measured and recorded.
- The period time T for a single oscillation is calculated and recorded.
- The experiment is repeated for other increasing values of *l*, and the corresponding values of 20T, T and $T^2$ calculated and tabulated.

| $l(m)$ | 20T(s) | T(s) | $T^2(s^2)$ |
|---|---|---|---|
| | | | |

A graph of $T^2$ against $l$ is plotted. It is a straight line graph through the origin and its slope, S is calculated.

A gragh of $T^2$ Against $\boldsymbol{l}$




$$Slope, S = \frac{\Delta T^2}{\Delta l}$$

The acceleration due to gravity,g. is then calculated from;

$$\boldsymbol{g = \frac{4\pi^2}{S}}$$

**NOTE:** *Experiments have shown that the periodic time T does not depend on the mass of the bob, but it depends on the length of pendulum l bob and acceleration due to gravity g at that point. i.e.:*

$$\boldsymbol{T = 2\pi\sqrt{\frac{l}{g}}}$$

**Example1:**
A stone falls from rest from the top of a high tower. Calculate the velocity after 2s.

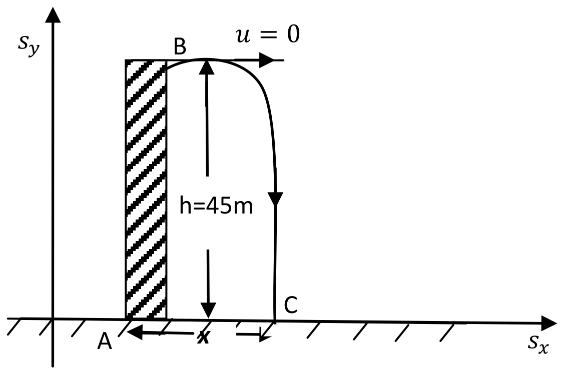
**Solution**

| | |
|---|---|
| u= 0 $ms^{-1}$<br>a = g =10 $ms^{-2}$<br>t = 2s | Then from:<br>$v = u + gt$<br>$v = 0 + (10 \times 2)$<br>$v = 20$<br>$\underline{\underline{v = 20\ ms^{-1}}}$ |

**Example 2:**
An object is dropped from a helicopter. If the object hits the ground after 2s, calculate the height from which the object was dropped.
**Solution**

| | |
|---|---|
| u= 0 $ms^{-1}$<br>a = g =10 $ms^{-2}$<br>t = 2s<br>Then from:<br>$s = \ ut + \frac{1}{2}gt^2$<br>$s = \ 0(2) + \frac{1}{2}(10)(2^2)$<br>$s = 20$<br>$\underline{\underline{s = 20\ m}}$ | **Alternatively**<br>From conservation of energy<br>$P.E_{Bottom} = K.E_{Top}$<br>$mgh = \frac{1}{2}mv^2$ |

**Example 3:**
An object is dropped from a helicopter at a height of 45m above the ground.
a) If the helicopter is at rest, how long does the object take to reach the ground and what is its velocity on arrival?
**Solution**
u= 0 $ms^{-1}$
a = g =10 $ms^{-2}$
t = ?
s = 45m




| | |
|---|---|
| $s = ut + \frac{1}{2}gt^2$<br>$45 = 0(t) + \frac{1}{2}(10)(t^2)$<br>$45 = 5t^2$<br>$t = 3\ s$ | Then from:<br>$v = u + gt$<br>$v = 0 + (10 \times 3)$<br>$v = 30$<br>$v = 30\ ms^{-1}$ |

b) If the helicopter had a velocity of $1ms^{-1}$ when the object was released, what would be the final velocity of the object?

**Solution**

| | |
|---|---|
| $u = 1\ ms^{-1}$<br>$a = g = 10\ ms^{-2}$<br>$t = ?$<br>$s = 45m$<br>$v = ?$ | $v^2 = u^2 + 2gs$<br>$v^2 = 1^2 + 2(10)(45)$<br>$v^2 = 901$<br>$v = \sqrt{901}$<br>$v = 30.02\ ms^{-1}$ |

**Example 4:**

An object is released from an aircraft traveling horizontally with a constant velocity of $200ms^{-1}$ at a height of 500m.

a) Ignoring air resistance, how long it takes the object to reach the ground?

**Solution**

| | |
|---|---|
| For vertical motion<br>$u = 0\ ms^{-1}$<br>$a = g = 10\ ms^{-2}$<br>$t = ?$<br>$s = 500\ m$<br>$v = ?$ | Then from:<br>$s = ut + \frac{1}{2}gt^2$<br>$500 = (0)t + \frac{1}{2}(10)(t^2)$<br>$500 = 5t^2$<br>$100 = t^2$<br>$t = 10\ s$ |

b) Find the horizontal distance covered by the object between leaving the aircraft and reaching the ground.

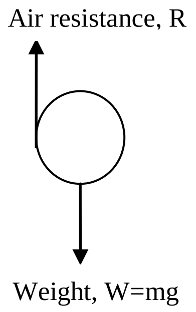
**Solution**

| | |
|---|---|
| For vertical motion<br>$u = 200\ ms^{-1}$<br>$a = g = 0\ ms^{-2}$<br>$t = 10s$<br>$s = 500\ m$<br>$v = ?$ | Then from:<br>$x = ut + \frac{1}{2}gt^2$<br>$x = 200(10) + \frac{1}{2}(0)(10^2)$<br>$x = 2000 + 0$<br>$x = 2000\ m$ |

**Note**: For a body thrown vertically up ward, the time taken to reach the maximum vertical height is equal to the time taken for the body to fall from maximum height.

**Note;** If a body is not falling freely but there is air resistance R then the acceleration of the body can be calculated from:-

**ma = mg – R ,** where m is the mass of the body




Resultant force, F

F= mg – R ……………. (i)

Then from:

F= ma …………………. (ii)

From equations (i) and (ii)

**ma = mg – R**

**Example 4:**

An object of 2kg is dropped from a helicopter at a height 45m above the ground. If the air resistance is 0.8N, calculate the:

i) acceleration of the body

ii) velocity with which the body hits the ground

**Solution**

For vertical motion

m = 2kg

$u = 0\ ms^{-1}$

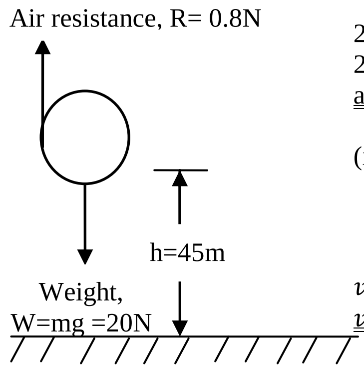
$g = 10\ ms^{-2}$

R = 0.8 N

s = 45m

a = ?

v = ?




(i) Then from**:**

**ma = mg – R**

2a = 2 (10) -0.8

2a = 19.2

$a = 9.6\ ms^{-2}$

(ii) Then from**:**

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(9.6)(45)$

$v^2 = 864$

$v = \sqrt{864}$

$v = 29.39\ ms^{-1}$

**Exercise**

**Qn: 1.** The table below shows the variation of velocity with time for a body thrown vertically upwards from the surface of a planet.

| | | | | | | |
|---|---|---|---|---|---|---|
| Velocity($ms^{-1}$) | 8 | 6 | 4 | 2 | 0 | -2 |
| Time(s) | 0 | 1 | 2 | 3 | 4 | 5 |

(a) What does the negative velocity mean?

(b) Plot a graph of velocity against time.

(c) Use the graph in (b) above to find the

i) Acceleration due to gravity on the planet.($= 2ms^{-2}$)

ii) Total distance travelled. .($= 17$m)

(d) If the body weighs34n on earth, what is its weight on the planet? ($= 6.8$N)

2. An aeroplane travelling at $200ms^{-1}$ at a height of 180m is about to drop an aid package of medical supplies onto an IDP camp in northern Uganda.

(a) At what horizontal distance before the target should the package be released? [x = 1200m]

(b) Find the time taken by the package to hit the target.[t=6s]

See UNEB

| | |
|---|---|
| 2000 Qn.20<br>1995 Qn.10<br>1987 Qn.12<br>1989 Qn.1 | 1992 Qn.23<br>1996 Qn.24<br>1991 Qn.2 |

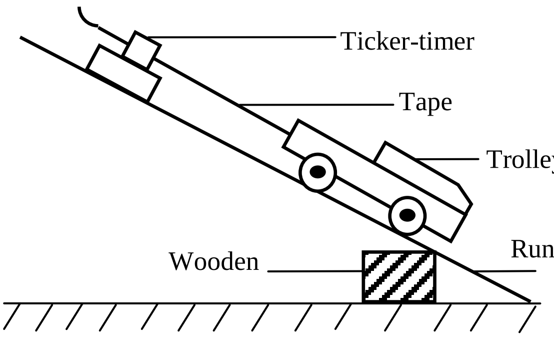
**THE TICKER- TAPE TIMER**

**DETERMINING THE VELOCITY AND ACCELERATION OF A BODY USING A TICKER TAPE TIMER:**

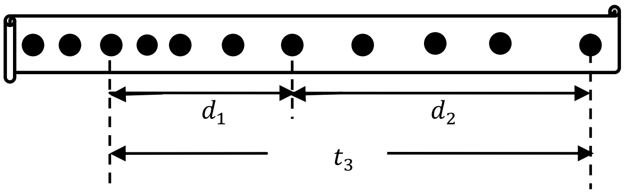
**COMPENSATION FOR FRICTION**

Before each experiment with a trolley, it is necessary to compensate for friction.

This can be done by tilting the runway with suitable packing pieces until it moves with uniform velocity after having been given a slight push.




- A paper tape is driven through a ticker timer connected to a mains supply of known frequency e.g. 50Hz by a trolley running freely on an inclined plane.
- After the trolley has reached the end of the run way, the tape is removed and marked every after 5dots.The first mark made is the zero time.




- The time 't' between n- spaces, is calculated from: $t = number\ of\ spaces \times Periodic\ time$

$$t = nT, \qquad \text{where. } T = \frac{1}{f}$$

- The speed or velocity at different times is the calculated by measuring the distances $d_1$ and $d_2$ covered in those times. Thus: $v_1 = \frac{d_1}{t_1}$ and $v_2 = \frac{d_2}{t_2}$
- The acceleration of the trolley is then calculated from: $acceleration, \boldsymbol{a} = \frac{v_2 - v_1}{t_3 - t_1}$: Whre $\boldsymbol{t_3} = n_3T$ is the total time taken to cover distances $d_1$ and $d_2$.

**OR**

- ❖ The procedures are repeated, various velocities determined and a graph of velocity against time plotted. The slope of the graph gives the acceleration of the body.

<u>Using the ticker tape timer to determine Acceleration After it has printed dots on A tape.</u>

**Frequency:** These are vibrations per second or number of dots per second. The S I unit is Hertz. (Hz).

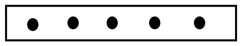
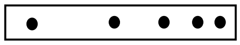
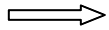
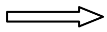
Example: A frequency of 60Hz mean 60 dots per second.

**NB:** Frequency is also number of dots printed per second.

**Period:** This is the time taken for a dot to be printed on a tape. The SI unit of period is seconds.

Period, T= $\frac{1}{\text{frequency,f}} \Leftrightarrow \mathbf{T} = \frac{\mathbf{1}}{\mathbf{f}}$

**Ticker tapes showing dots for bodies in motion**

| State of motion | Sample tape | Direction of motion |
|---|---|---|
| Uniform velocity |  |  |
| Uniform acceleration |  |  |
| Uniform deceleration |  |  |

**Example:**

Calculate the period for a frequency of 60 Hz

Frequency, f = 60Hz

Period time, T=$\frac{1}{f}$

T=$\frac{1}{60}$

T= 0.0167

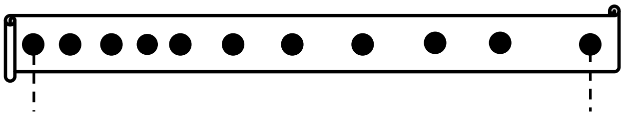
**<u>Calculating time taken from a tape</u>**

**Time taken,t=number of spaces(n)×Periodic time(T)**

Time taken,t=nT

**Example: 1**

Below is a tape printed by ticker- tape timer vibrating at 100Hz. Find the time taken to print these dots.



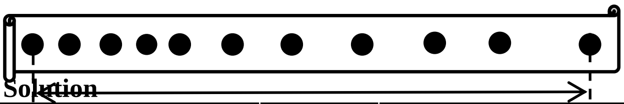
| | |
|---|---|
| Frequency,<br>f = 100Hz<br>Period time,T=$\frac{1}{f}$<br>T=$\frac{1}{100}$<br>T= 0.01 | $\begin{pmatrix}\text{Time}\\\text{taken}\end{pmatrix} = \begin{pmatrix}\text{number of}\\\text{spaces}\end{pmatrix} \times \begin{pmatrix}\text{Periodic}\\\text{time}\end{pmatrix}$<br>Time taken,t=nT<br>Time taken,t=10(0.01)<br><u>Time taken,t=0.1s</u> |

**<u>Calculating the average speed</u>**

Average speed=$\frac{\textbf{Distance, (d)}}{\textbf{Time taken, (t)}} = \frac{d}{t}$

**Example: 2**

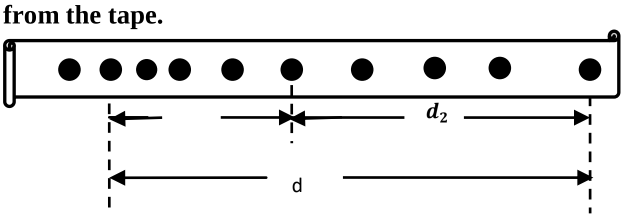
Below is a tape printed by a ticker –tape timer vibrating at 50Hz. Calculate the average speed.




**Solution**

| | |
|---|---|
| Frequency,<br>f = 50Hz<br>Period time, $T=\frac{1}{f}$<br>$T=\frac{1}{50}$<br><u>T= 0.02</u> | $\begin{pmatrix}\text{Time}\\ \text{taken}\end{pmatrix}=\begin{pmatrix}\text{number of}\\ \text{spaces}\end{pmatrix}\times\begin{pmatrix}\text{Periodic}\\ \text{time}\end{pmatrix}$<br>Time taken,t=nT<br>Time taken,t=10(0.02)<br><u>Time taken,t=0.2s</u><br>Distance=200cm=$\frac{200}{100}=2\text{m}$<br>Average speed=$\frac{\text{Distance, (d)}}{\text{Time taken, (t)}}$<br>Average speed=$\frac{d}{t}$<br>Average speed=$\frac{2}{0.2}$<br><u>Average speed=10$ms^{-1}$</u> |

**Calculating the initial velocity "u" and final velocity "v" from the tape.**



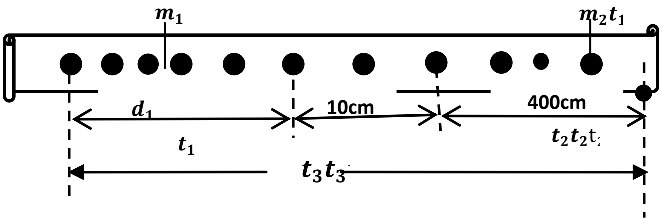

Initial velocity,u = Average speed for initial distance =initial distance "$d_1$" divided by time taken "$t_1$"

Initial speed,u=$\frac{d_1}{t_1}$; where, Time taken,$t_1=n_1T$

Final velocity (v) = Average speed for final distance = final distance "$d_1$" divided by time taken "$t_2$"
i.e.

Final speed,v=$\frac{d_2}{t_2}$; where, Time taken,$t_2=n_2T$

**Example:3**
Below is a tape printed by a timer vibrating at 50Hz



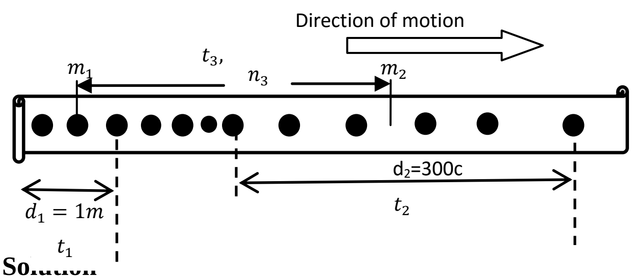

Calculate the;
i) Initial velocity
ii) Final velocity
iii) Acceleration

For the above, the following steps should be involved.
- ✓ identifying the frequency
- ✓ finding the periodic time from T= 1/f
- ✓ finding the time taken to cover given distances
- ✓ calculating the required velocities
- ✓ finding the time taken to cover distance between mid points of the distances
- ✓ calculating the required acceleration

**Solution**

| | |
|---|---|
| Frequency, f = 50Hz<br>Period time, $T=\frac{1}{f}$<br>$T=\frac{1}{50}$<br><u>T= 0.02s</u><br>When,$d_1$=200cm,<br>$=\frac{200}{100}$<br>$=2\text{m}$ | (i)<br>$\begin{pmatrix}\text{Time}\\ \text{taken}\end{pmatrix}=\begin{pmatrix}\text{number of}\\ \text{spaces}\end{pmatrix}\times\begin{pmatrix}\text{Periodic}\\ \text{time}\end{pmatrix}$<br>Time taken,$t_1=n_1T$<br>Time taken,$t_1$=5(0.02)<br><u>Time taken,$t_1$=0.1s</u><br>Initial speed, u=$\frac{d_1}{t_1}=\frac{200}{0.1}=2000\text{cms}^{-1}$<br>Or<br>Initial speed, u=$\frac{d_1}{t_1}=\frac{\left(\frac{200}{100}\right)}{0.1}=20\text{ms}^{-1}$ |
| (ii)<br>When,$d_2$=400cm,$=\frac{400}{100}=4m$<br>Time taken,$t_2=n_2$T<br>Time taken,$t_2$=3(0.02)<br><u>Time taken,$t_2$=0.06s</u><br>Final speed, v=$\frac{d_2}{t_2}=\frac{400}{0.06}=6666.67\text{cms}^{-1}$<br>Or<br>Final speed, v=$\frac{d_2}{t_2}=\frac{\left(\frac{400}{100}\right)}{0.06}=66.67\text{ms}^{-1}$ | |
| (iii)<br>$\begin{pmatrix}\text{Time taken}\\ \text{for change}\end{pmatrix}=\begin{pmatrix}\text{number of}\\ \text{spaces}\\ \text{between mid}\\ \text{points of}\\ d_1 \text{ and } d_2\end{pmatrix}\times\begin{pmatrix}\text{Periodic}\\ \text{time}\end{pmatrix}$<br>Time taken,$t_3=n_3$T<br>Time taken,$t_3$=6.5(0.02)<br><u>Time taken,$t_3$=0.13s</u> | |
| **Acceleration;**<br>Acceleration calculated applying v = u + at<br>**Acceleration, a = $\frac{\textbf{change in velocity}}{\textbf{Time for the change}}$**<br>Acceleration, a = $\frac{\boldsymbol{v-u}}{\boldsymbol{t_3}}$<br>Acceleration, a = $\frac{\mathbf{66.67-20}}{\mathbf{0.13}}$<br><u>Acceleration, a = 359$ms^{-2}$</u> | |

**Example II:**
Below is a tape printed by a ticker timer vibrating at 20Hz. Calculate the acceleration.

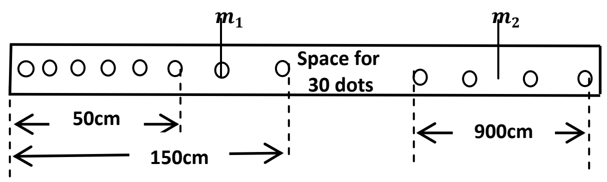



**Solution**

| | |
|---|---|
| Frequency,<br>f = 20Hz<br>Period time, $T=\frac{1}{f}$<br>$T=\frac{1}{20}$<br>T= 0.05 s<br><br>When, $d_1$= 1m<br>Time taken, $t_1=n_1T$<br>Time taken, $t_1$=2(0.05)<br>Time taken, $t_1$=0.1s<br><br>Initial speed, $u=\frac{d_1}{t_1}=\frac{1}{0.05}$<br>$=10ms^{-1}$<br>When, $d_2$=300cm,<br>$=\frac{300}{100}=3m$<br>Time taken, $t_2=n_2T$<br>Time taken, $t_2$=4(0.05)<br>Time taken, $t_2$=0.2s | Final speed, $v=\frac{d_2}{t_2}=\frac{3}{0.2}$<br>$=15ms^{-1}$<br><br>Time taken, $t_3=n_3T$<br>Time taken, $t_3$=7.5(0.05)<br>Time taken, $t_3$=0.375s<br><br>**Acceleration;**<br>Acceleration calculated<br>Applying: v = u + at<br>$\mathbf{a = \frac{change\ in\ velocity}{Time\ for\ the\ change}}$<br>Acceleration, $a = \frac{v-u}{t_3}$<br>Acceleration, $a = \frac{15-10}{0.375}$<br>Acceleration, $a = 13.33ms^{-2}$ |

**Example III**

The timer is vibrating at 20Hz. Calculate the acceleration




**Solution**

| | |
|---|---|
| Frequency, f = 20Hz<br>Period time, $T=\frac{1}{f}$<br>$T=\frac{1}{20}$<br>T=0.05s<br><br>$d_1 = 150cm - 50cm$<br>$d_1 = 100cm$<br>$d_1 = \frac{100}{100} = 1m$<br><br>$t_1 = n_1T$<br>$t_1 = 2(0.05)$<br>$t_1 = 0.1s$ | $t_2 = n_2T$<br>$t_2 = 3(0.05)$<br>$t_2 = 0.15s$<br><br>$v = \frac{d_2}{t_2}$<br>$v = \frac{9}{0.15}$<br>$v = 60ms^{-1}$<br><br>Time taken, $t_3=n_3$T<br>Time taken, $t_3$=31.5(0.05)<br>Time taken, $t_3$=1.575s<br><br>**Acceleration;**<br>Acceleration calculated applying v = u + at |

| | |
|---|---|
| $u = \frac{d_1}{t_1}$<br>$u = \frac{1}{0.1}$<br>$u = 10ms^{-1}$<br><br>$d_2 = 900cm$<br>$d_2 = \frac{900}{100}$<br>$d_2 = 9m$ | $\mathbf{Acceleration,\ a = \frac{change\ in\ velocity}{Time\ for\ the\ chang}}$<br>Acceleration, $a = \frac{v-u}{t_3}$<br>Acceleration, $a = \frac{60-10}{1.575}$<br>Acceleration, $a = 31.75ms^{-2}$ |

**NOTE:**

If there are n-dots, then there are (n-1) spaces.

i.e: $\boldsymbol{n_s = (n_d - 1).}$

Where $\mathbf{n_s}$ is the number of spaces and $\mathbf{n_d}$ is the number of dots.

**Example:**

A ticker timer is vibrating at 10Hz. Calculate the time taken if the timer prints 21 dots.

**Solution**

| | |
|---|---|
| Number of dots, $n_d$=21 dots<br>Number of spaces, $\boldsymbol{n_s = (n_d - 1).}$<br>Number of spaces, $\boldsymbol{n_s = (21 - 1).}$<br>Number of spaces, $\boldsymbol{n_s = 20}$ spaces<br><br>Frequency, f = 10Hz<br><br>Period time, $T=\frac{1}{f}=\frac{1}{10}$=0.1s<br><br>$\begin{pmatrix}\text{Time}\\\text{taken}\end{pmatrix}=\begin{pmatrix}\text{Number of}\\\text{spaces}\end{pmatrix}\times\begin{pmatrix}\text{Period}\\\text{time}\end{pmatrix}$ | t = nT<br>t = 20(0.1)<br>t = 2s |

**Example:**

A ticker timer prints 11 dots at 20Hz in a space of 2m. Calculate the average speed.

**Solution**

| | |
|---|---|
| Number of dots, $n_d$=11 dots<br>Number of spaces, $\boldsymbol{n_s = (n_d - 1).}$<br>Number of spaces, $\boldsymbol{n_s = (11 - 1).}$<br>Number of spaces, $\boldsymbol{n_s = 10}$ spaces<br><br>Frequency, f = 20Hz<br><br>Period time, $T=\frac{1}{f}=\frac{1}{20}$=0.05s<br><br>$\begin{pmatrix}\text{Time}\\\text{taken}\end{pmatrix}=\begin{pmatrix}\text{Number of}\\\text{spaces}\end{pmatrix}\times\begin{pmatrix}\text{Period}\\\text{time}\end{pmatrix}$ | $t = n_sT$<br>t = 10(0.05)<br>t = 0.5s<br><br>Average speed,<br>$v = \frac{\text{Distance}}{\text{Time taken}}$<br>$v = \frac{2}{0.5}$<br>$v = 4ms^{-1}$ |

**Note:**

In experiments with ticker timer being pushed by a trolley, the first dots are ignored because they are overcrowded for accurate measurements.

**Calculating Acceleration from given number of dots.**

If the distance is measured from $\mathbf{m^{th}}$ dot to $\mathbf{n^{th}}$ dot then the number of spaces can be calculated directly by subtracting m from n.

Number of spaces, $\boldsymbol{n_s} = (\boldsymbol{n^{th}}dot - \boldsymbol{m^{th}}dot)$.
Time taken, **t**=Number of spaces, $\boldsymbol{n_s}$×Period time, **T**

Period time,**T**= $\frac{1}{\text{frequency, } \mathbf{f}}$

**Example:**
The distance between 15th dot and 18th dot is 10cm. if the ticker timer is vibrating at 20Hz. Calculate the;

i) time taken

Number of spaces, $n_s = (n^{th}dot - m^{th}dot)$
Number of spaces, $n_s = (18 - 15)$
Number of spaces, $n_s = 3$ spaces

Frequency, f = 20Hz

Period time,T= $\frac{1}{f}$
T= $\frac{1}{20}$
T=0.05s

Time taken, **t**=Number of spaces, $\boldsymbol{n_s}$×Period time, **T**
Time taken, **t**=3(0.05)
**t**=0.15s

ii) average speed

Distance covered = 10cm = 0.1m

$$\text{Average speed, v} = \frac{\text{Distance}}{\text{Time taken}}$$

$$v = \frac{0.1}{0.15}$$

$$v = 0.67ms^{-1}$$

**Example:**
A trolley is pulled from rest with a constant force down an inclined plane. The trolley pulls a tape through a ticker timer vibrating at 50Hz. The following measurements were made from the tap.
Distance between 16th dot and 20th dot= $\mathbf{d_1}$=20cm
Distance between 20th dot and 30th dot = 34cm
Distance Q between 30th dots and 40th dot = 48cm
Distance between 40th dot and 50th dot = $\mathbf{d_2}$=62cm

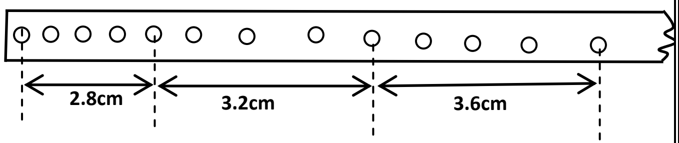
Calculate the acceleration of the trolley.

**Solution**

| | | | |
|---|---|---|---|
| $\begin{pmatrix}\text{Number of}\\ \text{spaces}, n_s\end{pmatrix}$ $= (n^{th}dot - m^{th}dot)$ $= (20 - 16)$ $= 4$ spaces<br>Number of spaces, $n = 4$ spaces<br><br>Frequency, f = 50Hz<br><br>Period time,T= $\frac{1}{f}$<br>T= $\frac{1}{50}$<br>T=0.02s<br><br>$d_1 = 20cm$<br>$d_1 = \frac{20}{100}$<br>$d_1 = 0.2m$<br><br>$t_1 = n_1T$<br>$t_1 = 4(0.02)$<br>$t_1 = 0.08s$<br><br>$u = \frac{d_1}{t_1}$<br>$u = \frac{0.2}{0.08}$<br>$u = 2.5ms^{-1}$<br><br>$d_2 = 62cm$<br>$d_2 = \frac{62}{100}$<br>$d_2 = 0.62m$ | $t_2 = n_2T$<br>$t_2 = 10(0.02)$<br>$t_2 = 0.2s$<br><br>$v = \frac{d_2}{t_2}$<br>$v = \frac{0.62}{0.2}$<br>$v = 3.1ms^{-1}$<br><br>For $\mathbf{d_1}$ last dot is 20th<br>For $\mathbf{d_2}$ last dot is 50th | $\begin{pmatrix}\text{Time taken}\\ \text{for change; } t_3\end{pmatrix} = (50^{th} - 20^{th}) \times 0.02$<br>$\begin{pmatrix}\text{Time taken}\\ \text{for change; } t_3\end{pmatrix}$ = 30×0.02<br>$t_3$= 0.6s<br><br>**Acceleration;**<br>Acceleration calculated applying v = u + at<br><br>**Acceleration, a** = $\frac{\textbf{change in velocit}}{\textbf{Time for the chan}}$<br><br>Acceleration, a = $\frac{\boldsymbol{v-u}}{\boldsymbol{t_3}}$<br><br>Acceleration, a = $\frac{\mathbf{3.1 - 2.5}}{\mathbf{0.6}}$<br><br>Acceleration, a =$1.0ms^{-2}$ | |

**Exercise**
1. A paper tape dragged through a ticker timer by a trolley has the first ten dots covering a distance of 4cm and the next ten dots covering a distance of 7cm. If the frequency of the ticker timer is 50Hz, calculate the acceleration of the trolley.(Ans:=**75cms**$^{-2}$or **0.75ms**$^{-2}$)

2. The ticker timer below was pulled by a decelerating trolley. The tape consists of 3 five dot spaces and the frequency of the timer is 50Hz.




Exercise: See UNEB

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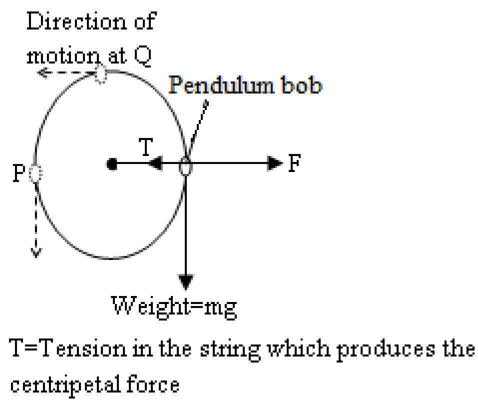
|---|---|
| 2003.Qn.26<br>1998.Qn.1(b) | 2001.Qn.25<br>2006.Qn.9 |

## 1: 11. CIRCULAR MOTION

Circular motion is motion in which a body moves in a circle about a fixed point.

For a body moving in a circle;
- ✓ Its direction and velocity are constantly changing.
- ✓ It has an acceleration called centripetal acceleration.
- ✓ It has a force called Centripetal force acting towards the centre of the circular path.




**Note:** When the object is released, it moves such that the direction of motion at any point is along a tangent to the circular path.

Forces acting on the body describing circular motion.

(i) **Tension:** Force acting towards the centre of the circular path. It provides the centripetal force.

(ii) **Centripetal force**: Force acting towards the centre of the circular path.

(iii) **Centrifugal force**: Force acting away from the centre of the circular path.

(iv) **Weight:** Force acting vertically down wards towards the centre of the earth.

Examples of circular motion

-Pendulum bob tied to a string whirled in a vertical or horizontal plane

-Planetary motion etc

Exercise: See UNEB

| | |
|---|---|
| 1999 Paper II Qn.1 | |

## 1: 12. NEWTON'S LAWS OF MOTION

These are three laws that summarize the behavior of particles in motion.

**1:12:1. Newton's First Law of motion**

Newton's first law of motion states that a body continues in its state of rest or uniform motion in a straight line unless acted upon by an external force.

**Inertia**

Inertia is the reluctance of a body to move, when at rest or to stop when moving.

Thus, when a force acts on a body, the body;
- ✓ Starts or stops moving.
- ✓ Increases or reduces speed depending on the direction of the force.
- ✓ Changes direction of motion.

**1:12:2. Newton's second law of motion**

Newton's second law states that the rate of change in momentum is directly proportional to the force acting on the body and takes place in the direction of the force.

$$F \,\alpha \frac{mv - mu}{t} \Leftrightarrow F \,\alpha\, m\left(\frac{v-u}{t}\right) \Leftrightarrow F \,\alpha\, ma \Leftrightarrow F = k\,ma$$

When we consider a force of 1N, mass of 1kg and acceleration of $1ms^{-2}$, then, **k=1**.Therefore;

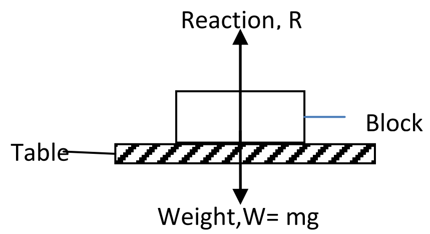
$$\mathbf{F = \ ma}$$

**A Newton;** Is the force which acts on a mass of 1kg to produce an acceleration of $1ms^{-2}$.

**1:12:3. Newton's third law of motion**

It states that action and reaction are equal but opposite.

When a body, A exerts a force on body B, body B also exerts an equal force in the opposite direction.




The block exerts a weight, W= mg on the table and the table also exerts an equal reaction R on the block. R= mg, so that the net force on the block is zero and therefore there is no vertical motion.

Applications of Newton's third law of motion

**(a) Rockets and jets**

Rockets and jet engines are designed to burn fuel in oxygen to produce large amounts of exhaust gases.

These gases are passed backwards through the exhaust pipes at high velocity (large momentum).

This in turn gives the Rocket or jet a high forward momentum which is equal but opposite to that of the exhaust gases.

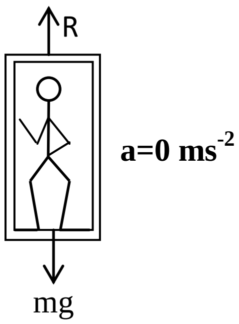
$$m_g v_g = -m_R v_R$$

Where $m_g v_g$ is the momentum of the exhaust gases, and $m_R v_R$ is momentum of the Rocket.

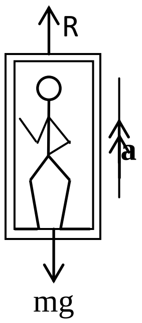
**(b) Motion in the lift**

Consider a person of mass m standing in a lift, when the;

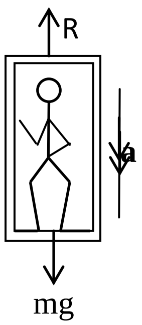
i) Lift is stationary or moving with uniform velocity

| | |
|---|---|
|   | The person exerts a weight, mg on the lift and at the same time, the lift exerts a reaction, R, on the person. **R= mg**. |

ii) Lift is moving upwards with acceleration, a.

| | |
|---|---|
|   | In this case, three forces act on the lift. i.e, the resultant accelerating force (**ma**), the weight, **(mg)** and the normal reaction or Apparent weight (**R**).<br><br>**Accelerating force = Net force**<br>ma = R – mg<br>R = mg + ma<br>**R = m (g+a)** |
| Thus, the reaction on the person (apparent weight, R) is greater than the actual weight of the person, mg.<br>This is why one feels **heavier** when the lift is just beginning its upward journey. | |

iii) Lift is moving down wards with acceleration, a.

| | |
|---|---|
|   | In this case, the resultant accelerating force (**ma**), and the weight, **(mg)** act down wards. The normal reaction or Apparent weight (**R**) act upwards.<br><br>**Accelerating force = Net force**<br>ma = mg - R<br>R = mg - ma<br>**R = m (g - a)** |
| Thus, the reaction on the person (apparent weight, R) is less than the actual weight of the person, mg.<br>This is why one feels **lighter** when the lift is just beginning its downward journey. | |

**Example:1**

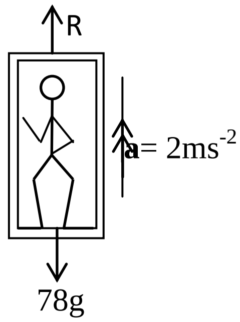
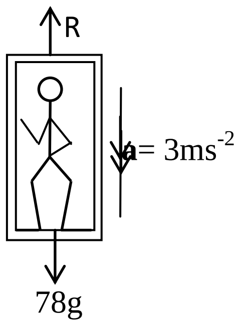
A person of mass 78kg is standing inside an electric lift. What is the apparent weight of the person if the;

a) Lift is moving upwards with an acceleration of $2ms^{-2}$?

b) Lift is descending with an acceleration of $2ms^{-2}$?

Solution

(a)

| | |
|---|---|
|   | m = 78kg<br>a = $2ms^{-2}$<br>R=?<br>R = mg + ma<br>**R = m (g+a)**<br>R = 78 (10+2)<br>R = 936N |
| (b)<br>  | m = 78kg<br>a = $3ms^{-2}$<br>R=?<br>R = mg - ma<br>R = 78(10 - 3)<br>R = 546N |

## 1: 12: 4. COLLISIONS AND MOMENTUM

**Linear Momentum:**

Momentum is the product of mass and its velocity.

$$\begin{pmatrix}\textbf{Linear Momentum}\\ \textbf{of a body}\end{pmatrix} = \begin{pmatrix}\textbf{Mass of}\\ \textbf{the body}\end{pmatrix} \times \textbf{Velocity}$$

**Impulse:**

Impulse is the change in the momentum of a body.

$$\textbf{Impulse} = \textbf{mv} - \textbf{mu}$$

Impulse can also be defined as the product of force and time of impact.

From Newton's second law of motion,

$$F = \frac{mv - mu}{t} \Leftrightarrow Ft = mv - mu$$

$$\textbf{Impulse} = \textbf{Ft} = \textbf{mv} - \textbf{mu}$$

The S.I unit of momentum and impulse is **$Kgms^{-1}$**

**Note**: Momentum and impulse are vector quantities.

**Principle of conservation of momentum**

It states that when two or more bodies collide, the total momentum remains constant provided no external force is acting.

It states that when two or more bodies collide, the total momentum before collision is equal to the total momentum after collision.

Suppose a body of mass $m_1$ moving with velocity $u_1$ collides with another body of mass $m_2$ moving with velocity $u_2$. After collision, the bodies move with velocities $v_1$ and $v_2$ respectively, then;

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

**Types of collisions**

✓ <u>Elastic collision</u>

Elastic collision is the type of collision whereby the colliding bodies separate immediately after the impact with each other and move with different velocities.

In short, for elastic collision,

$$\begin{pmatrix}\textbf{Total momentum}\\ \textbf{before collision}\end{pmatrix} = \begin{pmatrix}\textbf{Total momentum}\\ \textbf{after collision}\end{pmatrix}$$

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

✓ <u>Inelastic collision</u>

Inelastic collision is when the colliding bodies stay together and move with the same velocity after collision.

In short, for inelastic collision,

$$\begin{pmatrix}\textbf{Total momentum}\\ \textbf{before collision}\end{pmatrix} = \begin{pmatrix}\textbf{Total momentum}\\ \textbf{after collision}\end{pmatrix}$$

$$\mathbf{m_1u_1 + m_2u_2 = (m_1 + m_2)V}$$

<u>Comparisons between Elastic collision and Inelastic collision</u>

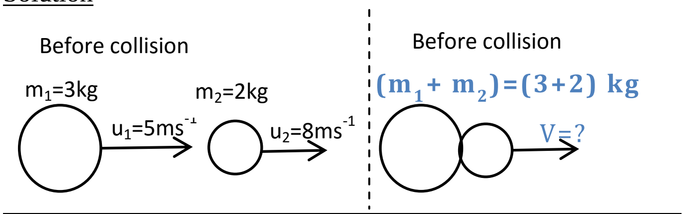
| **Elastic collision** | **Inelastic collision** |
|---|---|
| (i) Bodies separate after collision | Bodies stick together after collision. |
| (ii) Bodies move with different velocities after collision | Bodies move with same velocity after collision |
| (iii) Kinetic energy of the bodies is conserved | Kinetic energy of the bodies is not conserved |
| Momentum is conserved<br>**Total momentum before collision= Total momentum after collision**<br>$\mathbf{m_1u_1+m_2u_2=m_1v_1+m_2v_2}$ | Momentum is conserved<br>**Total momentum before collision= Total momentum after collision**<br>$\mathbf{m_1u_1+m_2u_2=(m_1+m_2)}V$ |

**NOTE**; For any stationary body or body at rest, the initial velocity is zero so the initial momentum of such a body before collision is zero.

**Example:1**

A body of mass 3kg traveling at $5ms^{-1}$ collides with a 2kg body moving at $8ms^{-1}$ in the same direction. If after collision the two bodies moved together, Calculate the velocity with which the two bodies move after collision.

<u>Solution</u>

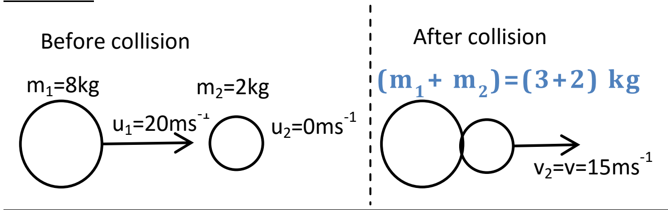



| | |
|---|---|
| $m_1$= 3kg, $m_2$= 2kg<br>$u_1=5ms^{-1}$, $u_2=8ms^{-1}$<br>$v_1$=V=?, $v_2$=V=?<br>(Total momentum before collision) = (Total momentum before collision) | $m_1u_1+m_2u_2= (m_1+m_2)V$<br>3(5)+2(8)= (3+2)V<br>15+16= 5V<br>31= 5V<br>$\frac{31}{5} = \frac{5V}{5}$<br>6.2 = V<br><u>V= 6.2 $ms^{-1}$</u> |

**Example: 2**

A body of mass 8kg traveling at 20 $ms^{-1}$ collides with a stationary body and they both move with velocity of $15ms^{-1}$. Calculate the mass of the stationary body.

<u>Solution</u>

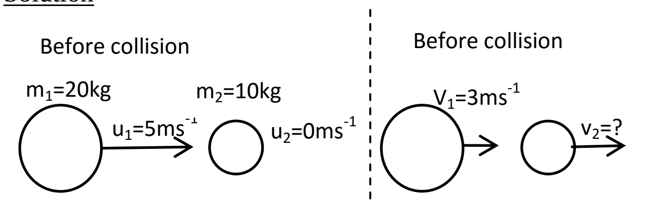



| | |
|---|---|
| $m_1$= 8kg, $m_2$= 2kg<br>$u_1=20ms^{-1}$, $u_2=8ms^{-1}$<br>$v_1$=V=1 $ms^{-1}$, $v_2$=V=15 $ms^{-1}$<br>(Total momentum before collision) = (Total momentum before collision) | $m_1u_1+m_2u_2= (m_1+m_2)V$<br>$8(20)+m_2(0)= (8+m_2)(15)$<br>$160+0= 8(15) + 15m_2$<br>$40= 15m_2$<br>$2.67 = m_2$<br>$m_2$ = 2.67kg |

**Example: 3**

A body of mass 20kg traveling at $5ms^{-1}$ collides with another stationary body of mass 10kg and they move separately in the same direction. If the velocity of the 20kg mass after collision was $3ms^{-1}$. Calculate the velocity with which the 10kg mass moves.

<u>Solution</u>




| | |
|---|---|
| $m_1$= 20kg, $m_2$= 10kg<br>$u_1=5ms^{-1}$, $u_2=0ms^{-1}$<br>$v_1=3ms^{-1}v_2$=?<br>(Total momentum before collision) = (Total momentum before collision) | $m_1u_1+m_2u_2= m_1v_1+m_2v_2$<br>$20(5)+10(0)= 20(3)+10(v_2$<br>$100+0= 60 + 10v_2$<br>$100 - 60 = 10v_2$<br>$\frac{40}{10} = \frac{10v_2}{10}$<br>$4 = v_2$<br>$\mathbf{v_2= 4ms^{-1}}$ |

**Exercise:**

**1.** A particle of mass 200g moving at $30ms^{-1}$ hits a stationary particle of mass100g so that they stick and move together after impact. Calculate the velocity with which they move after collision.**(Ans: V= 20ms$^{-1}$)**

2. A military tanker of mass 4tonnes moving at $12ms^{-1}$ collides head on with another of mass 3tonnes moving at $2oms^{-1}$. After collision, they stick together and move as one body. Ignoring the effect of friction, find their common velocity.

**(Ans: V=$1.7ms^{-1}$**in the direction of the 2nd tank**)**

3. A body of mass 10kg moving at $20ms^{-1}$ hits another body of mass 5kg moving in the same direction at $10ms^{-1}$. After collision, the second body moves separately forward with a velocity of $30ms^{-1}$. Calculate the velocity of the first body after collision. **.(Ans:$v_1$=$10ms^{-1}$)**

4. A car X of mass 1000kg travelling at a speed of 20 ms-I in the direction due east collides head-on with another car Y of mass1500kg, travelling at 15ms-1 in the direction due west. If the two cars stick together, find their common velocity after collision.

## EXPLOSIONS

Momentum is conserved in explosions such as when a riffle is fired. During the firing, the bullet receives an equal but opposite amount of momentum to that of the rifle.

Total momentum before collision = Total momentum after collision

$$m_g u_g + m_b u_b = m_g v_g + m_b v_b$$
$$m_g(0) + m_b(0) = m_g v_g + m_b v_b$$
$$0 = m_g v_g + m_b v_b$$
$$\mathbf{m_g v_g = -m_b v_b}$$

Where; $m_g$ is mass of the rifle (or gun), $V_g$ is velocity of the rifle which is also called recoil velocity. $m_{ob}$ is mass of the bullet, $V_b$ is velocity of the bullet.

For any explosion of bodies, the amount of momentum for one body is equal but opposite to that of another body.

The negative sign indicates that the momentum are in opposite directions.

**Example:1**

A bullet of mass 8g is fired from a gun of mass 500g. If the missile velocity of the bullet is $500ms^{-1}$. Calculate the recoil velocity of the gun.

Solution

$m_b$= 8g = $\frac{8}{1000} = 0.008kg$, $m_g$= 500g = $\frac{500}{1000} = 0.5$kg

$v_b$= $500ms^{-1}$, $v_g$ = ?

From, $m_g v_g = -m_b v_b$

$0.5V_g = -0.008(500)$

$0.5V_g = -4$

$$\frac{0.5V_g}{0.5} = \frac{-4}{0.5}$$

$$\mathbf{V_g = -8ms^{-1}}$$

The negative sign indicates that the recoil velocity, $V_g$ is in opposite direction to that of the bullet.

**Example:2**

A bullet of mass 200g is fired from a gun of mass 4kg. If the muzzle velocity of the bullet is $400ms^{-1}$, calculate the recoil velocity.

**Solution**

$m_b$= 200g = $\frac{200}{1000} = 0.2$kg, $m_g$= 4kg

$v_b$= $400ms^{-1}$, $v_g$ = ?

From, $m_g v_g = -m_b v_b$

$4V_g = -0.2(400)$

$4V_g = -80$

$$\frac{4V_g}{4} = \frac{-80}{4}$$

$$\mathbf{V_g = -20ms^{-1}}$$

**Example:3**

A bullet of mass 12.0g travelling at $150ms^{-1}$penetrates deeply into a fixed soft wood and is brought to rest in 0.015s. Calculate

(i) How deep the bullet penetrates the wood [**1.125m]**

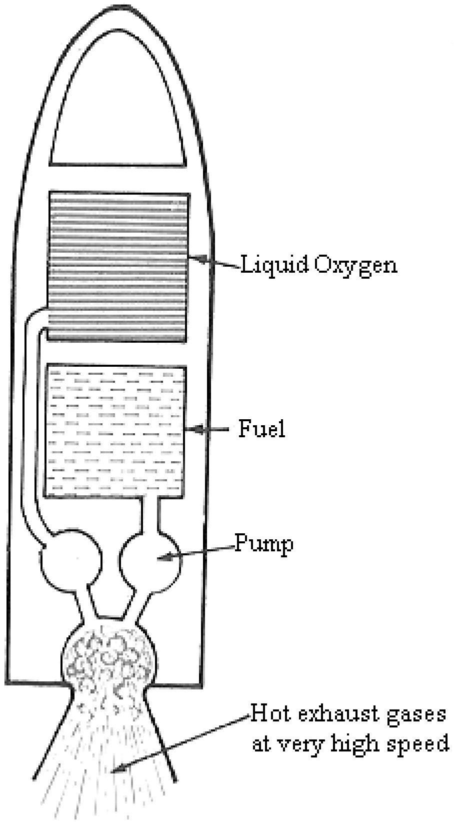
(ii) the average retarding force exerted by the wood on the bullet. [**120N**]

## ROCKET AND JET ENGINES

These work on the principle that in any explosion one body moves with a momentum which is equal and opposite to that of another body in the explosion. For the rocket and the jet engine, the high velocity hot gas is produced by the burning of fuel in the engine.

**Note: Rockets use liquid oxygen while jets use oxygen from air.**

**How a rocket engine work:**




Principle: the jet and rocket engines work on the principle that momentum is conserved in explosion.

High velocity: the high velocity of the hot gas results in the burning of the fuel in the engine.

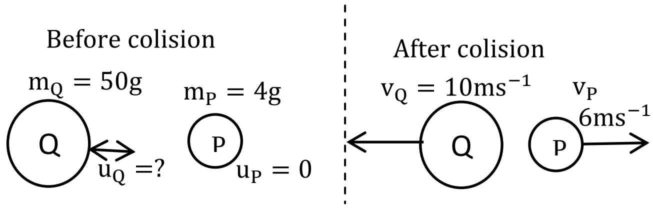
Large momentum: the large velocity of the hot gas results in the gas to leave the exhaust pipe with a large momentum.

Engine: the engine itself acquires an equal but opposite momentum to that of the hot gas.

**Note**: when the two bodies collide and they move separately after collision but in opposite directions then.
$m_1u_1+m_2u_2= m_1v_1+m_2(-v_2)$
$m_1u_1+m_2u_2= m_1v_1- m_2v_2$

**Example:**
A body Q of mass 50g collides with a stationary body "P" of mass 4g. If a body "Q" moves backward with a velocity of $10ms^{-1}$ and a body "P", moves forward with a velocity of $6ms^{-1}$. Calculate the initial velocity of a body Q.

Solution
$m_Q= 50g,=\frac{50}{1000} = 0.05kg$ $m_P= 4g =\frac{4}{1000} = 0.004kg$
$u_Q= ?$, $u_P= 0ms^{-1}$
$v_Q= 10\ ms^{-1}$, ← $v_P= 6ms^{-1}$ →



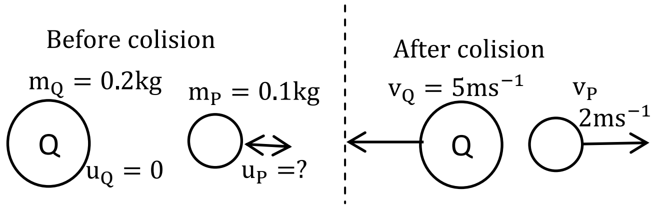

Total momentum before collision = Total momentum after collision

$$m_Qu_Q+m_Pu_P=m_Qv_Q+m_pv_P$$

$0.05u_Q+ 0.004(0)= 0.05(-10)+0.004(6)$
$0.05u_Q= -0.5+0.024$
$0.05u_Q= -0.476$
$\frac{0.05u_Q}{0.05} = \frac{-0.476}{0.05}$
$\mathbf{u_Q= -9.52ms^{-1}}$
Thus, the initial velocity of Q is $9.52ms^{-1}$ to the left

**Example:2**
A moving ball "P" of mass 100g collides with a stationary ball Q of mass 200g. After collision, P moves backward with a velocity of $2ms^{-1}$ while Q moves forward with a velocity of $5ms^{-1}$. Calculate the initial velocity of P.

Solution
$m_Q= 200g,=\frac{200}{1000} = 0.2kg$ $m_P= 100g =\frac{100}{1000} = 0.1kg$
$u_Q= 0$, $u_P= ?$
$v_Q= 5\ ms^{-1}$, ← $v_P= 2ms^{-1}$ →




Total momentum before collision = Total momentum after collision

$$m_Qu_Q+m_Pu_P= m_Qv_Q+m_pv_P$$

$0.2(0)+ 0.1u_P= 0.2(5)+0.1(-2)$
$0.1u_P= 1+ -0.2$
$0.1u_Q= -0.8$
$\frac{0.1u_Q}{0.1} = \frac{-0.8}{0.1}$
$\boldsymbol{u_P= -8ms^{-1}}$
Thus, the initial velocity of Q is $8ms^{-1}$ towards Q.

**Example: 3**.
A body of mass 10kg moves with a velocity of $20ms^{-1}$.Calculate its momentum.

**Solution**
$m=10kg$; $v=20ms^{-1}$
Linear Momentum = Mass × Velocity
= 10 × 20
= $200kgm^{-1}$

Initial Momentum = Mass × Initial Velocity= mu

Final Momentum = Mass × Final Velocity= mv

**Example:2**

A 20kg mass traveling at 5mls is accelerated to 8mls. Calculate the change in momentum of the body.

Solution

| | |
|---|---|
| m=10kg<br>u=5ms$^{-1}$<br>v=8ms$^{-1}$ | |
| Initial Momentum = mu<br>= 20×5<br>= $100kgms^{-1}$ | Final Momentum<br>= mv<br>= 20×8<br>= $160kgms^{-1}$ |
| Change in Momentum = mv - mu<br>= 160 - 100<br>$= 60kgms^{-1}$ | |

***Note: The change in momentum is called Impulse.***

**Example:3**

A one tonne car traveling at 20ms$^{-1}$ is accelerated at 2ms$^{-2}$ for five second. Calculate the;

(i) change in momentum
(ii) rate of change in momentum
(iii) Accelerating force acting on the body.

Solution

| | |
|---|---|
| m=1tonne =1000kg<br>u=20ms$^{-1}$<br>v= ?<br>a=2ms$^{-2}$<br>t= 5s<br>(i) change in momentum<br>Change in Momentum<br>= mv - mu<br>$= m(v-u)$<br>But ; $v = u + at$<br>$v = 20 + 2(5)$<br>$v = 30ms^{-1}$<br>Change in Momentum<br>= mv - mu<br>$= m(v-u)$<br>$= 1000(30-20)$<br>$= 1000(10)$<br>$= 10{,}000kgms^{-1}$ | (ii) Rate of change in momentum<br>Rate of change in momentum<br>$= \frac{\text{Change in mometum}}{\text{Time taken}}$<br>$= \frac{m(v-u)}{t}$<br>$= \frac{1000(30-20)}{5}$<br>$= \frac{10000}{5}$<br>=2000 N |

NOTE: The S.I unit for the rate of change in momentum is a newton.

(iv) Accelerating force acting on the body.

Accelerating force, F = Rate of change in momentum

$$= \frac{m(v-u)}{t}$$

$$= \frac{1000(30-20)}{5}$$

F =2000N

From above, the force applied is equal to the rate of change in momentum. This leads to Newton's second law of motion.

**Exercise**

1. A body of mass 600g traveling at 10mls is accelerated uniformly at 2 ms$^{-2}$ for four seconds. Calculate the;
   (i) change in momentum
   (ii) force acing on a body

**Solution**

| | |
|---|---|
| (i)<br>mass, $m = 600g$<br>$= \frac{600}{1000}$<br>$= 0.6kg$<br>u=10ms$^{-1}$<br>v=?<br>a=2 ms$^{-2}$<br>t = 4s<br>From;<br>$v = u + at$<br>$v = 10 + 2(4)$<br>$v = 18ms^{-1}$<br>Change in Momentum<br>= mv - mu<br>$= m(v-u)$<br>$= 0.6(18-10)$<br>$= 0.6(8)$<br>$= 4.8kgms^{-1}$ | (ii)<br>Rate of change in momentum<br>$= \frac{m(v-u)}{t}$<br>$= \frac{0.6(18-10)}{4}$<br>$= \frac{4.8}{4}$<br>=1.2 N<br>(iii) Force acting on the body<br>But ; $v = u + at$<br>But ; $18 = 10 + 4t$<br>But ; $a = 2ms^{-1}$<br>**F = ma**<br>=0.6(2)<br>F =1.2N<br>Thus, Force acting on the body is equal to the rate of change in momentum. |

**Example:4**

A van of mass 1.5 tonnes travelling at 20ms$^{-1}$, hits a wall and is brought to rest as a result in 0.5seconds. Calculate the;

(i) Impulse
(ii) Average force exerted on the wall.

Solution

| | |
|---|---|
| m=1.5tonnes<br>=1.5x1000<br>= 1500<br>u=20ms$^{-1}$<br>v=0ms$^{-1}$<br>t=0.5s | (i)Impulse:<br>Impulse=Change in Momentum<br>= mv - mu<br>$= m(v-u)$<br>$= 1500(0-20)$<br>$= 1500(-20)$<br>$= -30{,}000kgms^{-1}$ |
| ***The Negative sign means that the direction of the impulse is opposite to that in which the van was moving.*** | |
| .<br>(ii)Average force exerted on the wall:<br>From;Impuse = Force × Time = Ft<br>$-30000 = F \times 0.5$<br>$F = -60{,}000N$ | |

**Example:5**

A man of mass 60kg jumps from a high wall and lands on a hard floor at avelocity of 6ms.Calculate the force exerted on the man's legs if;

(i) He bends his knees on landing so that it takes 1.2s for his motion to be stopped.
(ii) He does not bend his knees and it takes 0.06s to stop his motion.

**Solution**

| | |
|---|---|
| (i)<br>m=60kg<br>u=6ms$^{-1}$<br>v=0ms$^{-1}$<br>t=1.2s<br><br>Force acting on the body<br>But ; $v = u + at$<br>But ; $0 = 6 + 1.2a$<br>But ; $a = -5\text{ms}^{-1}$<br><br>**F = ma**<br>=60(-5)<br>F =-300N | (ii)<br>m= 60kg<br>u=6ms$^{-1}$<br>v=0ms$^{-1}$<br>t=0.06s<br><br>Force acting on the body<br>But ; $v = u + at$<br>But ; $0 = 6 + 0.06a$<br>But ; $a = -100\text{ms}^{-1}$<br><br>**F = ma**<br>=60(-100)<br>F =-6000N |
| **Note:**<br>❖ The negative signs means the force acts to oppose that exerted by the man.<br>❖ Landing in (ii) exerts a larger force on the knees, which can cause injury compared to that in (ii). | |

**Exercise :**

1. An athlete of 80 kg moving at 5ms$^{-1}$, slides trough a distance of 10m before stopping in 4 seconds. Find the work done by friction on the athlete.

2. A car of mass 1500kg starts from rest and attains a velocity of 100ms$^{-1}$in 20 seconds. Find the power developed by the engine.
A. 750kW B. 3,000kW
C. 30, 000kW D. 750, 000kW

3. A ball of 3kg moves at 10ms$^{-1}$ towards a volley ball player. If the player hits the ball and the ball moves back with a velocity of 5ms$^{-1}$. Find the change in momentum.
A. $\frac{5\times3}{10}$ B. $\frac{10\times3}{5}$
C. $3(10-5)$ D. $3(10+5)$

1. A rubber bullet of mass 100g is fired from a gun of mass 5 kg at a speed of 200 ms$^{-1}$. Find the recoil velocity of the rifle.
A. $\frac{5\times200}{100\times1000}$ B. $\frac{5\times1000}{100\times200}$
C. $\frac{100\times200}{5\times1000}$ D. $\frac{200\times1000}{5\times100}$

4. **See UNEB**

| | |
|---|---|
| 2001. Qn.1 | 2006. Qn.32 |
| 1988. Qn.9 and Qn.20 | 2007. Qn.24 |
| 1994. Qn.5 and Qn. 3 | 1992. Qn.2 |
| 1995. Qn.8 | 2003. Qn.2 |